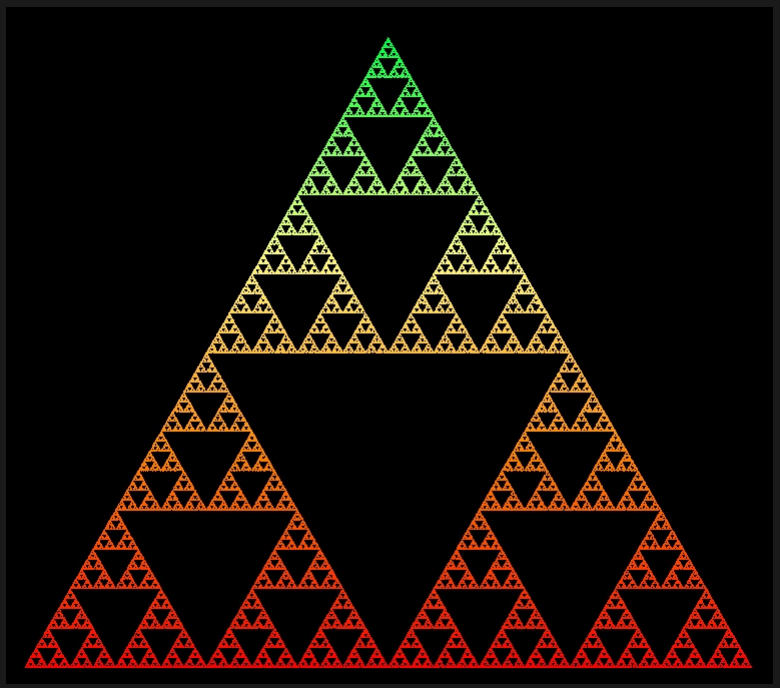
Princeton University
Algebraic Topology Seminar

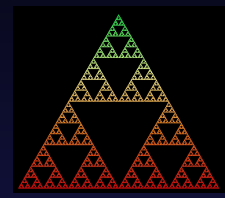


OpenAI and the 3-primary Kervaire invariant problem



Doug Ravenel
University of Rochester

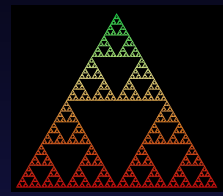
October 29, 2026



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The main point of this talk

The 3-primary Arf-Kervaire invariant problem was open until recently.



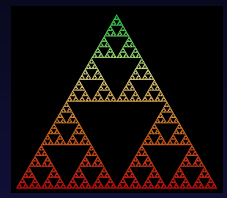
Doug Ravenel

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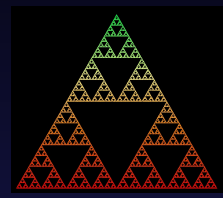
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Doug Ravenel

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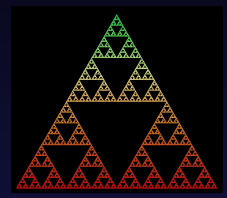
The Kervaire invariant problem at the prime three

OpenAI

September 24, 2026

Abstract

We solve the Kervaire invariant problem at the prime three for the standard Kervaire classes in the mod-three Adams spectral sequence. These classes survive precisely at indices 0, 2, and 3, and each surviving detection coset contains an element of additive order three. In particular, there is an order-three Kervaire element in stem 322.



Doug Ravenel

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The Kervaire invariant problem at the prime three

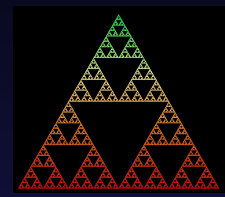
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Before getting into their paper, I will discuss the background and history of the problem.



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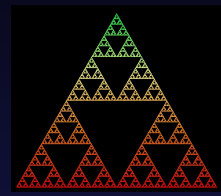
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Defining the problem

The Arf-Kervaire invariant problem for a prime p is to determine the fate of the elements

$$\theta_j = \left\{ \begin{array}{ll} h_j^2 & \text{for } p = 2 \\ b_{j-1} & \text{for } p > 2 \end{array} \right\} \in \mathrm{Ext}_{\mathcal{A}}^{2,2p^j(p-1)}(\mathbb{Z}/p, \mathbb{Z}/p)$$

for $j > 0$, where $\mathcal{A}$ denotes the mod p Steenrod algebra.



Doug Ravenel

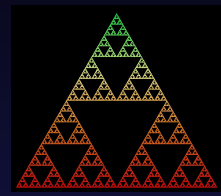
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for $j > 0$, where $\mathcal{A}$ denotes the mod p Steenrod algebra. This Ext group is the E_2-term for the classical Adams spectral sequence



Doug Ravenel

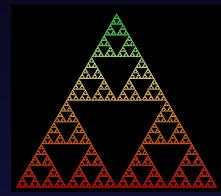
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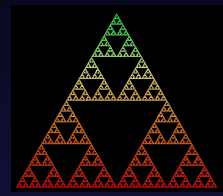
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These classes correspond to the Adem relations for $Sq^{2^j} Sq^{2^j}$ and $\mathcal{P}^{p^{j-1}} \mathcal{P}^{p^{j-1}(p-1)}$.



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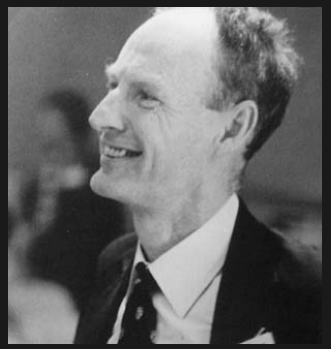
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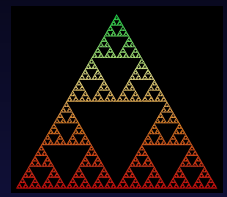
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These classes correspond to the Adem relations for $Sq^{2^j}Sq^{2^j}$ and $\mathcal{P}^{p^{j-1}}\mathcal{P}^{p^{j-1}(p-1)}$. In these bidegrees the groups are known to be isomorphic to $\mathbb{Z}/p$ in each case, generated by these elements.



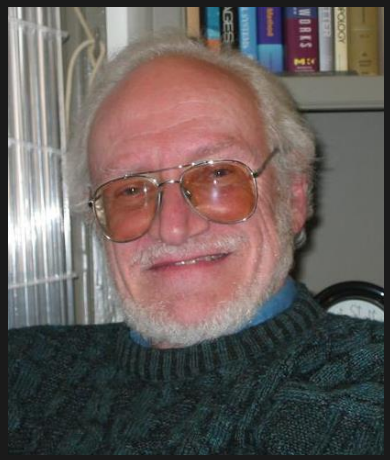
Frank Adams
1930–1989



Doug Ravenel

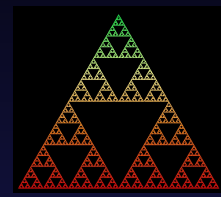
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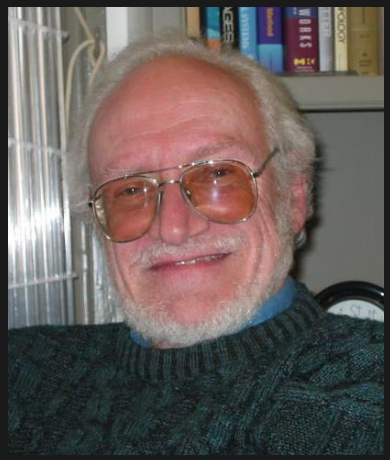


Bill Browder
1934–2025

Browder's Theorem of 1969 states that for $p = 2$, θ_j is a permanent cycle in the Adams spectral sequence if and only if there is a framed manifold with non-trivial Kervaire invariant in dimension $2^{j+1} - 2$.

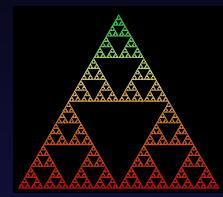


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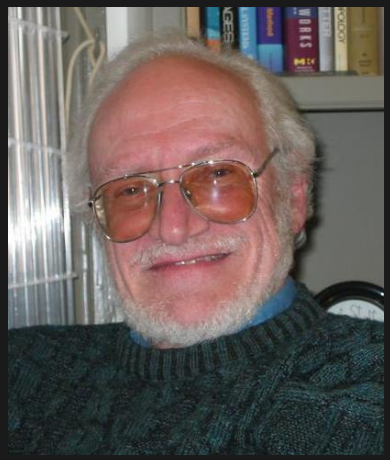


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In 2009 Mike Hill, Mike Hopkins and I showed that for θ_j does not exist for $j \geq 7$.

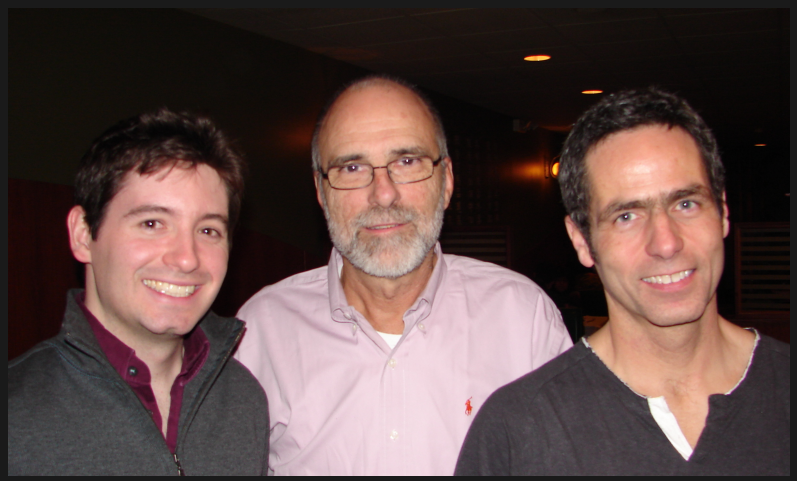
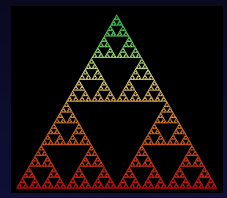


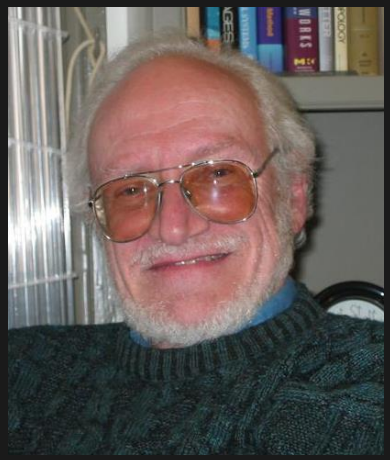
Photo by Bill Browder



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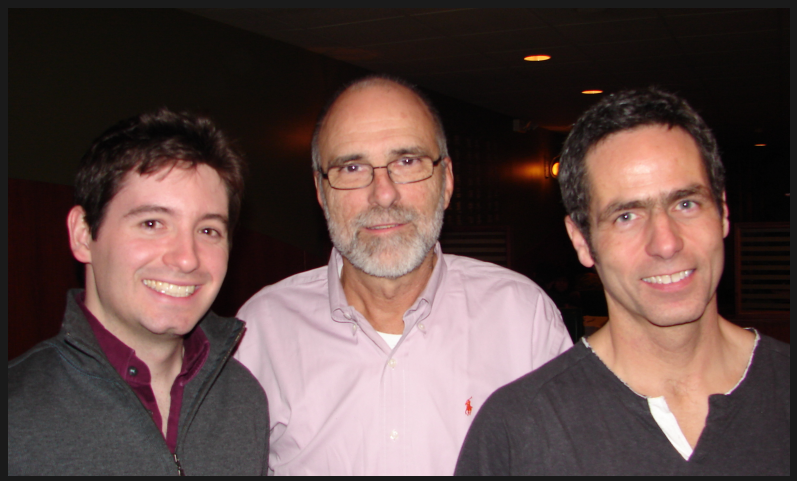
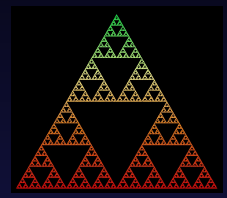


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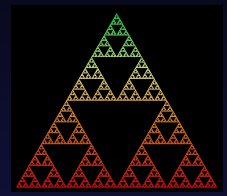


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The existence of θ_6 was shown by Weinan Lin, Guozhen Wang and Zhouli Xu in 2024,



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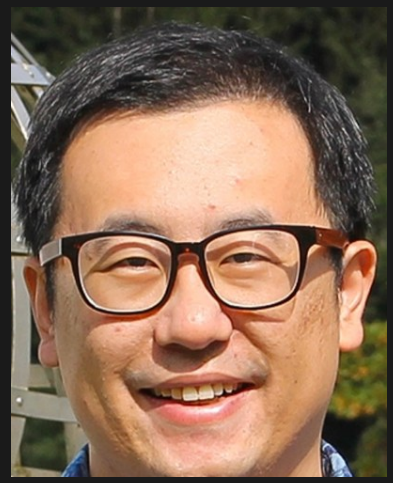
The existence of θ_6 was shown by Weinan Lin, Guozhen Wang and Zhouli Xu in 2024, thus finishing the story for $p = 2$.



