Competition and Free-Riding in Electoral Campaigns with Outside Spending*

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Abstract

Why do outside groups spend money on electoral campaigns? How do incentives to free-ride or, conversely, to compete for policy influence affect independent expenditures? We develop a game-theoretical model of elections as contests with competition both between and within coalitions of spending committees. Each member of the coalition (including the candidate herself) benefits from the candidate winning, but may also value access to policy favors that are distributed in proportion to campaign expenditures. We structurally estimate the model parameters using data on election outcomes, independent expenditures, and campaign spending in U.S. House elections from 2004 to 2020. Our estimates allow us to interpret electoral and spending data in terms of an underlying utility model for voters and spending committees. Observed spending patterns are consistent with intra-coalition competition among party organizations, 501(c) "dark money" groups, and traditional political action committees (PACs)—but not by Super PACs.

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Why do outside groups spend money on electoral campaigns? Since 2010, when the Supreme Court held in *Citizens United v. Federal Election Commission* that independent campaign expenditures cannot be limited by law, there has been intense public and scholarly interest in the effects of outside spending on political campaigns. Despite these concerns, empirical studies have generally found limited effects of outside spending on congressional races (Farrar-Myers, Gulati and Skinner 2013; Chaturvedi and Holloway 2017; Baker 2018; Cox N.d.).[1]

If outside spending does not move votes particularly effectively, then we face a puzzle as to why there is so much of it. Because candidates and the independent committees supporting them share a common goal—to get the candidate elected—we can think of campaign finance with outside spending as a classic free-rider problem.[2] From this strategic perspective, spending by the candidate or any individual outside group ought to crowd out spending by all the others. In the extreme, if expenditures from distinct sources were perfect substitutes, then we would only expect to see spending by the committee that can raise money most easily, with all others free-riding on that committee's efforts (Olson and Zeckhauser 1966). Clearly, this is not the case: about 45% of House candidates in contested elections since 2004 have had two or more distinct outside groups spending on their behalf.[3]

A potential resolution to this puzzle is that groups do not engage in outside spending solely for its electoral effects. Independent expenditures may also buy access to candidates, giving outside funders influence over policy favors in case of a victory by the candidate they backed. Competition for policy influence is a longstanding explanation for direct campaign contributions (Hall and Wayman 1990; Snyder 1990; Powell and Grimmer 2016; Stuckatz 2022; Thieme 2020), and a similar logic may apply to independent expenditures. Indeed,

[1]Petrova, Simonov and Snyder (N.d.) find an electoral effect of *Citizens United*, but show that it does not operate through partisan differences in advertising expenditures. Studies of state-level races have yielded mixed results, but generally find stronger effects than in the literature on federal races (La Raja and Schaffner 2014; Klumpp, Mialon and Williams 2016; Abdul-Razzak, Prato and Wolton 2020).

[2]The same issue arises in theories of lobbying, where individual firms may lobby for regulatory favors that yield sector-wide benefits (Bombardini and Trebbi 2012).

[3]See section 4 for details on the data.

recent research shows that the post-2010 deregulation of corporate campaign expenditures led to more distributive policy favors for corporations at the state level (Gilens, Patterson and Haines 2021).

The goal of this paper is to develop a model of outside spending that incorporates the incentive to free-ride, yet is consistent with observed data on spending patterns. To this end, we make both theoretical and empirical contributions. We develop a game-theoretical model of elections as contests with competition both between and within coalitions of spending committees. Each member of the coalition (including the candidate herself) benefits from the candidate winning, but may also value access to policy favors that are distributed in proportion to expenditures. To find the form of the model that best conforms with observed spending patterns, we structurally estimate its parameters using data on election outcomes, independent expenditures, and campaign spending in U.S. House elections from 2004 to 2020. Our estimates allow us to interpret electoral and spending data in terms of an underlying utility model for voters and spending committees. Additionally, we can use the equilibrium conditions of the model to address counterfactual questions about how spending and votes would change if the set of potential contributors changed.

Our initial estimates suggest that intra-coalition competition plays a small but nonzero role in the incentives for traditional political action committees (PACs) and 501(c) "dark money" groups. More surprisingly, we find the strongest incentives for intra-coalition competition by party organizations—even when accounting for the underlying competitiveness of the race, spending by party organizations appears to decrease relatively little with the extent of allied groups' spending. While it is unlikely that party organizations are seeking access to particularistic favors, our estimate here could reflect these institutions attempting to exert a moderating influence on candidates who would otherwise be reliant on special interest money. By contrast, spending by candidate committees is consistent with them being entirely office-motivated. We find that the same is true for Super PACs, the independent expenditure-only committees that came into being as a result of *Citizens United*.

Reduced form estimation of spending effectiveness is difficult because spending by one side may beget greater spending by the opposing side (Erikson and Palfrey 1998).[4] We thus follow in the tradition of structurally estimating game-theoretical models of campaign spending (Kawai 2014; Gordon and Hartmann 2016; Huang and He 2021; Zhao N.d.; Cox N.d.). The most closely related paper to this one is Cox (N.d.), which similarly estimates a structural model of outside spending in congressional elections. The key theoretical difference between our general election model and Cox's is that we introduce within-side competition over a private good.[5] Our analysis is also closely related to structural estimates of multiplayer contests in lobbying (Kang 2016) and international conflict (Kenkel and Ramsay 2022).

