HW: Random Number Generators and Monte Carlo Analyses

Please make sure your discussion and results are clear and easy to read. Turn in your well-commented code, along with your results and discussion.

1. Consider the Weibull distribution: $F(y) = 1 - e^{-(\lambda y)^{\alpha}}$, where $y \geq 0$, $\lambda > 0$, $\alpha > 0$.

   (a) Write a Weibull random number generator in R.

   (b) Write a procedure that calculates the Weibull pdf, given any value of y, λ, and α.

   (c) For some set of (λ, α), sample $n = 10,000$ observations from your random number generator and compare the density of the sample to the true density. You may do this graphically.

   (d) Calculate and compare the population vs sample mean and variance. Use the sample generated in the prior step.

2. Consider the regression $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$, where X_i and ϵ_i have all the usual (nice) properties of the Normal classical linear regression model. The purpose of this exercise will be to conduct a monte carlo analysis of omitted variable bias.

   (a) First, assume x_1 and x_2 are correlated at .8, but that x_2 is omitted from the regression. Conduct a monte carlo experiment with $R = 2,000$ iterations. Plot the density of the monte carlo $\hat{\beta}_1$s and calculate the mean. Compare the mean to the analytically-derived value of $\hat{\beta}_1$.

   (b) Now let the correlation between x_1 and x_2 vary from $-.9$ to .9 in steps of .2. Again, assume x_2 is omitted from the regression. Conduct a monte carlo analysis to demonstrate the bias in $\hat{\beta}_1$ as a function of the correlation. Plot the relationship. On the same graph, plot the analytical relationship.