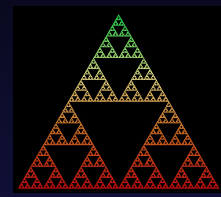
Weinan Lin



Guozhen Wang



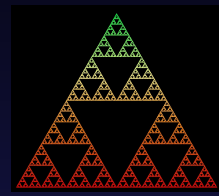
Zhouli Xu



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Again, we want to know the fate of the elements

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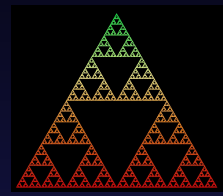
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The corresponding Adams-Novikov group, $\mathrm{Ext}_{BP_*(BP)}^{2,2p^j(p-1)}$, is more complicated.



Doug Ravenel

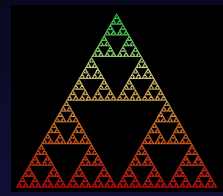
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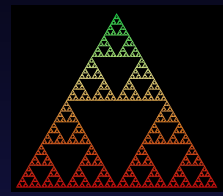
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$$\mathrm{Ext}_{BP_*(BP)}^{2,2p^j(p-1)} \xrightarrow{\Phi} \mathrm{Ext}_{\mathcal{A}}^{2,2p^j(p-1)}$$

is onto in all but one case, with



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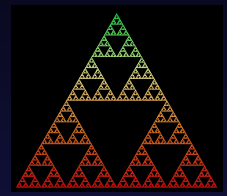
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$$\mathrm{Ext}_{BP_*(BP)}^{2,2p^j(p-1)} \xrightarrow{\Phi} \mathrm{Ext}_{\mathcal{A}}^{2,2p^j(p-1)}$$

is onto in all but one case, with

$$\widehat{\theta_j} = \beta_{p^{j-1}/p^{j-1}} \mapsto \left\{ \begin{array}{ll} 0 & \text{for } j = 1 \text{ and } p = 2 \\ \theta_j & \text{otherwise} \end{array} \right.$$



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Again, we want to know the fate of the elements

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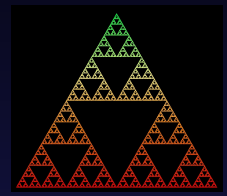
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A reformulation of the problem is the following:



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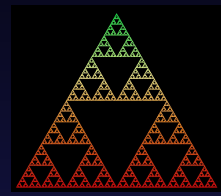
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A reformulation of the problem is the following:

> *Is any element of* $\mathrm{Ext}_{BP_*(BP)}^{2,2p^j(p-1)}$ *mapping to* θ_j *a permanent cycle?*

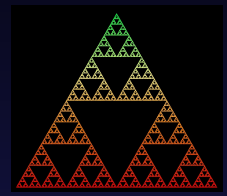


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There is no known intrepretation of the problem at odd primes in terms of manifolds.

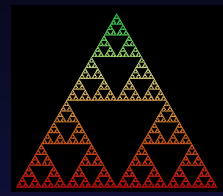


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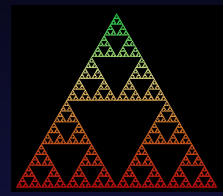
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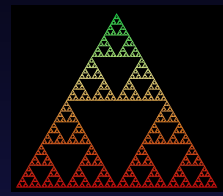
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Modulo some indeterminacy, there are differentials

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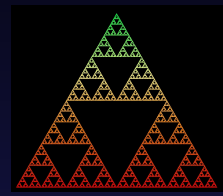
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where $h_0 \in \mathrm{Ext}_{\mathcal{A}}^{1,2p-1}$ represents $\alpha_1 \in \pi_{2p-3}\mathbb{S}$. Thus the only remaining case is $p = 3$.

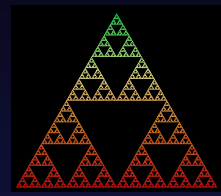


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The role of the Morava stabilizer group

In order to describe the difficulties at $p = 3$, we need to recall the methods of of HHR for $p = 2$ and myself for $p \geq 5$.

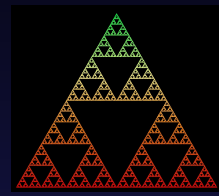


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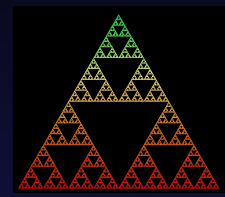
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Hirosi Toda
1928–2026

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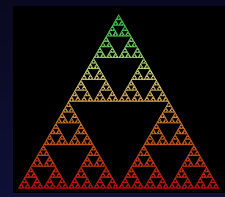
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We also show that there are relations

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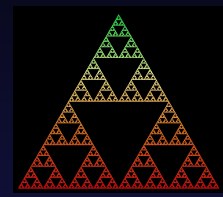
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The hard part is to use chromatic methods to show that these targets are all nontrivial.



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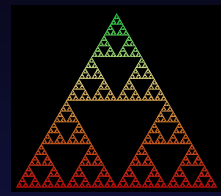
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The role of the Morava stabilizer group (continued)



Jack Morava
1944–2025

We now know (but only suspected in the late 70s) that the extended Morava stabilizer group $\mathbb{G}_n$

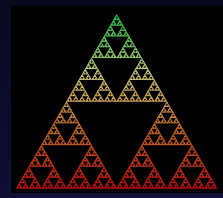


The role of the Morava stabilizer group (continued)



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We now know (but only suspected in the late 70s) that the extended Morava stabilizer group $\mathbb{G}_n$ acts on the Morava spectrum E_n in such a way that the homotopy fixed point set $E_n^{h\mathbb{G}_n}$ is $L_{K(n)}\mathbb{S}$,



Doug Ravenel

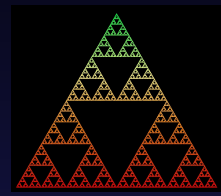
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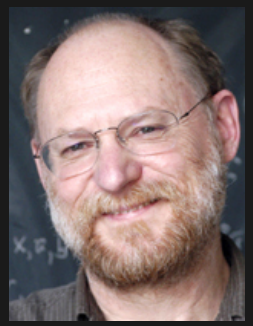
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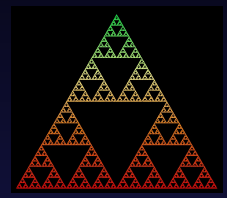
This is a corollary of the Hopkins-Miller theorem.



Mike
Hopkins



Haynes
Miller



The role of the Morava stabilizer group (continued)



Jack Morava
1944–2025

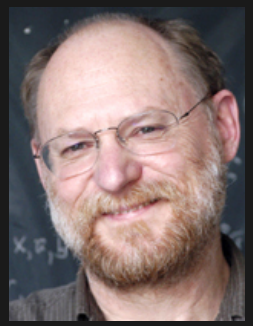
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This is a corollary of the Hopkins-Miller theorem. For any closed subgroup $H \subset \mathbb{G}_n$ there is a homotopy fixed point spectral sequence

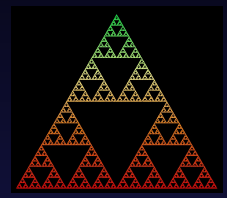
$$H^*(H; \pi_* E_n) \Longrightarrow \pi_* E_n^{hH}$$



Mike
Hopkins



Haynes
Miller



The role of the Morava stabilizer group (continued)



Jack Morava
1944–2025

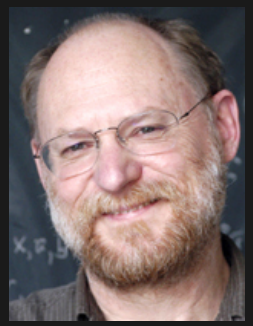
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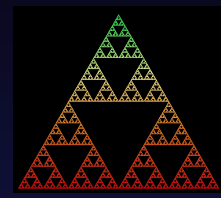


Mike Hopkins



Haynes Miller

which coincides with the Adams-Novikov spectral sequence for E_n^{hH}.



The role of the Morava stabilizer group (continued)



Jack Morava
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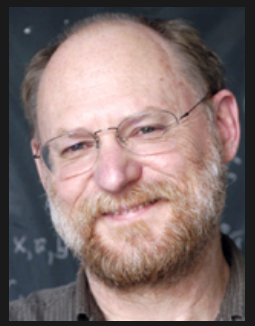
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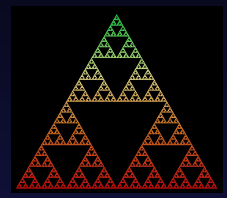


Mike Hopkins



Haynes Miller

which coincides with the Adams-Novikov spectral sequence for E_n^{hH}. One has the expected restriction maps for subgroups.

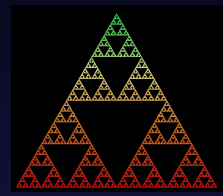


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The role of the Morava stabilizer group (continued)

The group $\mathbb{G}_n$ is known to have a subgroup of order p (unique up to conjugacy) when $p-1$ divides n.



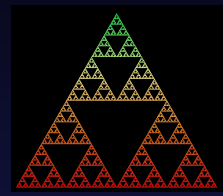
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The role of the Morava stabilizer group (continued)

The group $\mathbb{G}_n$ is known to have a subgroup of order p (unique up to conjugacy) when $p-1$ divides n. This leads to a composite homomorphism, the detection map

$$\mathrm{Ext}_{BP_*(BP)} \longrightarrow H^*(C_p; \pi_* E_{p-1}) \longrightarrow H^*(C_p; \mathbb{F}_{p^{p-1}}[u, u^{-1}])$$



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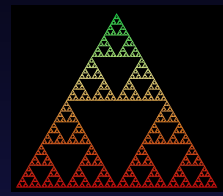
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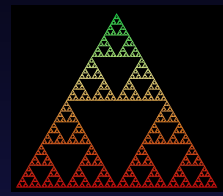
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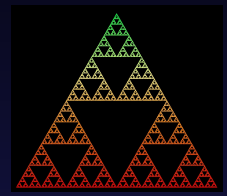
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$$E(\alpha) \otimes P(\beta) \otimes \mathbb{F}_{p^{p-1}}[u, u^{-1}]$$

where $\alpha \in H^{1,0}$ and $\beta \in H^{2,0}$ each have topological degree 0.



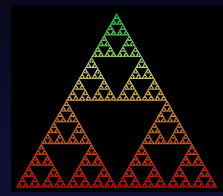
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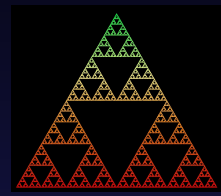
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We showed that under this map we have

$$\begin{aligned} \alpha_1 &\mapsto u^{1-p}\alpha \\ \widehat{\theta_j} &\mapsto u^{-p^j(p-1)}\beta \end{aligned}$$

up to unit scalar.



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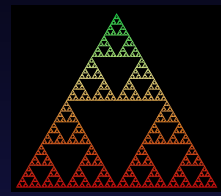
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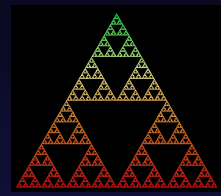
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$$d_{2p-1}(\widehat{\theta}_j) = \alpha_1 \widehat{\theta}_{j-1}^p$$

are nontrivial as desired.