1 Formal Model

We model an electoral contest between two coalitions of spending committees, each trying to win votes for its preferred candidate from a large electorate. The two sides are labeled D and R (for Democrat and Republican), consisting of $N_D \geq 1$ and $N_R \geq 1$ committees respectively. Each spending committee is indexed by a pair ki, where $k \in \{D, R\}$ denotes party affiliation and $i \in \{1, \ldots, N_k\}$ counts the committees on each side. Let $\mathcal{K}$ denote the set of all committee indices. The electorate is a unit mass of voters, indexed by $h \in [0, 1] \equiv \mathcal{H}$.

The sequence of play is as follows:

1. **Committee shocks.** Nature draws each committee's marginal cost of fundraising for this election, $C_{ki} > 0$. These are publicly revealed to all players.

2. **Spending.** Each committee $ki \in \mathcal{K}$ simultaneously chooses how much to spend on the election, $S_{ki} \geq 0$.

3. **Voter shocks.** Nature draws a common shock to all voters' utility for voting for the

[4] Similar issues arise in the empirical study of lobbying (Bombardini and Trebbi 2020).

[5] Cox rationalizes spending by multiple players per side by assuming there are technological diminishing returns to spending by each individual committee, such that it is more electorally efficient for two separate committees to each spend \$5,000 than for a single committee to spend \$10,000.

Democrat, $\eta_D \in \mathbb{R}$, as well as idiosyncratic individual voter shocks, $\epsilon_{Dh} \in \mathbb{R}$. Nature also draws analogous shocks to utility for voting for the Republican, η_R and ϵ_{Rh}.

4. **Voting.** Each voter $h \in \mathcal{H}$ simultaneously votes for a candidate, $V_h \in \{D, R\}$.

Let Y_k denote the vote share of candidate k, so that $Y_k = \int_0^1 I(V_h = k)\, dh$. The Democrat wins the election if and only if $Y_D \geq \frac{1}{2}$.[6]

A voter's utility from voting for a candidate is a function of (1) the amount spent on the candidate, (2) the electorate-level bias in favor of (or against) that candidate, and (3) the voter's own idiosyncratic bias. Formally, voter h's utility from voting for candidate k given each committee's spending decision is

$$U_h^k(S) = \underbrace{\log\left(1 + \sum_{i=1}^{N_k} \theta_{ki} S_{ki}\right)}_{\text{spending}} + \overbrace{b_k + \eta_k}^{\text{electorate bias}} + \underbrace{\epsilon_{kh}}_{\text{individual bias}},$$

where $S = (S_{ki})_{ki \in \mathcal{K}}$ is the vector of all committees' spending choices. The electoral impact of each committee's spending is weighted by its *effectiveness*, $\theta_{ki} > 0$. Total spending is logged in the voter utility function, reflecting the diminishing marginal return of campaign spending (Hill et al. 2013; Jacobson 1990).[7] Moving on to the electorate-level bias, the term $b_k \in \mathbb{R}$ reflects district-level influences on the candidate's electoral prospects that are observable *ex ante*, such as incumbency. Finally, as noted above, the terms η_k and ϵ_{kh} are stochastic district-level and voter-level shocks respectively. We assume the electorate-level shocks η_k have a type 1 extreme value distribution. For the voter-level shocks, we only assume that each ϵ_{kh} is symmetrically distributed about 0.[8] All of these random shocks are independent of each other.

Victory by candidate k results in two separate benefits for each committee on k's side.

[6] We assume the shocks to voter utility have full support, so the tie-break rule is immaterial to equilibrium spending.

[7] This specification ensures that each committee's expected utility is strictly concave in its own spending, which in turn ensures that a pure strategy equilibrium exists.

[8] We do not currently place a distributional assumption on the voter shocks, but we may do so in the future to improve the efficiency of our estimates of spending effectiveness.

First, each committee receives a net benefit of value 1 when its preferred candidate wins the election. This is a public good from the perspective of the committees supporting the candidate, as the consumption value to each committee does not depend on the number of committees sharing the prize, nor on how much (or little) the committee spent on the election. The second component is a private good distributed among the committees on the winning side, which each committee values at a potentially different level, $\alpha_{ki} \geq 0$. We assume shares of this component are divided according to the effective amount of spending by each committee. This is a reduced-form representation of a process where spoils are divvied up among participants in rough proportion to their financial contribution to the electoral victory.[9]

Formally, conditional on k winning, the share of private good accruing to each supporting committee ki as a function of spending is

$$Q_{ki}(S) = \frac{\frac{1}{N_k} + \theta_{ki} S_{ki}}{\sum_{j=1}^{N_k} (\frac{1}{N_k} + \theta_{kj} S_{kj})} = \frac{\frac{1}{N_k} + \theta_{ki} S_{ki}}{1 + \sum_{j=1}^{N_k} \theta_{kj} S_{kj}}. \tag{1}$$

We add $\frac{1}{N_k}$ to effectiveness-weighted spending as a normalizing constant to ensure the model is continuous even at zero spending.

