Cooperation without Monitoring*

Yu Awaya†

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Abstract

By means of an example, this note shows that monitoring is not necessary to achieve cooperation. More precisely, I consider an environment in which no player gets *any* information about past behavior of the other players. Despite this, there is an equilibrium in which players take myopically suboptimal actions. This is in sharp contrast to the body of literature on repeated games, where observability of others' past behavior is crucial to sustain cooperation.

1 Introduction

It is well observed that individuals, organizations and living things often cooperate with each other, sacrificing their own myopic interest when they are engaging in a long-term relationship. The literature on repeated games provides an explanation (see Mailath and Samuelson (2006) for a survey). The literature on repeated games provides the following explanation : cooperation can be achieved as a consequence of agents being able to monitor each other's actions. Moreover, it has been noted that monitoring is in fact crucial in achieving cooperation. Thus, naturally it is expected that when monitoring is poor—as in, for example, large societies where each agent has negligible influence on others' payoffs—it is hard to sustain cooperation. Green (1980) and its consequent papers show that this is indeed the case for various environments (see Section 2.7 and 7.8 of). That is, for a fixed discount factor, as the population tends to infinity the equilibrium set of the repeated games converges to that of the one-shot games.[1] This idea appears quite often in

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†Department of Economics, Penn State University, yxa120@psu.edu

[1] Hence, the order of the quantifiers is different from "folk theorems" as in Sugaya (2013).

macroeconomics. A few examples include the capital taxation problem where an action of a citizen is observed neither by the government or other citizens (Chari and Kehoe (1990)), and monetary economics, where money plays a role when there is no device to keep track of individuals behavior (Wallace (2010)).

The purpose of this note is to present that, even though the idea sounds very intuitive, there is a counterexample to it. The example shows that even when *no* player gets *any* information about past behavior of the other players, there is still an equilibrium in which players take myopically suboptimal actions. The model is a simple two-player and two-period example[2] where prisoner's dilemma is followed by coordination game. After actions are taken in the first period, each player gets a signal which is observed privately. The distribution of the signal depends on the action profile. The signal, however, does not contain *any* information about the action profile, because the *marginal* distribution does not depend on the action profile. It is also assumed that the joint distribution does depend on the action profile, and moreover correlation is higher when they take the same action.[3]

In this case, players can use the signal as a coordination device in the second period. Suppose a player cooperates in the first period, and then chooses his action depending on the signal he gets. Because the stage game in the second period is coordination game, the other player will be better-off by knowing what her rival will do. To get the information, a player is willing to scarify her short-term gain, even though her action does not affect his opponent's future play.

2 Preliminaries

Consider the following two-period game played by two players $i \in \{1, 2\}$, where the prisoner's dilemma in Figure 1 is followed by the coordination game in Figure 2:

At the end of the first stage, each player receives a private signal $z_i \in \{\underline{z}, \overline{z}\}$. The joint distribution depends on the action profile, and given by Figures 3 (in the case where action profile is CC) and 4 (otherwise).

I emphasize that there is *no* monitoring, because the *marginal* probability that a player gets a signal is always one half regardless of action profile, and hence a player cannot infer the other player's action at all. Joint distribu-

[2]It is easy to extend the idea to an infinitely repeated game.

[3]Aoyagi (2002) is first to introduce the monitoring structure in which correlation between signals depends on actions, and then explored by Zheng (2008), Awaya (2014) and Awaya and Krishna (2014). Awaya (2014) and Awaya and Krishna (2014), like this note, assume that marginal distributions of signals are similar across action profiles.

	C	D
C	$3,3$	$-1,4$
D	$4,-1$	$0,0$

Figure 1: Prisoner's Dilemma

	G	B
G	$6,6$	$0,0$
B	$0,0$	$4,4$

Figure 2: Coordination Game

	$\underline{z}$	$\overline{z}$
$\underline{z}$	$1/2$	0
$\overline{z}$	0	$1/2$

Figure 3: CC

	$\underline{z}$	$\overline{z}$
$\underline{z}$	$1/4$	$1/4$
$\overline{z}$	$1/4$	$1/4$

Figure 4: Otherwise

tion, however, does vary a lot with action profiles. In particular, signals are perfectly correlated when the action profile is CC, and they are independent otherwise.[4]

There is no discounting. Players receive their payoffs at the end of the second period, so first period payoffs do not convey information to the players. The solution concept is sequential equilibrium.

3 Result and Proof

The result of the note is given as follows:

Theorem 1. *There exists an equilibrium in which players cooperate in the first period.*

Proof

Strategies

I will prove the theorem by construction. I will claim that the following strategy profile constitutes an equilibrium. In the first period, play C. In the second period, play G if (i) he played C and observed $\overline{z}$, or (ii) he played D. Otherwise, play B.

[4]It will be clear from the proof that these extreme assumptions can be weakened to get the result.

Incentives

First, check the optimality of the second period behavior. Consider a player who has played C in the first period. Because equilibrium strategies are common knowledge, he thinks the other player also has played C, and hence gets the same signal as he himself did. This shows that it is optimal to follow the strategy. In this case, his payoff in the second period is $6 \times 1/2 + 4 \times 1/2 = 5$.

Next take a player (say i) who has deviated and played D in the first period. Given the action profile, for each $z_i \in \{\underline{z}, \overline{z}\}$

$$\Pr(z_{-i} = \underline{z} \mid z_i) = \Pr(z_{-i} = \overline{z} \mid z_i) = 1/2$$

This means, from his perspective, player $-i$ will play G and B with the same probability. Given the strategy, it is optimal for him to play G. In this case, his expected payoff is 3.

Now turn to the first period. It is shown that if a player plays C, then his expected payoff in the second period is 5, while plays D, it is 3. Thus, the total expected payoff of playing C is $3 + 5 = 8$, while that of playing D is $4 + 3 < 8$. This completes the proof.

Notice, the equilibrium payoff exceeds the maximum payoff attained by trivial equilibrium—a repetition of the stage game equilibrium—which is 6.

4 Conclusion

In this note, I provide a simple numerical example in which no monitoring does not necessarily mean no cooperation. It can also be easily seen that it is not possible to achieve the first best equilibrium—to attain such an outcome, each player must play C in the first period and then play G *for sure*. In this case, it is profitable to deviate and play D, because even if a player does so, the other player will play G in the second period. In Awaya and Krishna (2014), we generalize this observation and provide a method to provide an upper bound on the equilibrium payoffs, which only depends on the marginal distribution of signals.

References

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