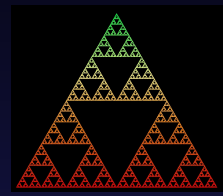


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To summarize:



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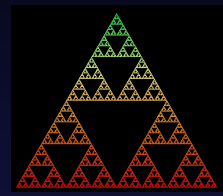
To summarize:

- The existence of an element of order p in $\mathbb{S}_{p-1}$ leads to the detection map above,

$$\begin{array}{ccc} \mathrm{Ext}_{BP_*(BP)} & \longrightarrow & H^*(C_p; \mathbb{F}_{p^{p-1}}[u, u^{-1}]) \\ & & \| \\ & & E(\alpha) \otimes P(\beta) \otimes \mathbb{F}_{p^{p-1}}[u, u^{-1}] \end{array}$$

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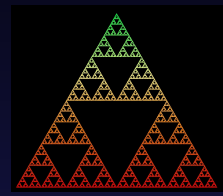
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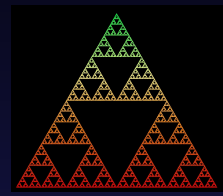
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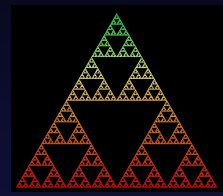


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Difficulties at $p = 3$

Why does this approach fail for $p = 3$?



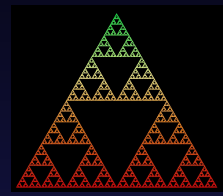
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The group $\mathrm{Ext}^{2,2p^j(p-1)}_{BP_*(BP)}$ is known to have $\lfloor (j-1)/2 \rfloor$ other generators besides $\widehat{\theta_j}$.



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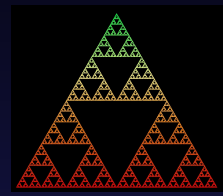
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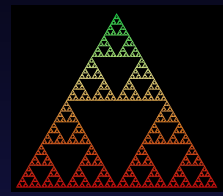
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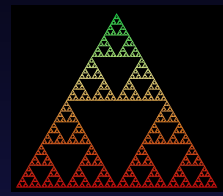
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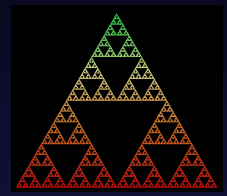
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The argument above establishes the nonexistence of $\widehat{\theta_j}$ (in the Adams-Novikov spectral sequence) for $j > 1$, but not that of θ_j in the classical Adams spectral sequence.

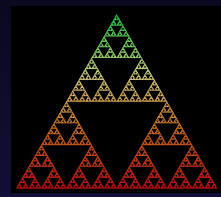


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Difficulties at $p = 3$ (continued)

It turns out that the way out of these difficulties is to use the subgroup $C_9 \subseteq \mathbb{S}_6$ rather than $C_3 \subseteq \mathbb{S}_2$.



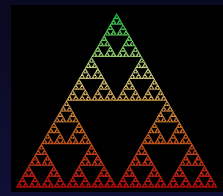
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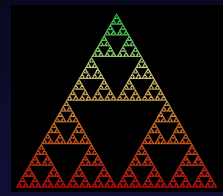
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For $p = 2$ similar considerations suggest using $C_8 \subseteq \mathbb{S}_4$, which is the approach used by HHR. Our strategy was to construct a ring spectrum Ω with a unit map $\mathbb{S} \to \Omega$ satisfying three properties:



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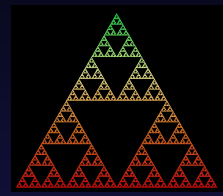
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(i) DETECTION THEOREM. If θ_j exists, its image in $\pi_*\Omega$ is nontrivial.



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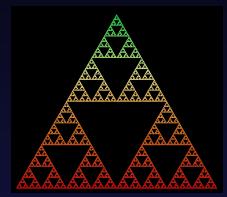
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(i) DETECTION THEOREM. If θ_j exists, its image in $\pi_*\Omega$ is nontrivial.

(ii) PERIODICITY THEOREM. $\pi_k\Omega$ depends only on the congruence class of k modulo 256.



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Difficulties at $p = 3$ (continued)

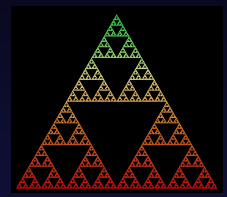
It turns out that the way out of these difficulties is to use the subgroup $C_9 \subseteq \mathbb{S}_6$ rather than $C_3 \subseteq \mathbb{S}_2$.

For $p = 2$ similar considerations suggest using $C_8 \subseteq \mathbb{S}_4$, which is the approach used by HHR. Our strategy was to construct a ring spectrum Ω with a unit map $\mathbb{S} \to \Omega$ satisfying three properties:

(i) DETECTION THEOREM. If θ_j exists, its image in $\pi_*\Omega$ is nontrivial.

(ii) PERIODICITY THEOREM. $\pi_k\Omega$ depends only on the congruence class of k modulo 256.

(iii) GAP THEOREM. $\pi_{-2}\Omega = 0$.



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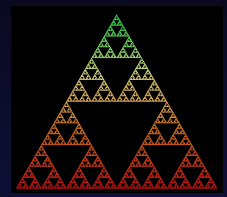
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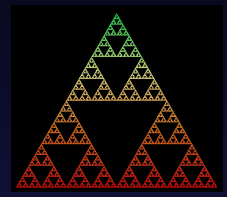
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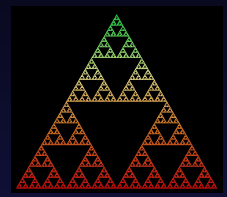
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Ever since the discovery of the Hopkins-Miller theorem, it has been possible to prove that $E_4^{hC_8}$ satisfies the first two of these properties without the use of equivariant stable homotopy theory.

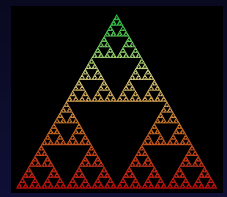


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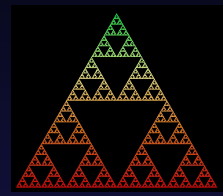


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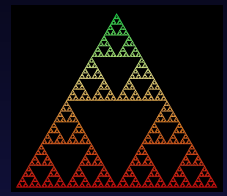


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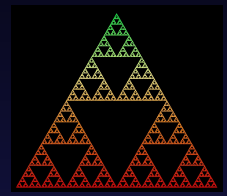
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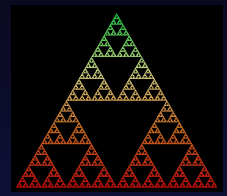
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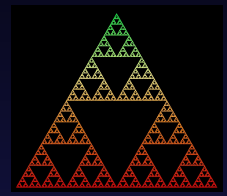
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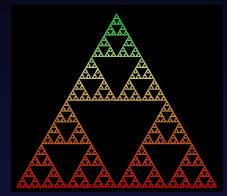
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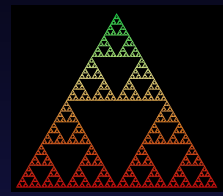
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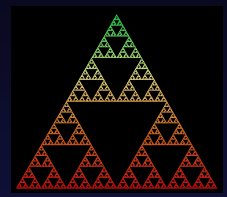
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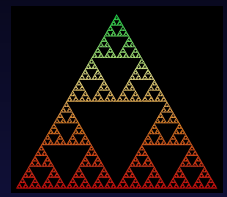
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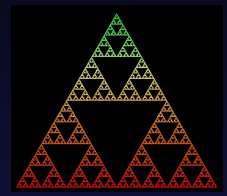


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Here is a quote from page 87.



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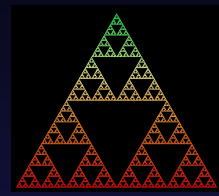
Difficulties at $p = 3$ (continued)

Here is a quote from page 87.

At weight 92 there are 2269 slots, of total Ext dimension 6341, and 1820 differential pairs with 23384 entries. The Leibniz assembly uses 15453 factor multiplication matrices, including successive factors in (A.31). There are 137471 scalar zero and Leibniz equations, 110060 of them having nonzero augmented row. Coefficient and augmented ranks are both 23119. Thus the initial affine dimension is 265. The successive square eliminations have the following checkpoints:

parameters	scalar square equations	polynomial row rank	new affine restrictions
265	16412	72	38
227	16412	20	1
226	16412	19	0

In each row quadratic monomials precede linear ones, as specified after (A.33); after the final refinement the affine rank is 23158. Apply (A.34) and (A.35) to its projected matrices. They give the sixteen rows of Table 5. The two additional stacked ranks are 1 and 2. Equations (A.36)–(A.38), using (A.40) in the second case, give $(4, 5, 5)$ and $(6, 5, 5)$. Equation (A.39) gives the pairs $(3, 3)$, $(2, 2)$, $(4, 4)$.



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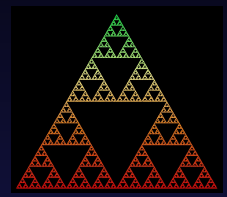
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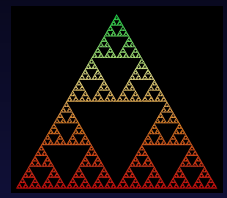


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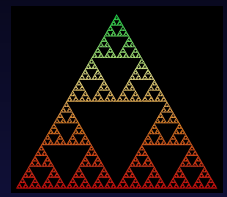


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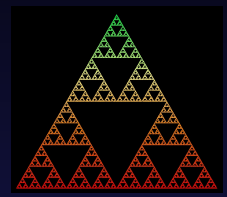


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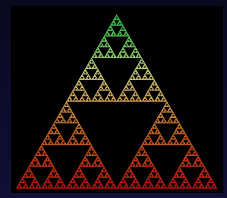


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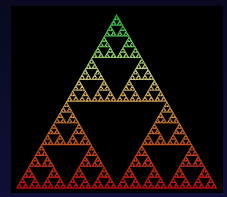
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We do know how to do it for $MU_{\mathbb{R}}$, which is MU as a C_2-spectrum via complex conjugation,



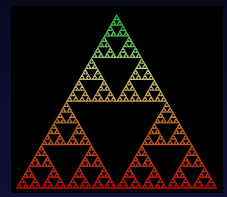
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Difficulties at $p = 3$ (continued)

The proof of the Gap Theorem for $p = 2$ requires the use of equivariant stable homotopy theory and the slice filtration, which is an equivariant analogue of the classical Postnikov filtration. Analyzing it for a general equivariant spectrum is difficult. We do not know how to do it directly for the case of interest, the Morava E_6 at $p = 3$ as a C_9-spectrum, or for E_4 at $p = 2$ for the group C_8.

We do know how to do it for $MU_{\mathbb{R}}$, which is MU as a C_2-spectrum via complex conjugation, and for $N_2^{2^{n+1}} MU_{\mathbb{R}}$, which is underlain by $MU^{(2^n)}$ with a $C_{2^{n+1}}$-action.

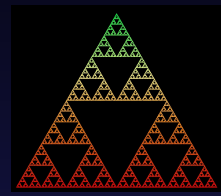


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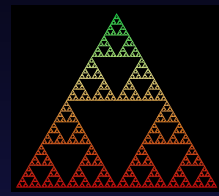
Difficulties at $p = 3$ (continued)

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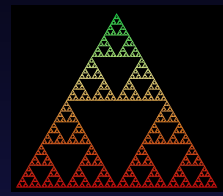
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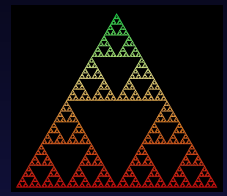
The real example

Difficulties at p = 3 (continued)

In order to do a similar thing at an $p = 3$ we need a C_3-analog MU_{Λ} (*MU* spoke) of the C_2-spectrum $MU_{\mathbb{R}}$. It should be underlain by $MU^{(2)}$ and have two properties:

(i) It has a tractable slice filtration with a certain description.

(ii) Its geometric fixed point spectrum $MU_{\Lambda}^{gC_3}$ is a wedge of suspensions of $H/3$, the mod 3 Eilenberg-MacLane spectrum.



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For $p = 2$, we have $MU_{\mathbb{R}}^{gC_2} = MO$, the unoriented cobordism spectrum, which fits a similar description.