Combining the public and private components of victory, along with the marginal costs of spending, gives us the following expected utility function for each committee:

$$U_{ki}(S) = W_k(S) \left[1 + \alpha_{ki} Q_{ki}(S)\right] - C_{ki} S_{ki}, \tag{2}$$

where $W_k(S)$ is the probability of candidate k winning from the committees' perspective (before the shocks over voter preferences are drawn).

The final requirement to close the model is to specify the stochastic component of the committees' marginal cost of fundraising. These are revealed publicly before spending de-

[9]See Skaperdas (1998) for a model of alliances in contests with this structure, and Kenkel and Ramsay (2022) for closely related structural estimates in an international relations context.

cisions are made, meaning their distributional properties are immaterial to equilibrium behavior, but the stochastic specification is important for the estimation and interpretation of the structural model. We assume that each $C_{ki} = \bar{C}_{ki} + \nu_{ki}$, where $\bar{C}_{ki} > 0$ is a fixed constant and where each ν_{ki} is an independent draw from a distribution with zero mean, finite variance σ_ν^2, and support bounded below by $-\bar{C}_{ki}$.

Appendix A summarizes all notation in the model.

2 Analysis

We solve for subgame perfect equilibria via backward induction. Proofs appear in Appendix B.

2.1 Equilibrium

In the final stage of the game, all stochastic shocks have been realized, and each voter selects the candidate who provides the voter with greater utility. The Democratic candidate's vote share is therefore

$$Y_D = \Pr(U_h^R(S) \leq U_h^D(S))$$

where the probability is taken with respect to the distribution of idiosyncratic voter shocks ϵ_{Dh} and ϵ_{Rh} across the electorate. We can order voters' preferences by the difference between their shocks; the median voter, for whom $\epsilon_{Dh} - \epsilon_{Rh} = 0$, is decisive. Therefore, the Democrat wins the election if and only if

$$\log\left(1 + \sum_{i=1}^{N_D} \theta_{Di} S_{Di}\right) + b_D + \eta_D \geq \log\left(1 + \sum_{i=1}^{N_R} \theta_{Ri} S_{Ri}\right) + b_R + \eta_R. \tag{3}$$

Now we move up the game tree to analyze the committees' decision of how much to spend. At the time of choosing S_{ki}, the committees know the prior distribution of the electorate-wide shocks η_D and η_R, but not their realization. Following Equation 3, the Democrat's probability of victory after spending decisions have been made but before the electorate shocks have been realized is

$$W_D(S) = \Pr\left[\eta_R - \eta_D \leq \log\left(\frac{1 + \sum_{i=1}^{N_D} \theta_{Di} S_{Di}}{1 + \sum_{i=1}^{N_R} \theta_{Ri} S_{Ri}}\right) + b_D - b_R\right].$$

To economize on notation in the remainder of the analysis, define $B_D \equiv \exp(b_D)$ and $B_R \equiv \exp(b_R)$. Because the electorate-level shocks η_D and η_R have type 1 extreme value distributions, we obtain a closed-form expression for each candidate's *ex ante* victory probability as a function of spending.

Lemma 1. *The* ex ante *probability of victory by candidate k as a function of all committees' spending is*

$$W_k(S) = \frac{B_k(1 + \sum_{i=1}^{N_k} \theta_{ki} S_{ki})}{\sum_{l \in \{D,R\}} B_l(1 + \sum_{i=1}^{N_l} \theta_{li} S_{li})}. \tag{4}$$

A nice implication of this result is that in the absence of spending, each candidate's chance of winning is proportional to the exponent of their bias parameter:

$$W_k(0) = \frac{B_k}{B_k + B_{-k}}.$$

From here on, we will simplify the expressions in the analysis by working with a change of variables. Let s_{ki} denote the *effective* spending of each committee, accounting for the committee's effectiveness, the district bias in favor of the candidate it supports, and the normalizing constant:

$$s_{ki} = B_k\left(\frac{1}{N_k} + \theta_{ki} S_{ki}\right).$$

There is a one-to-one relationship between S_{ki} and the normalized s_{ki}, so without loss of generality we can formulate the committees' decision problem in terms of the normalized value. The change of variables allows us to express the private good share of each individual committee (Equation 1) and the probability of victory by candidate k (Equation 4) more simply as:

$$Q_{ki}(s) = \frac{s_{ki}}{\sum_{j=1}^{N_k} s_{kj}};$$
$$W_k(s) = \frac{\sum_{j=1}^{N_k} s_{kj}}{\sum_{l \in \{D,R\}} \sum_{j=1}^{N_l} s_{lj}}.$$

By rewriting the marginal cost of spending as $c_{ki} = \frac{C_{ki}}{B_k \theta_{ki}}$, we obtain the following expected utility function for committees in the spending stage:[10]

$$U_{ki}(s) = W_k(s)[1 + \alpha_{ki} Q_{ki}(s)] - c_{ki} s_{ki} = \frac{\alpha_{ki} s_{ki} + \sum_{j=1}^{N_k} s_{kj}}{\sum_{lj \in \mathcal{K}} s_{lj}} - c_{ki} s_{ki}.$$

We characterize the Nash equilibrium of the spending stage in terms of the first-order condition for each committee. Each candidate's expected utility is strictly concave in its own spending, so any equilibrium is in pure strategies, and a strategy profile is an equilibrium if and only if the first-order condition is satisfied for each committee.

