PSC 407 Mathematical Modeling

Fall, 2018
MW 10:00-11:50
Harkness 112

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Hours: by appointment

This course is the first half of a two-course sequence consisting of PSC 407 and PSC 408. The goal of the sequence is to give a rigorous introduction to the main concepts and results in positive political theory. At the same time, we will teach you the mathematical tools necessary to understand these results, to use them, and (if it suits you) to surpass them in your own research in political science. The sequence emphasizes rigorous logical and deductive reasoning — this skill will prove valuable, even to the student primarily interested in empirical analysis rather than modeling.

The sequence is designed to be both a rigorous foundation for students planning on taking further courses in the positive political theory field and serve as a self-contained overview of the field for students who do not intend to do additional coursework in the field. PSC 407 is mainly concerned with social choice applications and the mathematics behind them, drawing mainly on logic, real analysis, and calculus, whereas PSC 408 will focus on strategic interaction and game-theoretic analysis.

Students should have, at a minimum, a sound familiarity with basic algebra (solving equations, graphing functions, etc.) and a knowledge of basic calculus. Consistent with department policy, students are required (unless explicitly exempted) to attend the "math camp" offered in the weeks before the fall semester.

Homeworks, a midterm, and a final will be assigned to help develop and test your mathematical modelling skills. Students are allowed to collaborate on homework, but after discussion with others, each student is expected to write up his or her answers independently. The date and time of the final are set by the University Registrar: it will take place on **Tuesday, December 18, at 12:30,** and you will have three hours to complete it. This date is firm, so keep it in mind when making your travel plans for winter break.

The primary source for material covered in the lectures will be two manuscripts, the first coauthored with Mark Fey:

- Duggan and Fey (DF) *Analytical Methods in Political Economy: The Mathematics of Voting*
- Duggan (D) *Analytical Methods in Political Economy: The Mathematics of Spatial Modeling*

In addition, I will assume students own or have access two textbooks for the course.

- Simon and Blume (SB), *Mathematics for Economists*

- Ordeshook (O), *Game Theory and Political Theory*

Simon and Blume is a valuable compendium of the mathematics used in the social sciences, but the applications are oriented toward economics. Nevertheless, it will be a valuable complement to the main text. Ordeshook is not a math book, but it draws together many of the social-choice theoretic applications we cover in class.

The teaching assistant for the course is Jacque Gao, who will hold a weekly recitation and office hour. Keep in mind that the TA's primary responsibility during recitation is to answer your questions, so come prepared.

An outline of the topics to be covered is as follows. Next to each, I list readings from the texts.

1. Logic, Set Theory, and Functions [DF 2–6]
    - logical connectives, necessary and sufficient conditions, direct proof, proof by contradiction, quantifiers, set operations, natural numbers, real numbers, maximum and supremum, relations, transitivity, weak orders, maximal elements, monotonicity, invertibility
2. Rational Choice Model and Social Choice [DF 7–10; O 1.1–1.2, 2.1–2.2]
    - strict and weak preference, utility functions, majority preference, core, Pareto dominance, Pareto optimality, aggregation rules, voting paradoxes, May's theorem, Arrow's Theorem
3. Multidimensional Analysis [D 4–7; SB 2–3, 5, 10, 13–15, 21, A2]
    - continuous/differentiable functions, vector addition, scalar multiplication, dot products and orthogonality, lines and hyperplanes, convex sets, Euclidean norm, open and closed sets, multivariate functions and level sets, concavity and quasi-concavity, gradients, maximization
4. Spatial Model [D 8–11; O 1.3–1.4, 2.3, 4.6–4.7]
    - single-peakedness, median voter theorem, convex and continuous preferences, indifference curves, utility representations, Pareto optimality, Plott's theorem, McKelvey's theorem and the top cycle set

*All assignments and activities associated with this course must be performed in accordance with the University of Rochester's Academic Honesty Policy; visit `www.rochester.edu/college/honesty` for more information.

** The University of Rochester respects and welcomes students of all backgrounds and abilities. In the event you encounter any barrier(s) to full participation in this course due to the impact of disability, please contact the Office of Disability Resources. Visit `disability@rochester.edu` for more information.

*** This course follows the College credit hour policy for four-credit courses. This course meets twice per week for a total of four hours per week; in addition, the course includes one hour of recitation per week.