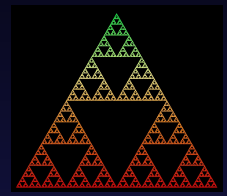
Difficulties at p = 3 (continued)

In order to do a similar thing at an $p = 3$ we need a C_3-analog $MU_{\bigtriangleup}$ (*MU* spoke) of the C_2-spectrum $MU_{\mathbb{R}}$. It should be underlain by $MU^{(2)}$ and have two properties:

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For $p = 2$, we have $MU_{\mathbb{R}}^{gC_2} = MO$, the unoriented cobordism spectrum, which fits a similar description. This identification is a pivotal step in determining differentials in the slice spectral sequence needed to prove the Periodicity Theorem.



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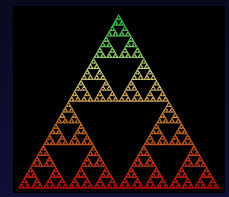
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Alas, we do not know how to construct $MU_{\bigcirc}$.





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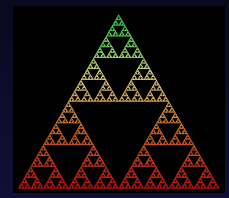
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Alas, we do not know how to construct $MU_{\bigcirc}$.

It is our missing piece.



Princeton University Algebraic Topology Seminar
OpenAI and the 3-primary Kervaire invariant problem



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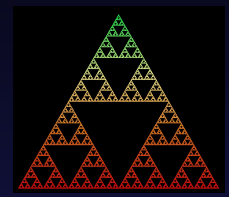
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Alas, we do not know how to construct $MU_{\bigcirc}$.

It is our missing piece. Maybe you can find it.



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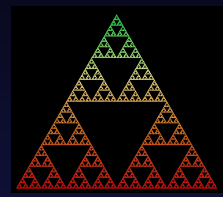
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Let's suppose the hypothetical $MU_{\bigcirc}$ exists as described above.



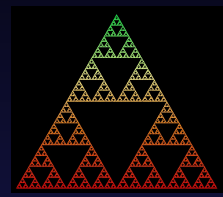
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Let's suppose the hypothetical $MU_{\bigcirc}$ exists as described above. For convenience we will work with its BP analog, $BP_{\bigcirc}$.



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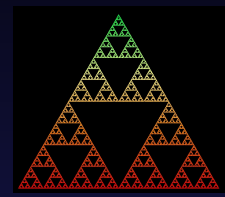
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Let's suppose the hypothetical $MU_{\bigcirc}$ exists as described above. For convenience we will work with its BP analog, $BP_{\bigcirc}$.

There is an element $Nv_1 \in \pi_{4\rho} BP_{\bigcirc}$.



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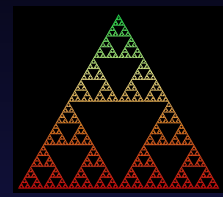
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Let's suppose the hypothetical $MU_{\bigcirc}$ exists as described above. For convenience we will work with its BP analog, $BP_{\bigcirc}$.

There is an element $Nv_1 \in \pi_{4\rho} BP_{\bigcirc}$. We can invert it and throw away the higher generators.



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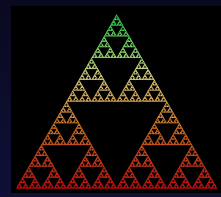
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Let's suppose the hypothetical $MU_{\bigcirc}$ exists as described above. For convenience we will work with its BP analog, $BP_{\bigcirc}$.

There is an element $Nv_1 \in \pi_{4\rho} BP_{\bigcirc}$. We can invert it and throw away the higher generators. The resulting fixed point spectrum looks a lot like *tmf*, but with periodicity in dimension 36 instead of 72.

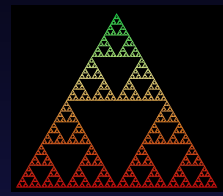


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Here is its slice spectral sequence.

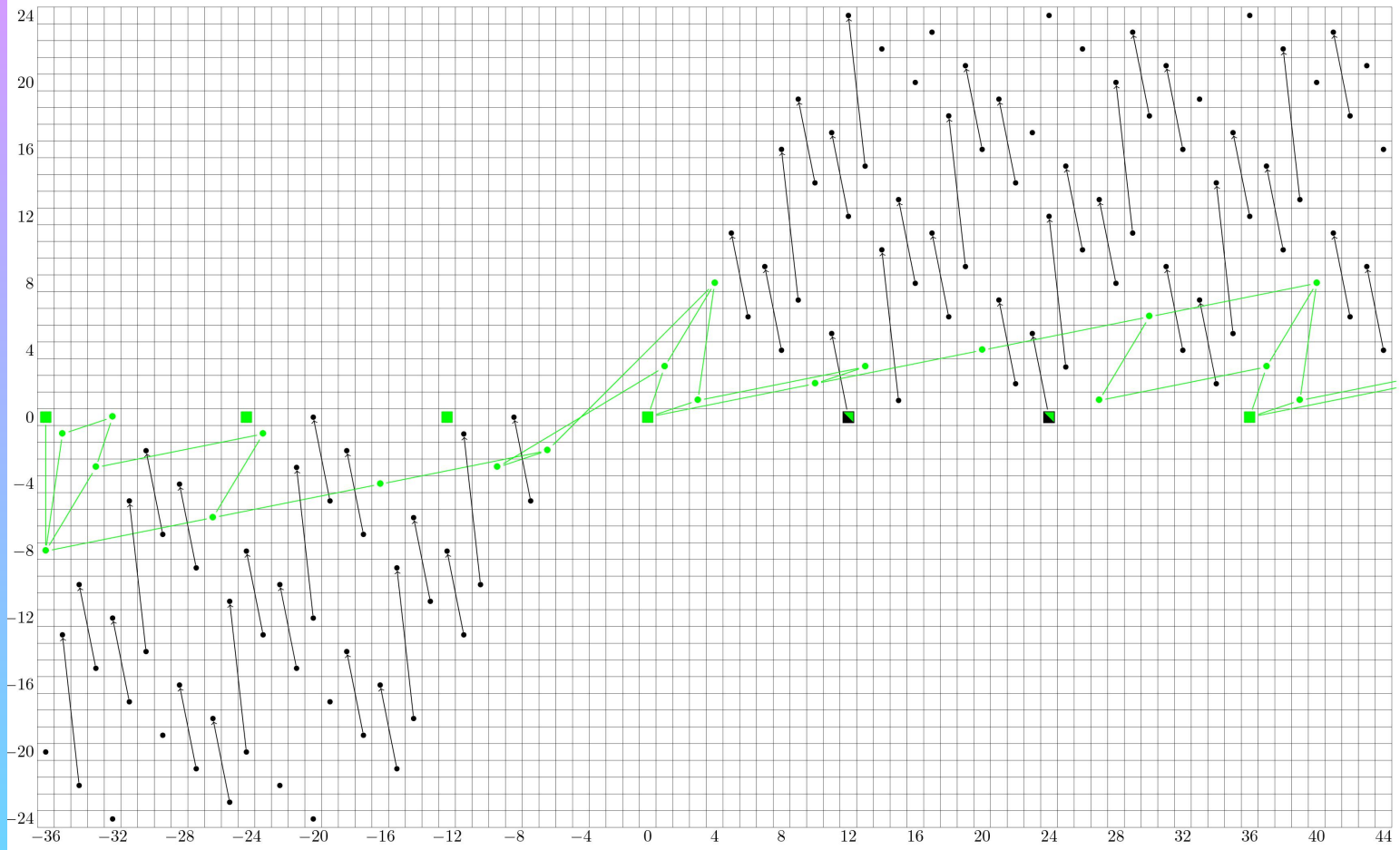


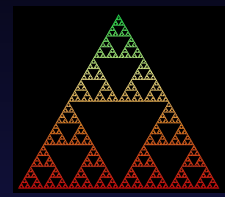
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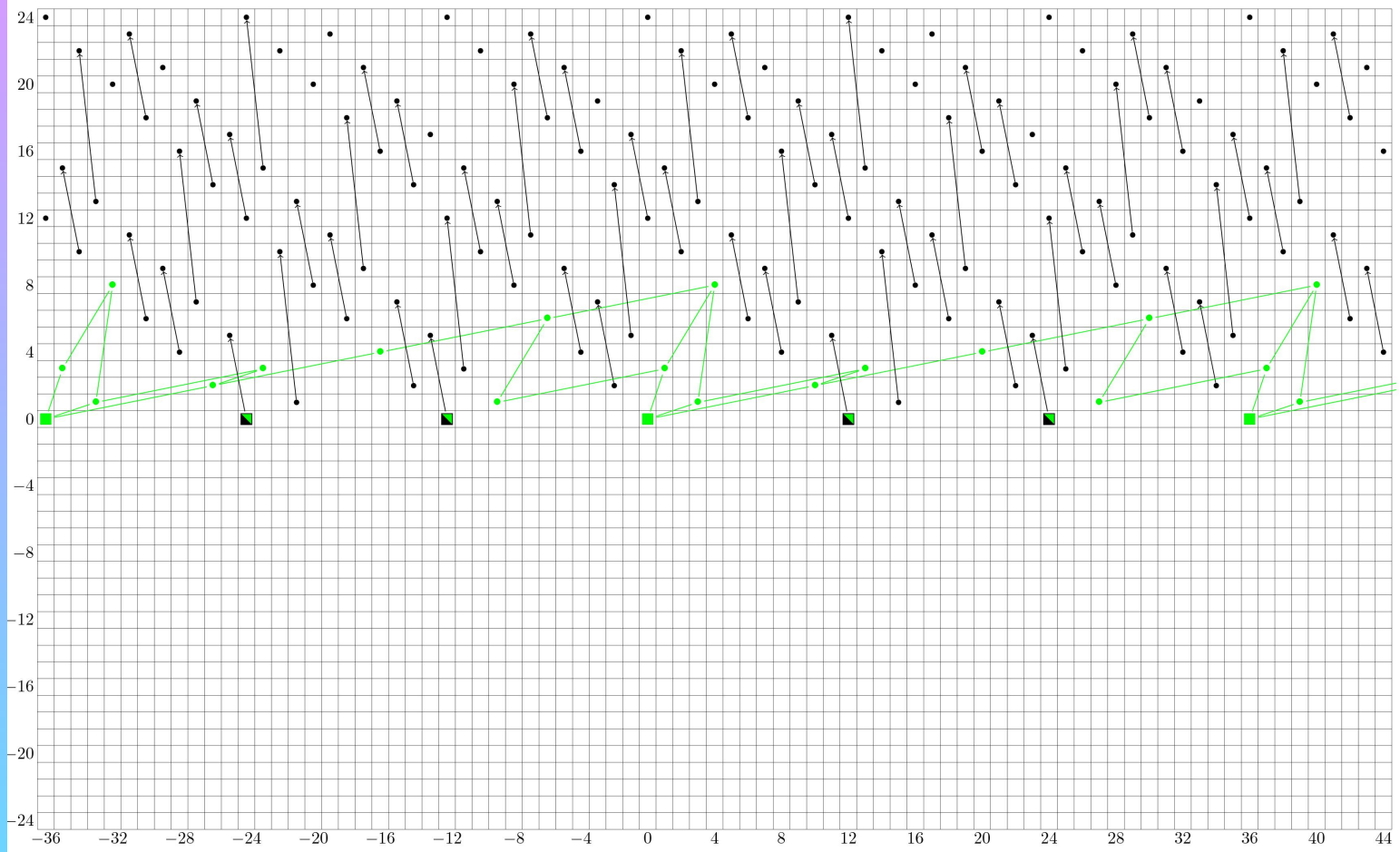
Here is its slice spectral sequence. Surviving elements are shown in green.

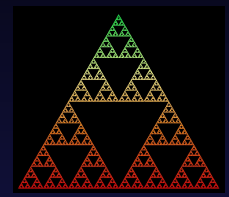




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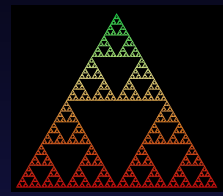
Here is its homotopy fixed point spectral sequence.





Back to reality

After 15 years, there is no construction of $MU_{(3)}$,

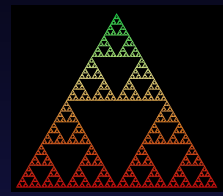


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Back to reality

After 15 years, there is no construction of $MU_{(3)}$, so we do not have a slice spectral sequence.