Proposition 1. *s^* is an equilibrium of the spending stage if and only if*

$$\frac{\Sigma_{-k}^* + \alpha_{ki}(\Sigma^* - s_{ki}^*)}{(\Sigma^*)^2} = c_{ki} \tag{5}$$

for each ki such that $s_{ki}^* > 0$, and

$$\frac{\Sigma_{-k}^* + \alpha_{ki}(\Sigma^* - \frac{B_k}{N_k})}{(\Sigma^*)^2} \leq c_{ki} \tag{6}$$

for each ki such that $s_{ki}^* = 0$, where Σ^* denotes total overall effective spending ($\Sigma^* = \sum_{lj \in \mathcal{K}} s_{lj}^*$) and Σ_{-k}^* denotes total effective spending by the side opposite k ($\Sigma_{-k}^* = \sum_{lj: l \neq k} s_{lj}^*$).

Standard results guarantee the existence of a pure-strategy Nash equilibrium in the spending stage.[11] However, as in other contests with spillovers (Chowdhury and Sheremeta 2011),

[10]This differs from the original utility function by a constant $-\frac{C_{ki}}{N_k \theta_{ki}}$, but that is irrelevant to equilibrium behavior.

[11]Spending $S_{ki} > \frac{1+\alpha_{ki}}{C_{ki}}$ is strictly dominated, so without loss of generality we could assume the strategy

there is not necessarily a unique equilibrium.[12] Consequently, in the structural estimation of the model we use estimating equations that do not require computation of equilibrium spending at the given parameter values.

3 Structural Estimation

Our goal is to estimate the exogenous parameters of the model as functions of observable characteristics of districts, candidates, and committees in US Congressional races. We wish to find the values of the model parameters that best conform with the data we observe on election outcomes, committee spending, and exogenous characteristics of these races.

We observe a sequence of $m = 1, \ldots, M$ electoral contests. Each contest has $N_D^m \geq 1$ committees that spend on behalf of the Democrat, and $N_R^m \geq 1$ for the Republican. In each contest, we observe the Democrat's two-party share $Y_D^m \in [0, 1]$, as well as spending $S_{ki}^m > 0$ by each committee $ki \in \mathcal{K}^m$. The goal of the analysis is to estimate the following model parameters:

- The net district-level bias in favor of the Democrat, b_D^m (we normalize each $b_R^m = 0$ without loss of generality)
- The effectiveness of each spending committee, θ_{ki}^m
- The value each spending committee places on the excludable private good in case of electoral victory, α_{ki}^m
- The expected marginal cost of fundraising for each spending committee, $\bar{C}_{ki}^m$

For each race in the data, we observe a vector x^m of district- and candidate-level characteristics, as well as a collection of vectors $(z_{ki}^m)_{ki \in \mathcal{K}^m}$ of spending committee characteristics.

sets are $S_{ki} \in [0, \frac{1+\alpha_{ki}}{C_{ki}}]$.

[12] Trivially, there are infinitely many equilibria with the same total spending per side when $N_D = N_R > 1$, each $\alpha_{ki} = 0$, each committee has identical effectiveness and cost parameters, and the cost parameter is sufficiently low. We have not yet identified an example of multiplicity in the generic setting, but we cannot rule it out either.

Let $\mathcal{X}$ and $\mathcal{Z}$ denote the length of each x^m and each z_{ki}^m respectively. As detailed below, we write the model parameters as functions of these characteristics. We assume independence of all stochastic shocks across races. In addition, we assume strict exogeneity of our covariates: $(\eta_D^m, \eta_R^m, (\nu_{ki}^m))$ is independent of $(x^m, (z_{ki}^m))$.[13]

We estimate the parameters of the model separately in two stages. Because the estimates from the first stage are plugged into the second-stage model, we estimate standard errors via a cluster bootstrap (clustered by race).

3.1 Voter Model

We use the outcome of each electoral race to estimate the determinants of district Democratic bias and committee spending effectiveness. We model the bias as a linear function of district characteristics:

$$b_D^m = x^m \cdot \omega_b,$$

where ω_b is a coefficient vector to be estimated. We use a log-linear model for spending committee effectiveness:

$$\theta_{ki}^m = \exp(z_{ki}^m \cdot \omega_\theta),$$

where again ω_θ is an estimand. The log-linear specification ensures that all $\theta_{ki}^m > 0$.

