

PSCI 589 ADVANCED FORMAL METHODS IN POLITICAL ECONOMY

Spring, 2023
MW 10am–12pm
Harkness 112

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Hours: by appointment

The course takes as its starting point two canonical optimization problems: the problem of a proposer who offers a policy recommendation, subject to approval by one or more voters (this is analogous to a bid in a first-price auction), and the problem of a contestant who chooses how much effort to exert to win a prize (analogous to a bid in an all-pay auction). Many models in political economy are based directly on—or endogenize parameters in, or otherwise evolve from—these problems.

The focus of the course is on the mathematical tools needed to analyze these canonical problems and the models that grow from them. It is organized around different sets of tools, beginning with maximality for binary relations, then progressing to more advanced mathematical/theoretical topics. Along the way, the tools will be applied to the proposer and contest problems, and we delve into foundational theories of bargaining and elections. The course will consist of a mix of lectures, discussion, student presentations, and a final exam.

Topics covered will be selected from the list below. The list includes the main mathematical tools needed to understand and contribute to research in theoretical political economy at the highest level. Allocation of time to the topics will be determined by instructor and student interest.

- 1) Existence of maximal elements, acyclicity, and social choice
- 2) Median voter theorem and representative agents
- 3) Mathematical background: metric spaces, vector spaces, and normed linear spaces
- 4) Theorem of the maximum and deterministic dynamic choice
- 5) Continuous dynamic games: subgame perfect equilibria with perfect information
- 6) KKM theorem, general existence of maximal elements, and core in the spatial model
- 7) Fixed point theorems and Nash equilibrium existence

- 8) Separating hyperplane theorem and linear characterizations
- 9) Implicit function theorem and comparative statics
- 10) Lattice theory and monotone comparative statics
- 11) Transversality theorem and generic core emptiness
- 12) Mathematical background: measure spaces, transition probabilities
- 13) Transition probabilities and stochastic dynamic choice
- 14) Weak* topology and mixed-strategy equilibrium existence
- 15) Stationary equilibria in bargaining games
- 16) Bayesian games and accountability
- 17) Endogenous sharing rules and two-stage games
- 18) Continuous dynamic games: subgame perfect equilibria with almost perfect information
- 19) Measurable dynamic games: existence of subgame perfect equilibria
- 20) Stochastic Games: Stationary Markov perfect equilibria

I have created a Dropbox folder with a number of documents that we will draw on in the course, and readings will be added throughout the semester. In addition to the articles and notes available in the Dropbox folder, a number of books are noteworthy for their systematic coverage of different mathematical topics.

- Aliprantis and Border (2006) *Infinite Dimensional Analysis: A Hitchhiker's Guide*, Springer
- Austen-Smith and Banks (1999) *Positive Political Theory I*, Michigan press
- Austen-Smith and Banks (2005) *Positive Political Theory II*, Michigan press
- Border (1985) *Fixed Point Theorems*, Cambridge press
- Guillemin and Pollack (1974) *Differentiable Topology*, Prentice-Hall
- Hildenbrand (1974) *Core and Equilibria of a Large Economy*, Part 1, Princeton press,
- Mas-Colell (1985) *The Theory of General Economic Equilibrium*, Chapter 1, Econometric Society

- Ok (2007) *Real Analysis with Economic Applications*, Princeton press
- Simon and Blume (1994) *Mathematics for Economists*, Norton
- Sundaram (1996) *A First Course in Optimization Theory*, Cambridge press.