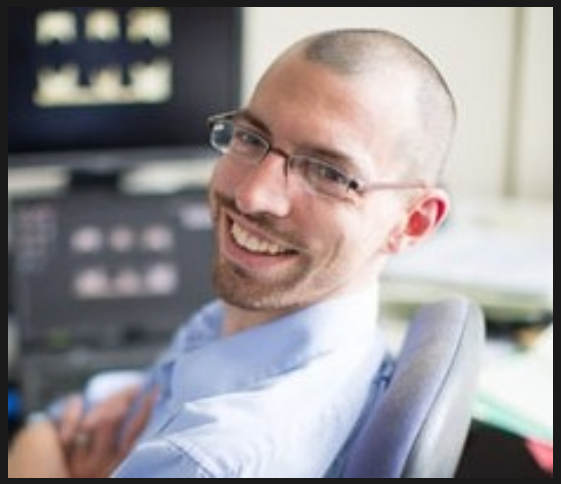


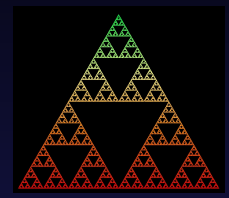
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After 15 years, there is no construction of $MU_{\bigcirc}$, so we do not have a slice spectral sequence. Gabriel Angelini-Knoll, Mark Behrens, Max Johnson, and Hana Jia Kong may be close to one,

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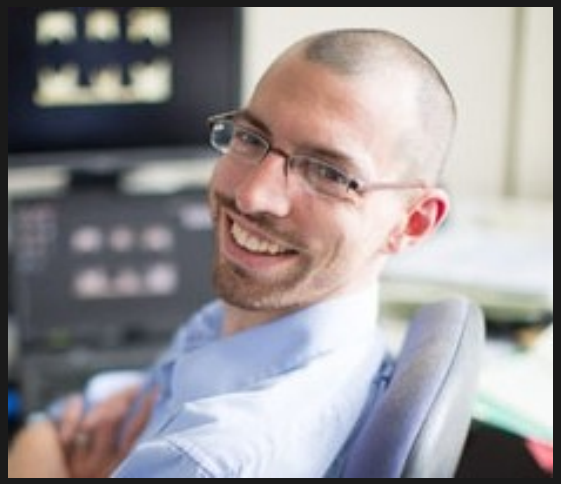
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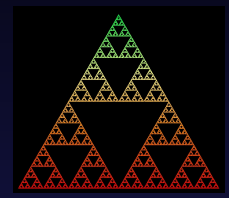
The real example

Back to reality

After 15 years, there is no construction of $MU_{\bigcirc}$, so we do not have a slice spectral sequence. Gabriel Angelini-Knoll, Mark Behrens, Max Johnson, and Hana Jia Kong may be close to one,


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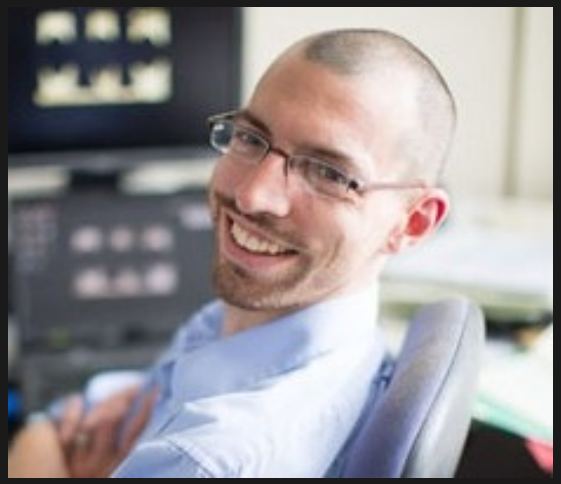


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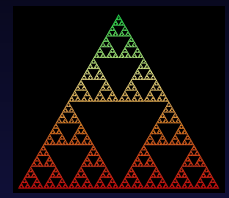
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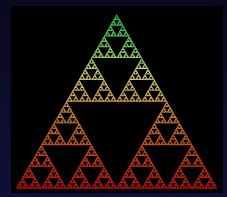
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Back to reality (continued)

OpenAI studies the homotopy fixed point and Tate spectral sequences

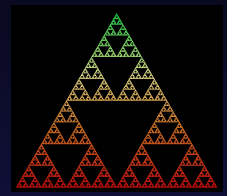


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Back to reality (continued)

OpenAI studies the homotopy fixed point and Tate spectral sequences for the Devinatz-Hopkins actions of the subgroups



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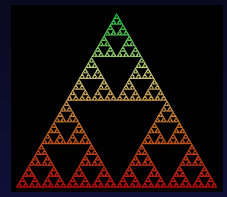
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Back to reality (continued)

OpenAI studies the homotopy fixed point and Tate spectral sequences for the Devinatz-Hopkins actions of the subgroups

$$\begin{array}{ccccc} \mathbb{G}_6 & \hookleftarrow & G & \hookleftarrow & H \\ & & \| & & \| \\ & & C_9 & & C_3 \end{array}$$

on the Morava spectrum E_6.



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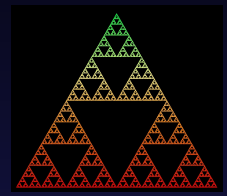
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Back to reality (continued)

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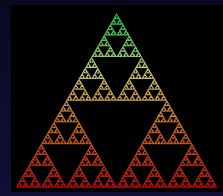
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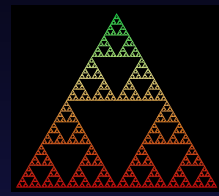
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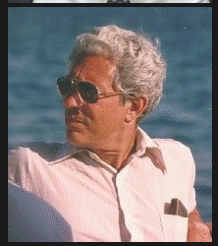
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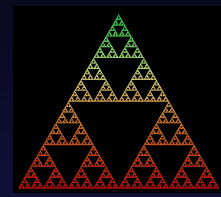
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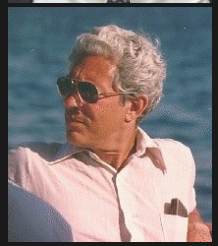
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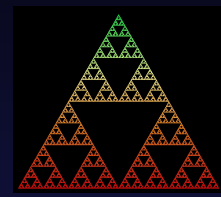




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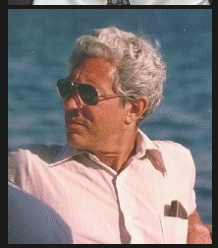
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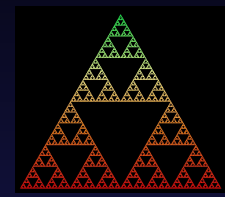




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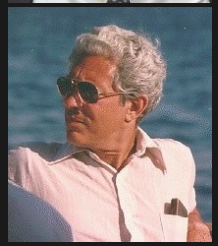
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The Tate spectral sequence

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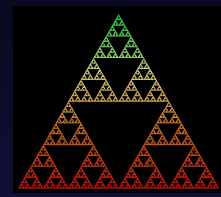




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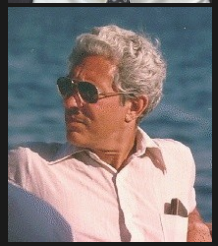
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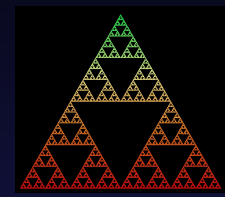




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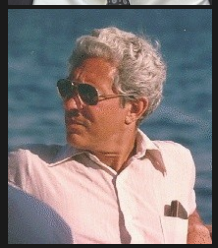
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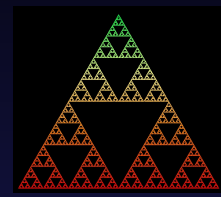




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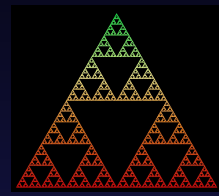


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The Tate spectral sequence (continued)

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We have known since 1996 that the Tate constructions E_6^{tH} and E_6^{tG} are both contractible.

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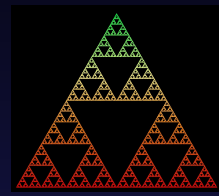
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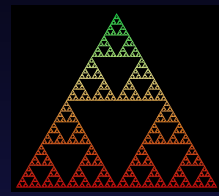
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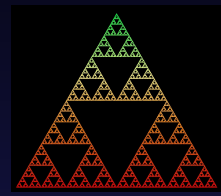
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The homotopy fixed point spectral sequence occupies the upper half plane,



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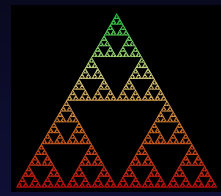
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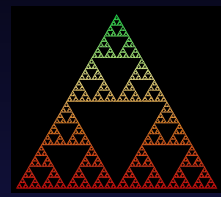
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The Tate spectral sequence (continued)

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The homotopy fixed point spectral sequence occupies the upper half plane, and the Tate spectral sequence occupies the full plane. The two agree above the x-axis.

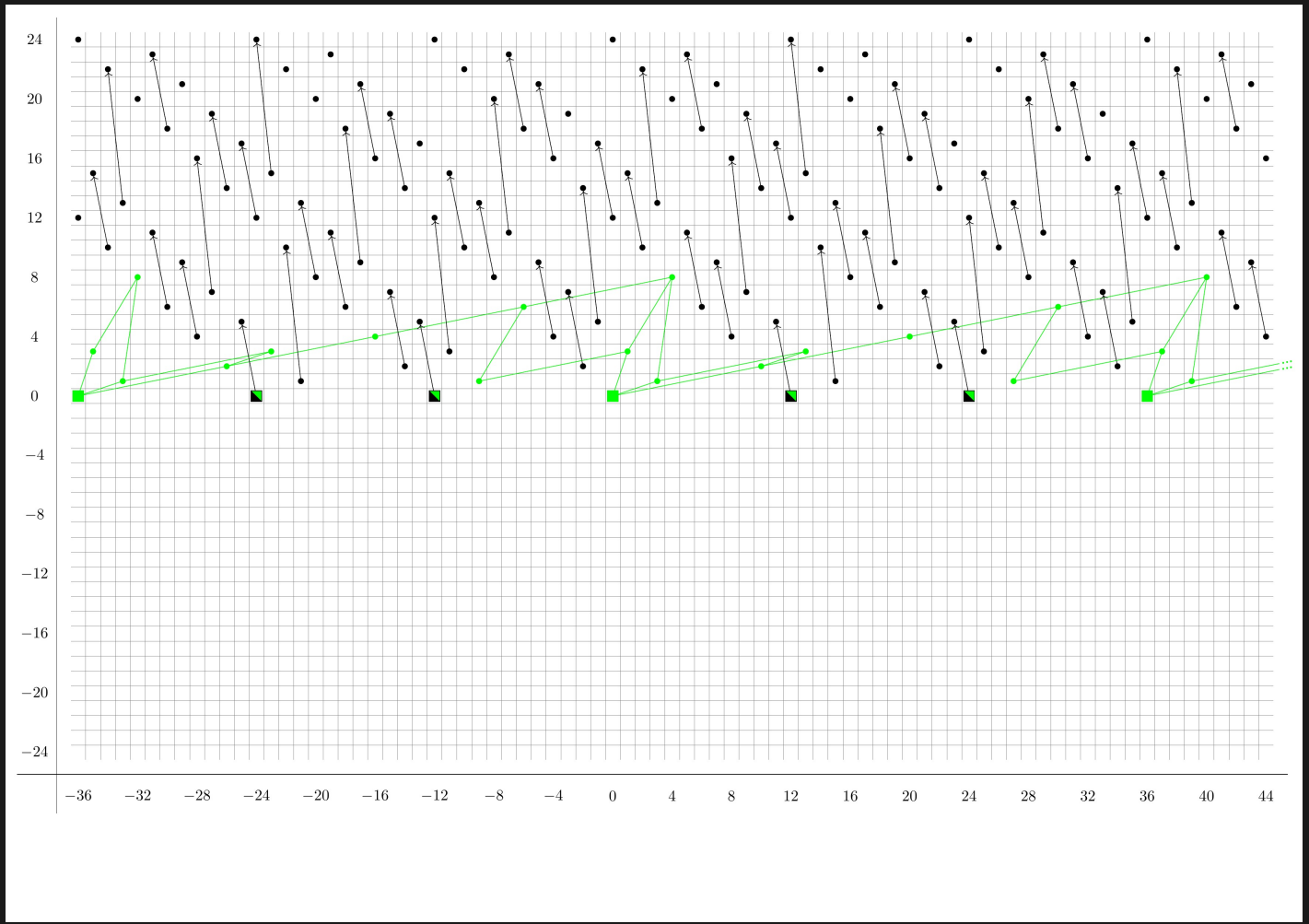


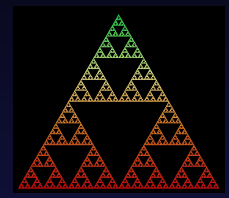
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A toy example

As an illustration, here again is the homotopy fixed point spectral sequence for the C_3-action on E_2 at $p = 3$.