We estimate the coefficients $\omega \equiv (\omega_b, \omega_\theta)$ by maximum likelihood. Let $W_D(S^m \mid \omega)$ denote the probability of Democratic victory in the m'th race as a function of the unknown parameters, taking all committees' spending as given at the observed values. From Lemma 1 above, we have

$$W_D(S^m \mid \omega) = \frac{\exp(x^m \cdot \omega_b)\left[1 + \sum_{i=1}^{N_D^m} \exp(z_{Di}^m \cdot \omega_\theta) S_{Di}^m\right]}{\exp(x^m \cdot \omega_b)\left[1 + \sum_{i=1}^{N_D^m} \exp(z_{Di}^m \cdot \omega_\theta) S_{Di}^m\right] + \left[1 + \sum_{i=1}^{N_R^m} \exp(z_{Ri}^m \cdot \omega_\theta) S_{Ri}^m\right]}.$$

[13] Our estimation framework can easily incorporate instrumental variables, so in principle we could relax this assumption if valid instruments were available.

We use this to define the log-likelihood function:

$$\log \ell(\omega \mid S, Y_D) = \sum_{m=1}^{M} \left[\mathbf{1}_{Y_D^m \geq \frac{1}{2}} \log W_D(S^m \mid \omega) + \mathbf{1}_{Y_D^m < \frac{1}{2}} \log(1 - W_D(S^m \mid \omega)) \right].$$

We obtain estimates $\hat{\omega}$ by numerically maximizing this function.

3.2 Spending Model

After obtaining estimates of the voter model parameters, we estimate the determinants of committee weights on the private good, α_{ki}^m, and of the expected marginal cost of fundraising, $\bar{C}_{ki}^m$. We model each of these as linear functions of committee characteristics,[14]

$$\alpha_{ki}^m = z_{ki}^m \cdot \gamma_\alpha,$$
$$\bar{C}_{ki}^m = z_{ki}^m \cdot \gamma_C,$$

where now $\gamma \equiv (\gamma_\alpha, \gamma_C)$ is the set of coefficients to estimate.

Whereas committee spending S_{ki}^m acted as an independent variable in the first-stage voter model, we now treat it as the dependent variable to be explained in the second-stage model. We estimate the relevant parameters by exploiting the first-order conditions for optimal spending (see Gordon and Hartmann 2016). For the ki'th committee in the m'th race, let $s_{ki}^m = B_k^m(\frac{1}{N_k^m} + \theta_{ki}^m S_{ki}^m)$, and define Σ^m and Σ_{-k}^m analogously. The first-order condition for positive spending by this committee (Equation 5) can be rearranged to:

$$\frac{B_k^m \theta_{ki}^m \Sigma_{-k}^m}{(\Sigma^m)^2} + \frac{B_k^m \theta_{ki}^m [\Sigma^m - s_{ki}^m]}{(\Sigma^m)^2} \alpha_{ki}^m - \bar{C}_{ki}^m = \nu_{ki}^m.$$

Let DU_{ki}^m denote the left-hand side of this equality. Our strict exogeneity assumption on ν_{ki}^m

[14] The implicit assumption that all committee-level covariates enter both equations is merely for expositional ease.

implies

$$\mathbb{E}\left[f(x^m, z^m) DU_{ki}^m\right] = 0 \tag{7}$$

for any (potentially vector-valued) function f of the associated district and committee characteristics.

We estimate the coefficients for the private good weights and the marginal costs of fundraising via a sample analogue to the population moment condition (Equation 7). First, we plug in the estimates of the district bias and effectiveness parameters from the first-stage model to create an empirical analogue of DU_{ki}^m:

$$\widehat{DU}_{ki}^m = \frac{\hat{B}_k^m \hat{\theta}_{ki}^m \hat{\Sigma}_{-k}^m}{(\hat{\Sigma}^m)^2} + \frac{\hat{B}_k^m \hat{\theta}_{ki}^m [\hat{\Sigma}^m - \hat{s}_{ki}^m]}{(\hat{\Sigma}^m)^2}(z_{ki}^m \cdot \gamma_\alpha) - (z_{ki}^m \cdot \gamma_C),$$

where $\hat{B}_k^m = \exp[\mathbf{1}_{k=D}(x^m \cdot \hat{\omega}_b)]$, $\hat{\theta}_{ki}^m = \exp(z_{ki}^m \cdot \hat{\omega}_\theta)$, and $\hat{s}_{ki}^m$, $\hat{\Sigma}^m$, and $\hat{\Sigma}_{-k}^m$ are defined in terms of observed spending and these estimated quantities. Notice that $\widehat{DU}_{ki}^m$ is an affine function of the model coefficients: $\widehat{DU}_{ki}^m = a_{ki}^m + h_{ki}^m \cdot \gamma$. We stack these components across all spending committees to form the $\mathcal{M} \times 1$ vector $\mathbf{a}$ and the $\mathcal{M} \times 2\mathcal{Z}$ matrix $\mathbf{H}$, where $\mathcal{M} \equiv \sum_{m=1}^{M}(N_D^m + N_R^m)$ denotes the total number of spending committees.