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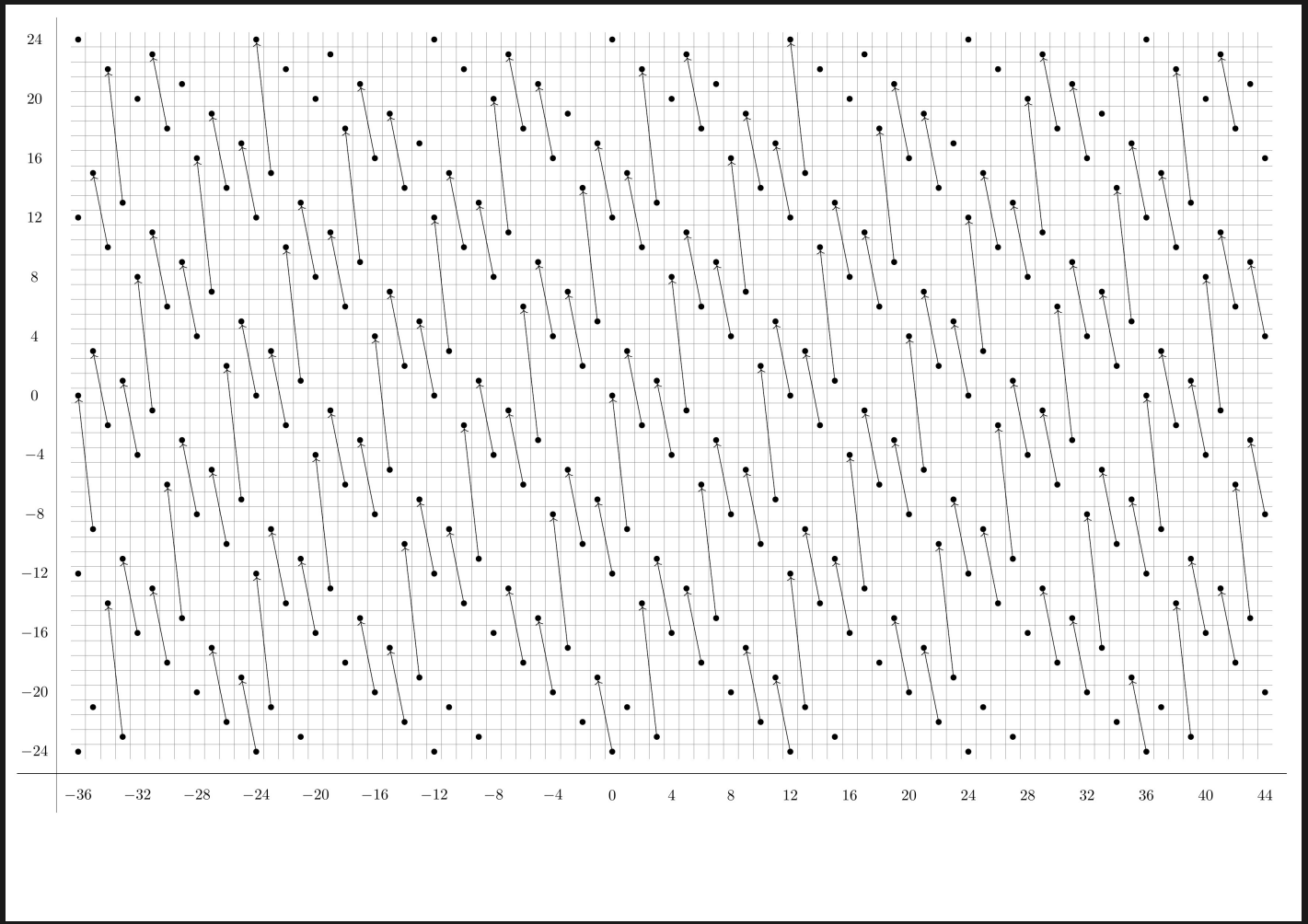
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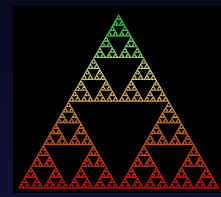
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A toy example (continued)

Here is the corresponding Tate spectral sequence.





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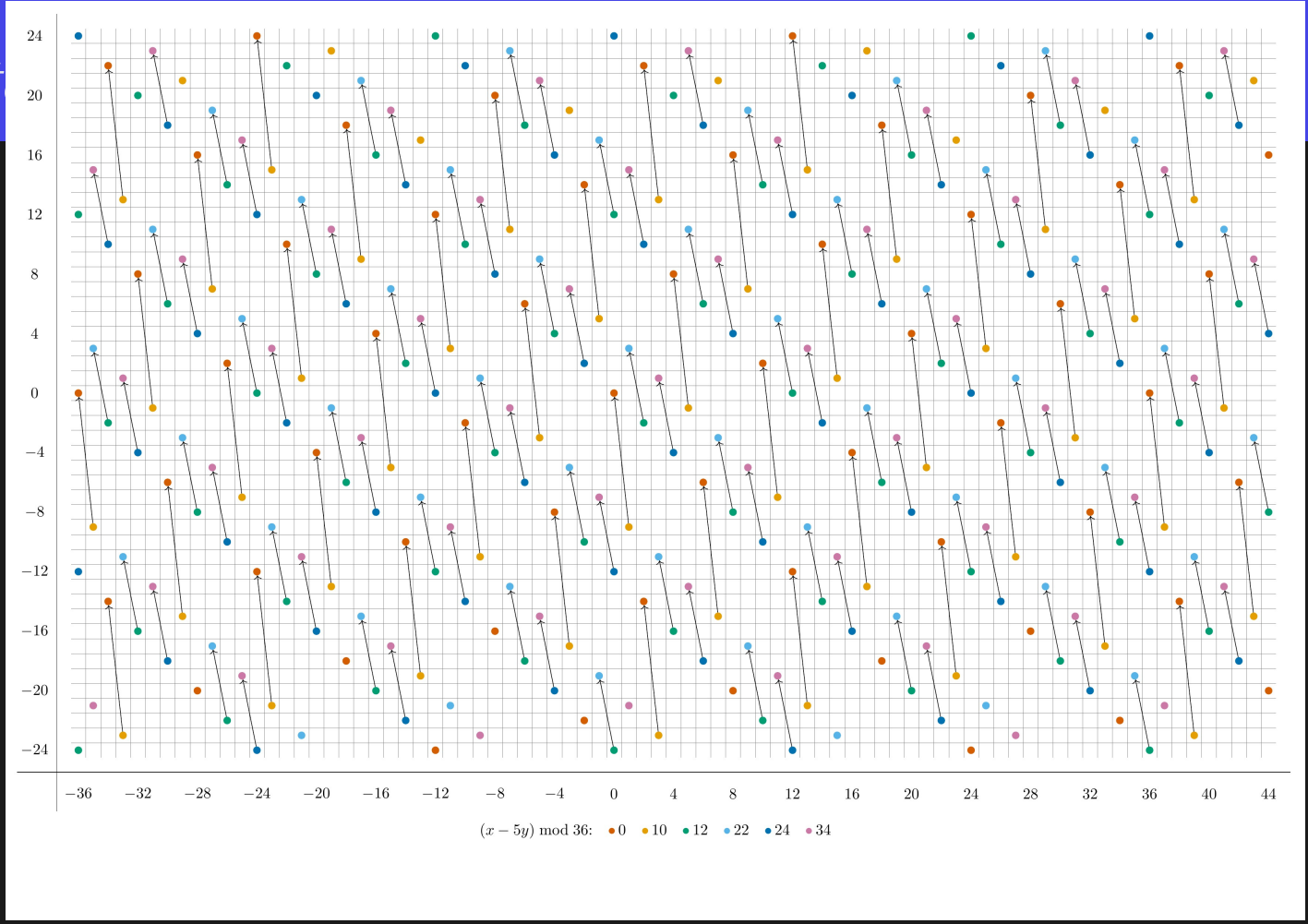
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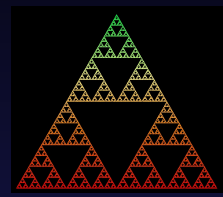
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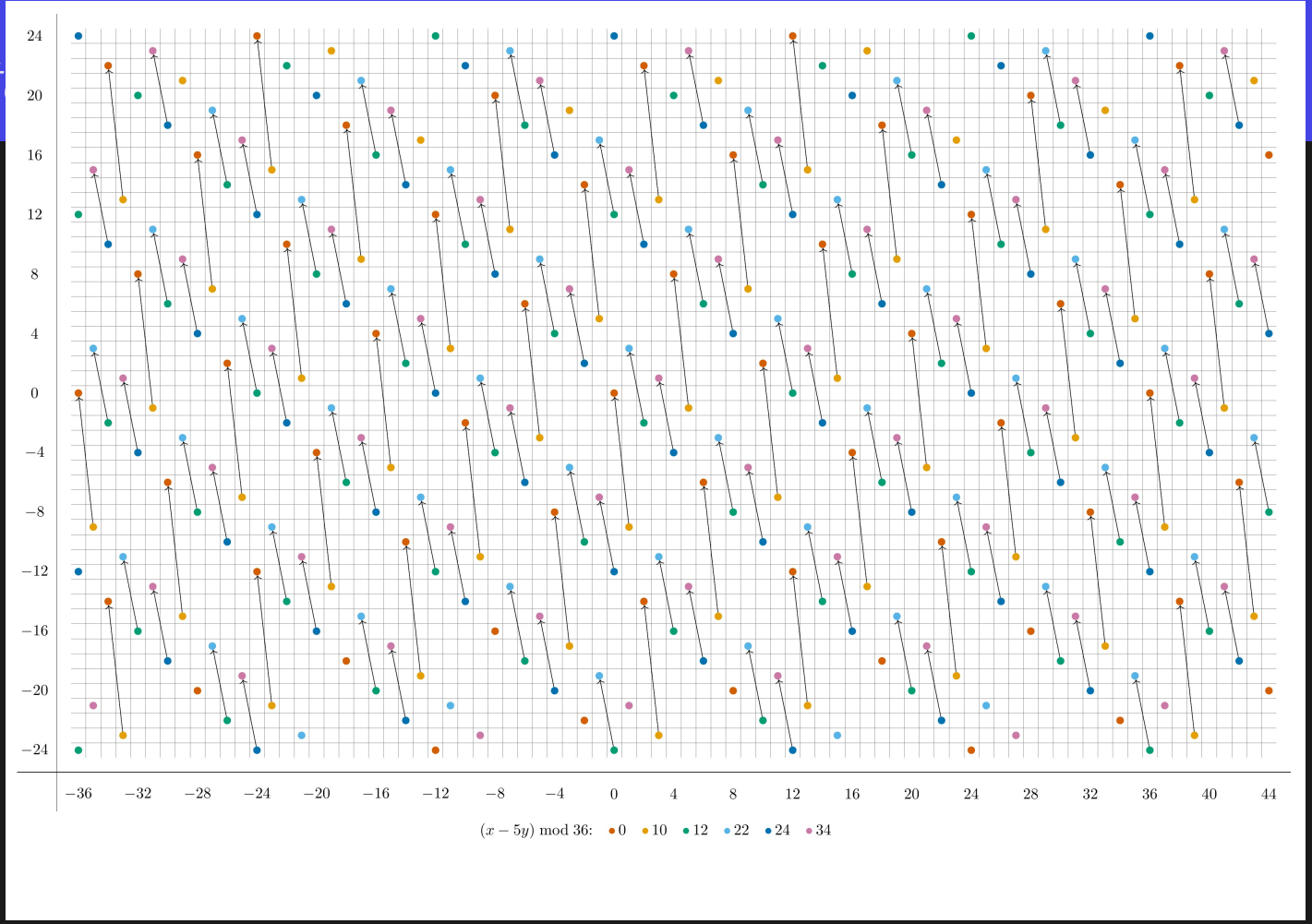
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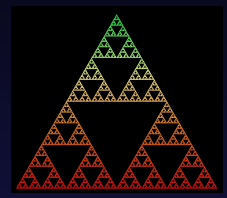
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Note that this chart is periodic in two directions:

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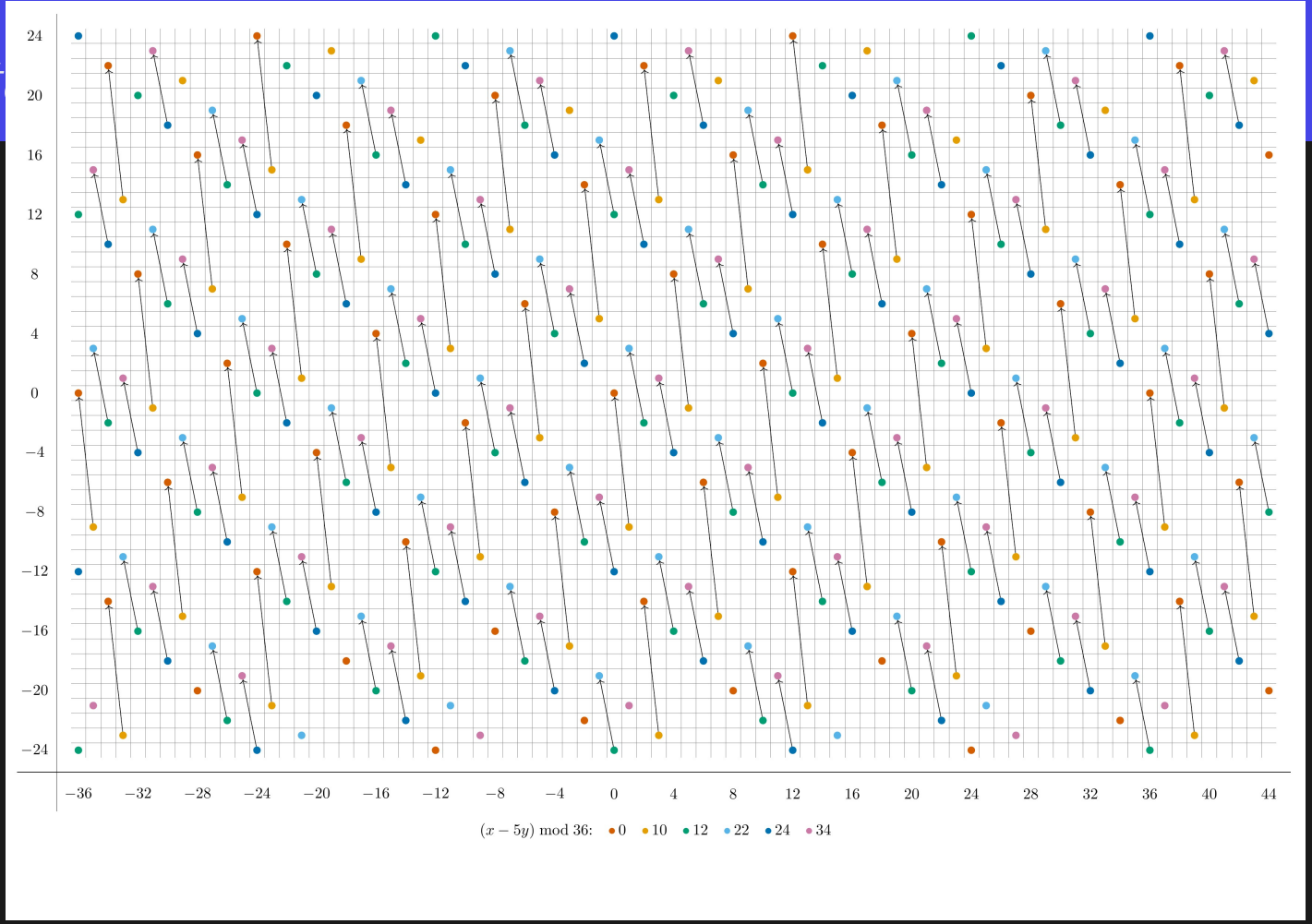
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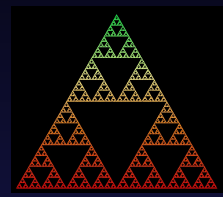
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Note that this chart is periodic in two directions: $(36, 0)$, chromatic periodicity,

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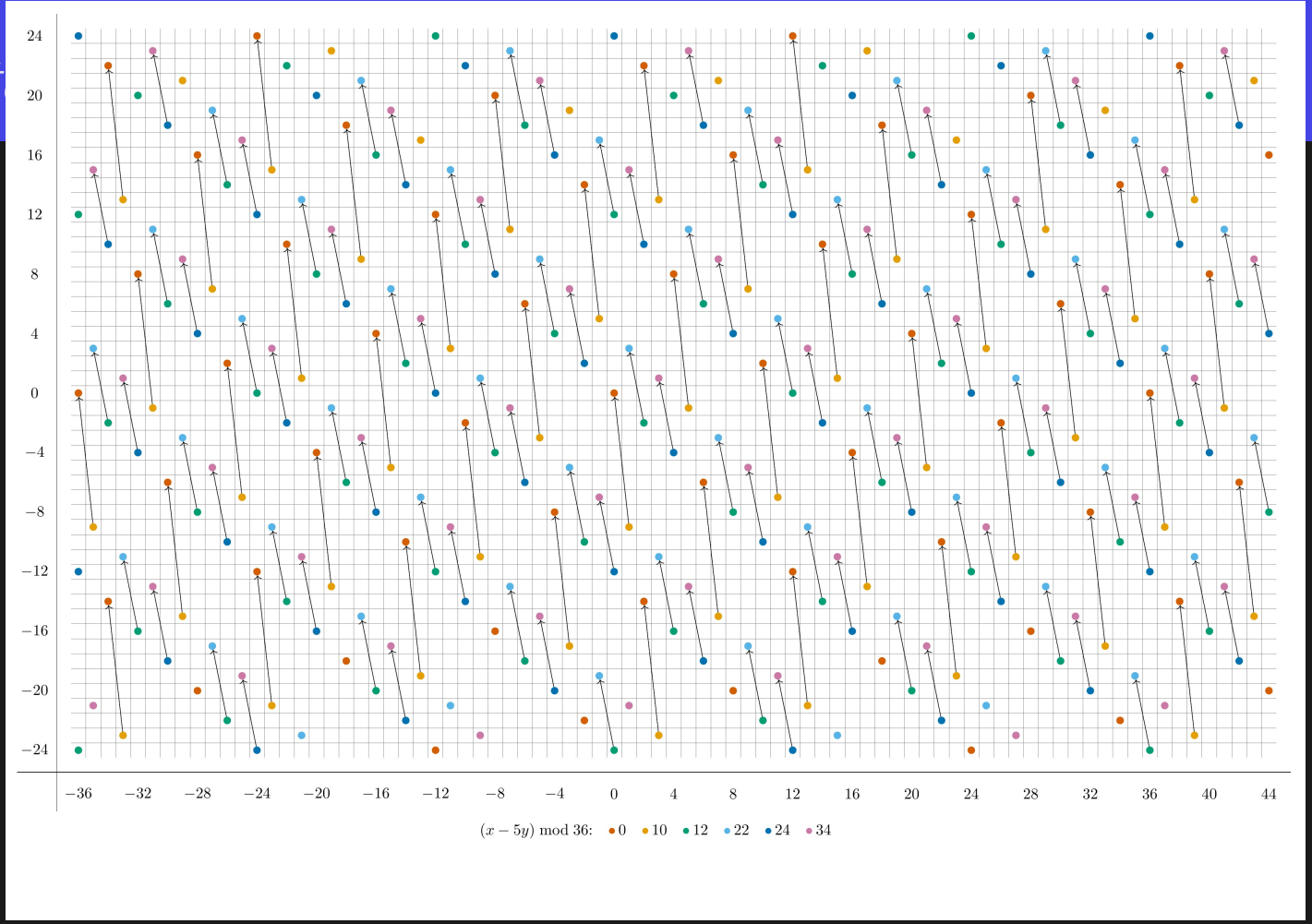
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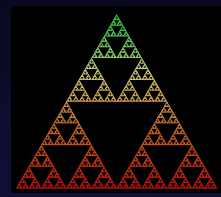
A toy example

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Note that this chart is periodic in two directions: $(36, 0)$, chromatic periodicity, and $(6, -6)$, Tate periodicity.

Princeton University Algebraic Topology Seminar
OpenAI and the 3-primary Kervaire invariant problem