We define a method of moments estimator with this sample analogue of the first-order condition. Because $\mathbf{H}$ contains endogenous spending decisions, we cannot employ the typical moment condition for a linear model, $\mathbf{H}^\top[\mathbf{a} + \mathbf{H}\gamma] = \mathbf{0}$. The problematic part is the multiple on each covariate in the equation for α_{ki}^m, which expands to

$$\frac{\hat{B}_k^m \hat{\theta}_{ki}^m}{\sum_{lj \in \mathcal{K}^m} \hat{B}_l^m (\frac{1}{N_l^m} + \hat{\theta}_{lj}^m S_{lj}^m)} \cdot \frac{\sum_{lj \neq ki} \hat{B}_l^m (\frac{1}{N_l^m} + \hat{\theta}_{lj}^m S_{lj}^m)}{\sum_{lj \in \mathcal{K}^m} \hat{B}_l^m (\frac{1}{N_l^m} + \hat{\theta}_{lj}^m S_{lj}^m)}.$$

We instrument for this term using each committee's bias-weighted spending effectiveness relative to others in the same race:

$$\pi_{ki}^m = \frac{\hat{B}_k^m \hat{\theta}_{ki}^m}{\sum_{lj \in \mathcal{K}^m} \hat{B}_l^m \hat{\theta}_{lj}^m}.$$

Specifically, we construct an $\mathcal{M} \times 2\mathcal{Z}$ instrument matrix $\mathbf{P}$ by stacking the vectors $(\pi_{ki}^m(1 - \pi_{ki}^m)z_{ki}^m, -z_{ki}^m)$ across all spending committees. This gives us the sample moment condition

$$\mathbf{P}^\top[\mathbf{a} + \mathbf{H}\gamma] = \mathbf{0}. \tag{8}$$

Identification requires the usual full rank condition. At a minimum, this requires that the matrix of committee-level covariates have full rank and that π_{ki}^m not be constant across observations.

We may estimate γ by solving for it in Equation 8. Depending on the sample, however, this may yield negative values of α_{ki}^m or $\bar{C}_{ki}^m$ for some committees in the data. To ensure valid estimates of the underlying utility parameters for all observations, we instead estimate γ by solving the quadratic program

$$\begin{aligned} \min_{\gamma} \quad & [\mathbf{a} + \mathbf{H}\gamma]^\top \mathbf{P}\mathbf{P}^\top [\mathbf{a} + \mathbf{H}\gamma] \\ \text{s.t.} \quad & z_{ki}^m \cdot \gamma_\alpha \geq 0 \quad \forall ki \; \forall m \\ & z_{ki}^m \cdot \gamma_C \geq 0 \quad \forall ki \; \forall m. \end{aligned} \tag{9}$$

We estimate inferential statistics via a nonparametric bootstrap clustered at the race level.[15]

4 Data

We structurally estimate the formal model using race- and committee-level datasets for each competitive general election for the U.S. House from 2004 through 2020.[16] Our sample consists of $M = 2{,}276$ races, with spending from a total of $\mathcal{M} = 15,776$ committees.

[15]Bootstrap estimates may be invalid when the constraints in Equation 9 bind (Andrews 2000). A task for future iterations of the project is to use newly developed methods for inference on constrained estimators (e.g., Hsieh, Shi and Shum 2022; Li 2023).

[16]2004 is the earliest election cycle for which the FEC has published expenditure data.

4.1 Sample and Sources

We include House races between a single Republican and a single Democrat between 2004 and 2020. After dropping uncontested races and those with more than one candidate from a single major party, we are left with 200–300 races for each of the nine electoral cycles in our sample.[17] For each of these races, we then collect Federal Election Commission (FEC) data on general election spending by every campaign committee, as well as every committee that reported outside spending in the race. Besides the campaign committees, our sample includes political party committees, traditional political action committees (PACs), Super PACs, and "dark money" 501(c) organizations.

Our key dependent variable in the race-level dataset is whether the Democratic candidate won, which we obtain using vote share data from the MIT Election Data and Science Lab (2022). Covariates include race and candidate characteristics that might affect the district-level bias in favor of one side or the other. We use the FEC codings to categorize each race as one of three types: Democratic incumbent, Republican incumbent, or open seat. We also include district ideology estimates from Tausanovitch and Warshaw (2013).

To create our committee-level spending dataset, we merge FEC records on candidate filings, committee filings, independent expenditures, and campaign spending for each race in the sample. The key dependent variable in the committee-level dataset is total spending on the race by each committee, measured in millions of current-year USD. For campaign committees, we code spending as total advertising expenditures in the race.[18] The vast majority of candidates in our sample recorded a nonzero level of advertising-related spending.

[17] The data on candidates include a number of problematic cases, such as candidates who appear twice in one cycle as two different candidate types (e.g. incumbent and open-seat) as well as races wherein one candidate is classified as an open-seat and the other is classified as a challenger. These constitute less than 10% of cases. We drop them in the current iteration of the project but plan to correct and include them in future iterations.

[18] Like others (e.g., Ansolabehere and Gerber 1994; Cox N.d.), we focus on campaign committees' advertisement spending because 1) direct campaign communication, not total spending, is the relevant quantity when considering campaign spending effects, and 2) virtually all independent expenditures are for the purpose of advertising, so this maximizes the comparability to the spending of other committee types. We use string matching to identify likely advertising expenditures from the designated purpose in campaign committees' disbursement reports.

In addition to the campaign committees of candidates in our races, we include groups that made independent expenditures, which expressly advocate the election or defeat of a given candidate without any coordination with candidates in the race. While party committees and traditional political action committees were subject to fundraising limits during our entire period of study, 2010 marked a turning point for the ease with which Super PACs and non-PAC groups could engage in this type of spending. Prior to *Citizens United v. Federal Election Commission*, groups such as labor unions, social welfare organizations, and business associations were forbidden from using general treasury funds to make independent expenditures, and the ruling allowed them to do so.[19] The same year, *SpeechNOW v. FEC* allowed for the formation of independent expenditure–only Super PACs, to which individuals and corporations may contribute unlimited amounts.

Because groups that make independent expenditures must file a report disclosing the transaction amount and which candidate the expenditure advocates for or against, we are able to aggregate the independent expenditures for every group engaged in outside spending in our races. Using FEC committee codes, we classify groups as party committees, PACs, Super PACs, or nonprofit "dark money" groups.[20] Thus, each of the roughly 11,500 non-candidate entries in the committee dataset contains the amount that a given group independently spent on the side of a candidate in a given district in a given year.[21] Our committee-level covariates are the committee type and the candidate alignment (Democratic or Republican) of the committee in a given race. Table 1 reports the distribution of committee types by partisan alignment, as well as total spending in each case. With the exception of traditional PACs, there tend to be greater numbers of committees spending on behalf of

[19] 501(c)(4) social welfare nonprofits, 501(c)(5) labor groups, and 501(c)(6) business associations are allowed to engage in political activities so long as it is not their primary purpose. They are commonly referred to as "dark money" groups given that they are not required to disclose their donors as their purpose is primarily non-political.

[20] In future iterations, we plan on breaking down and re-classifying committees to examine, for instance, how the motivations of party leadership–tied Super PACs differ from those of other Super PACs, as well as differences across 501(c) labor, business, and social welfare organizations.

[21] For example, in 2010, NARAL Pro-Choice America, a 501(c)(4) "dark money" group, independently spent a total of $4,985 on the side of the Democratic candidate in NY-20.

Democratic candidates. However, except for PACs and to a lesser degree party committees, the Republican-aligned groups tend to spend more in total.

	# Committees		Total Spending	
Type	Dem	Rep	Dem	Rep
Candidate	2115	2115	605.9	630.1
Party	516	452	441.1	385.5
Super PAC	2234	1336	297.8	393.7
PAC	2138	2459	204.9	52.5
Dark Money	1345	1067	43.3	79.6

Table 1. Number of committees and total spending (millions USD) by committee type and partisan alignment.

4.2 Specification

We must decide which race- and/or committee-level covariates to include in the equations for each model parameter we wish to estimate: district bias b_D^m, spending effectiveness θ_{ki}^m, private good weight α_{ki}^m, and expected fundraising cost $\bar{C}_{ki}^m$. Our goal is to include the most substantively important factors in each equation, but to avoid unnecessarily adding terms that reduce interpretability or statistical power.

District Democratic bias. We include three sets of terms in the equation for b_D^m, the district's bias toward the Democratic candidate. The first is the race type; we expect a positive bias for Democratic incumbents, a negative bias for Republican incumbents, and closer to even for open seats. We also include district ideology, where we expect higher values (more conservative) to result in bias against the Democrat. Finally, to account for cyclical national trends, we include election cycle fixed effects in the bias equation.

Spending effectiveness. Due to statistical power concerns, we only include candidate-level covariates in the equation for spending effectiveness, θ_{ki}^m.[22] Specifically, we include

[22] Pre-testing showed that the estimator was unable to recover committee-level effectiveness determinants even in relatively clean simulated data. We suspect that this is due to the two-step nature of the estimator,

the partisan alignment of the committees, so that the marginal effects of spending on vote share may differ for committees allied with Democrats versus Republicans. Additionally, to account for inflation as well as potential secular declines in campaign spending effectiveness,[23] we include election cycle fixed effects in the effectiveness equation as well.

Private good weight. We model the committees' utility weight on the excludable private good, α_{ki}^m, simply as a function of the type of committee: candidate, party, traditional PAC, Super PAC, or dark money. *Ex ante* we expect candidates and parties to be relatively office-motivated, and thus for their values to be relatively low. Committees' private good weight may also vary by the ease with which candidates can identify the group's policy aims and, consequently, dole out policy favors. Because dark money committees are typically established, easily-identified organizations that exist for the welfare of a given group such as small business or agricultural workers, their positions on policy proposals that pertain to their group may be more easily inferred than, for instance, independent Super PACs with names such as 'No Labels Action'.[24]

Cost of fundraising. We model the expected marginal cost of campaign spending, $\bar{C}_{ki}^m$, as a function of committee and cyclical factors. We include the partisan alignment of the committee to capture the possibility—suggested by Table 1—that Democratic groups find it more difficult to raise funds. Again to control for inflation and other potential secular trends in the fundraising environment, we include election cycle fixed effects. Finally, we include the interaction between partisan alignment and the cycle fixed effects.[25]

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where we are identifying the determinants of effectiveness solely from vote shares, ignoring the information in the optimal choice of spending itself. We plan to experiment with full-information estimators in future iterations of the project, though these pose severe numerical challenges as our moment conditions are highly nonlinear in θ_{ki}^m. As an intermediate step, we may impose a type 1 extreme value model on the idiosyncratic voter shocks, similar to Gordon and Hartmann (2016) and Cox (N.d.), so that we can extract information from vote share data instead of just election winners.