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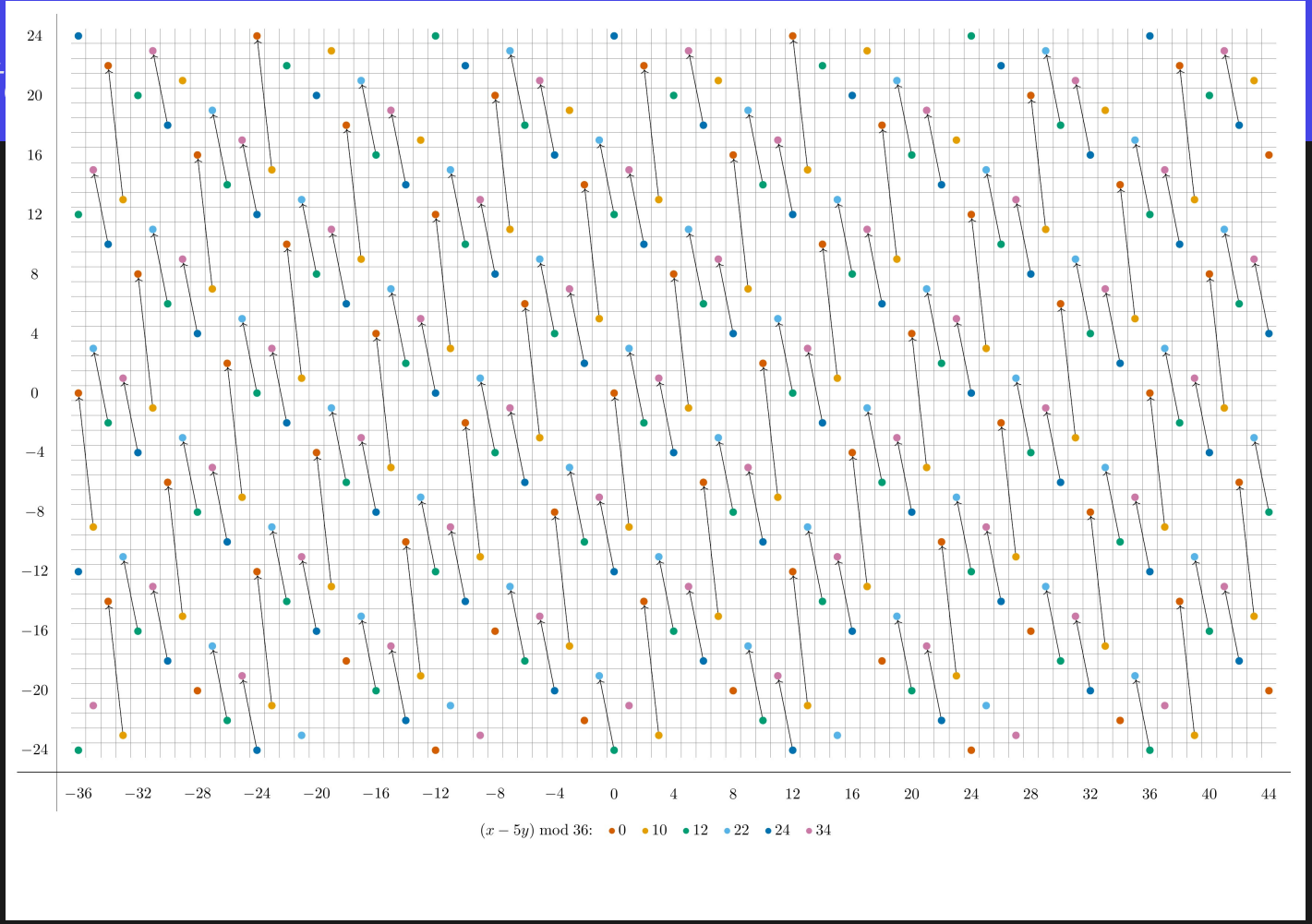
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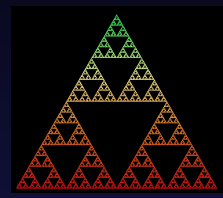
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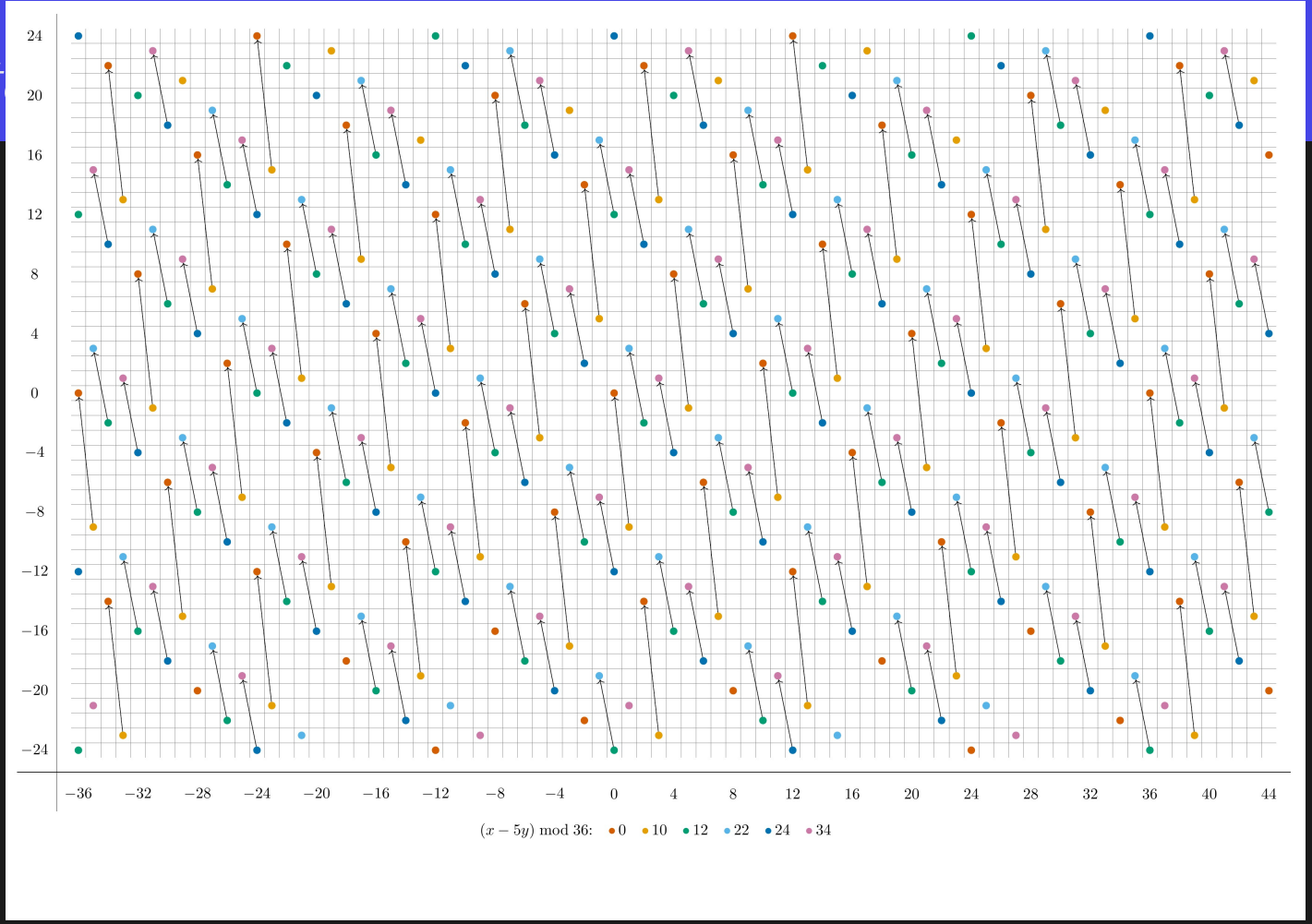
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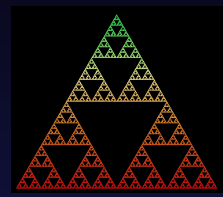
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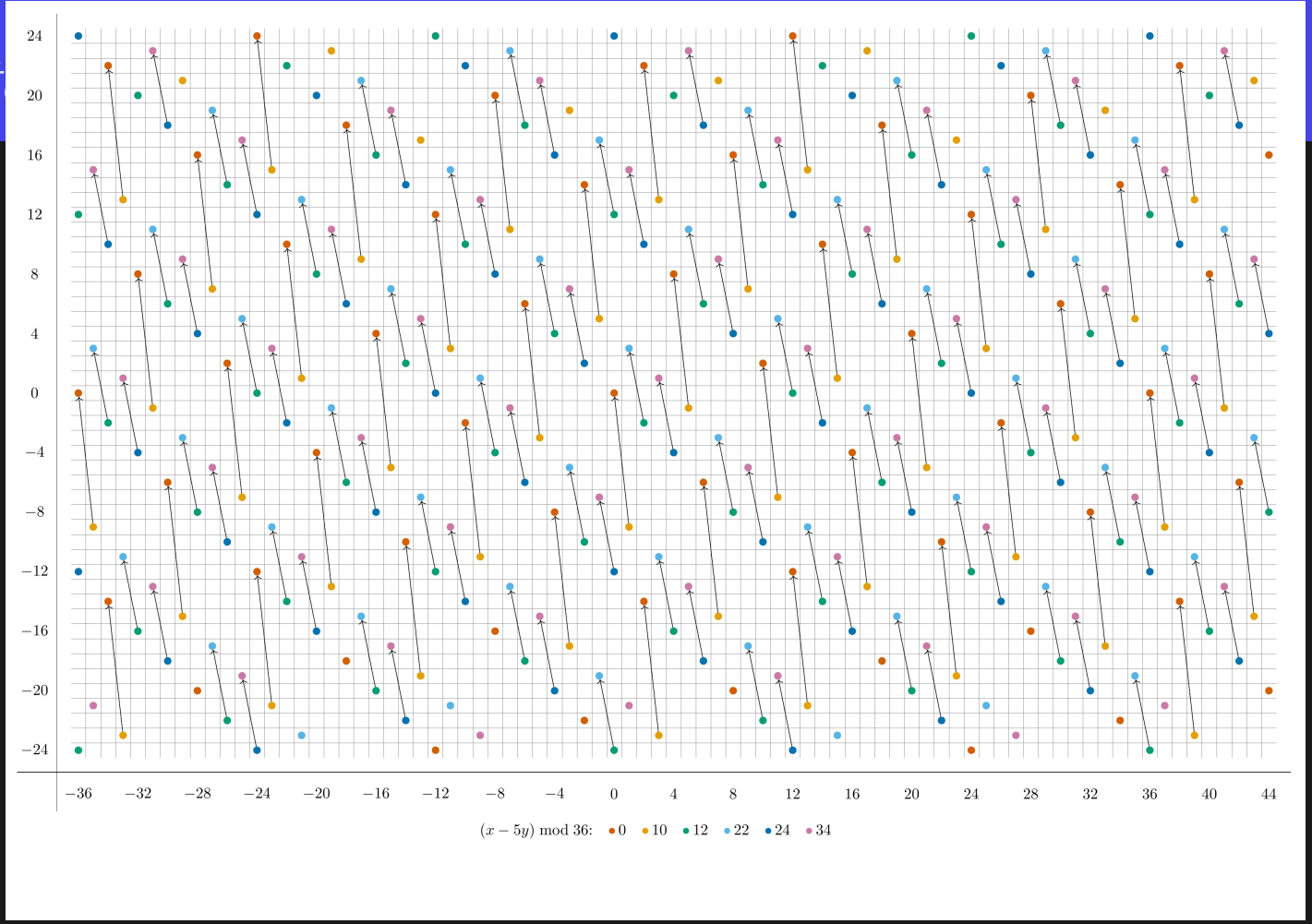
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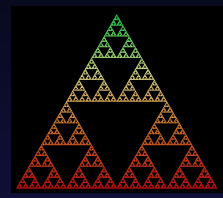
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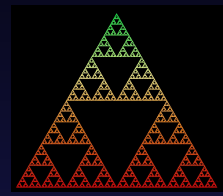


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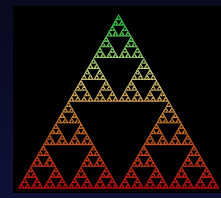
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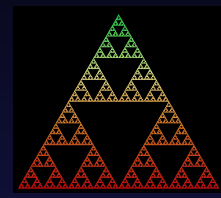
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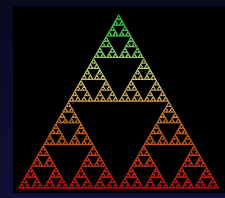
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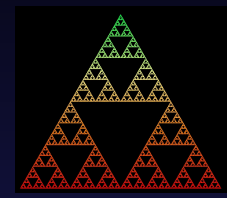
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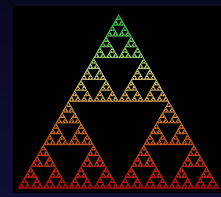




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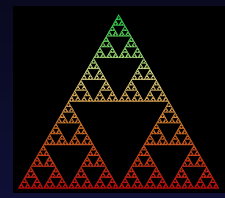




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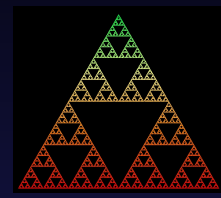




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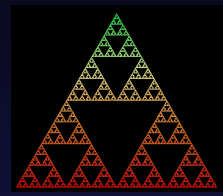
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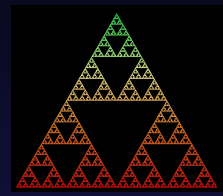
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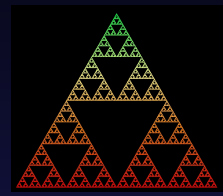
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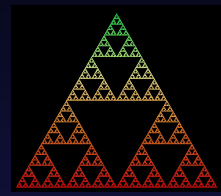
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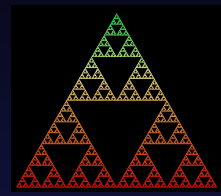
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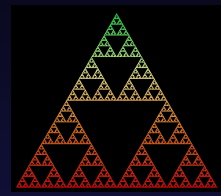
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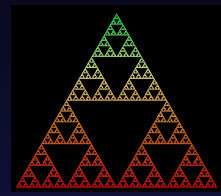
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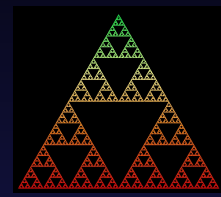
I cannot vouch for all these details.



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THANK YOU
FOR YOUR ATTENTION



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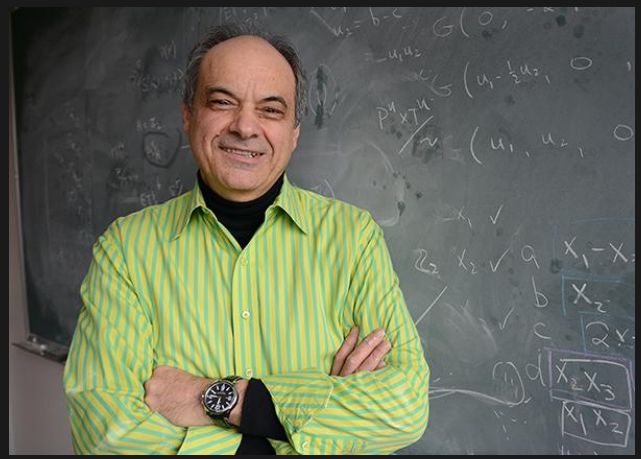
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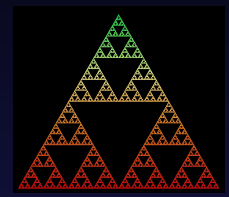
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