[23] `https://split-ticket.org/2022/04/04/the-declining-value-of-a-dollar/`

[24] `https://www.opensecrets.org/outsidespending/detail.php?cmte=C00680983&cycle=2018`

[25] Given how we construct the moment conditions for estimation, the saturated model ensures that the positivity constraint in Equation 9 will never bind.

Term	Estimate	Std. Err	95% Interval
b_D: *District-level Democratic bias*			
Intercept	2.32	0.48	[1.35, 3.29]
Open Seat	−3.47	0.31	[−4.18, −2.96]
R Incumbent	−5.92	0.34	[−6.82, −5.49]
District Ideology	−8.63	0.65	[−10.27, −7.74]
+ Cycle fixed effects			
$\log \theta_{ki}$: *Committee-level effectiveness*			
Intercept	0.60	20.59	[−18.11, 2.45]
D-aligned Committee	1.27	20.05	[−0.56, 10.84]
+ Cycle fixed effects			
α_{ki}: *Committee-level private good weight*			
Candidate Committee	0.00	0.00	[0.00, 0.00]
Party Committee	0.22	0.45	[0.05, 1.05]
Dark Money	0.02	0.05	[0.00, 0.18]
Traditional PAC	0.01	0.03	[0.00, 0.09]
Super PAC	0.00	0.07	[0.00, 0.21]
$\bar{C}_{ki}$: *Committee-level expected fundraising cost*			
+ Cycle–partisanship fixed effects			
Races	2276		
Committees	15776		

Table 2. Structural estimates for U.S. House races, 2004–2020. Standard errors are estimated via bootstrap and clustered at the race level. Confidence interval is the bootstrap percentile interval.

5 Structural Estimation Results

Table 2 reports the results of the structural analysis.[26] The top two panels are estimated from the vote share data (treating spending as fixed), while the bottom two are estimated from the spending data (plugging in the estimates from the vote share model).

We have strong and intuitive findings for the determinants of district-level biases. There is a substantial incumbent advantage, with a slightly larger magnitude for Democrats (relative to an open-seat race) than for Republicans. Note from Equation 4 that the district bias essentially acts as a force multiplier (or deflator, if net negative) for the Democrat in the spending contest. Because equilibrium effort in contests is greatest when the two sides are

[26] Full results appear in Appendix C.

equally effective, the coefficients here imply relatively high spending on open-seat races. We also find, again in line with intuition, a negative relationship between ideological conservatism of the constituency and the district-level Democratic bias.

In our estimates of spending effectiveness, we find a Democratic advantage. However, all spending effectiveness parameters are highly imprecisely estimated, including the cycle fixed effects not reported here.

Turning to the equation for α_{ki}, the weight each committee places on the private-good component of its utility, we see some intuitive patterns and some surprising ones. Again, most of these parameters are estimated imprecisely, so they should be interpreted cautiously.[27] We find $\alpha_{ki} = 0$ for candidate committees, meaning their spending is consistent with pure office motivation. As we show below, this implies that candidate committees always have strategic substitutes with aligned committees—i.e., additional spending by a friendly committee decreases a candidate's incentive to fundraise. Also intuitively, we find positive (albeit small) private good weights for dark money groups and traditional political action committees.

The more surprising results concern Super PACs and party committees. For Super PACs, we find zero weight on the private good, meaning their incentive structure looks identical to that of candidate committees. The little weight that Super PACs place upon the private good component may be due to many Super PACs' general partisan aim of protecting copartisan incumbents, helping to flip seats held by out-partisans, and winning competitive open seats. On the other hand, for party committees, we find a large private good weight, indicating that spending by copartisans does relatively little to dissuade party committees from contributing to a race. Though the standard error is large, the confidence interval does not include a private good weight of 0 for party committees—the only type of committee for which this is the case. Of course, it is unlikely that party committees are competing with coalition partners for distributive favors. However, to the extent that parties use general

[27]The moment conditions used to estimate the spending model are functions of the first-stage voter model estimates, so imprecision in the effectiveness parameter estimates carries through to the estimates of α_{ki} and $\bar{C}_{ki}$.

Term	Estimate	Std. Err	95% Interval
α_{ki}: *Committee-level private good weight*			
D Candidate Committee	0.00	0.02	[0.00, 0.00]
R Candidate Committee	0.00	0.06	[0.00, 0.02]
D Party Committee	0.33	0.77	[0.08, 2.21]
R Party Committee	0.05	6.50	[0.00, 0.73]
D Dark Money	0.02	0.07	[0.00, 0.25]
R Dark Money	0.04	0.14	[0.01, 0.43]
D Traditional PAC	0.01	0.07	[0.00, 0.19]
R Traditional PAC	0.01	0.04	[0.00, 0.05]
D Super PAC	0.00	0.15	[0.00, 0.46]
R Super PAC	0.04	0.20	[0.00, 0.18]

Table 3. Estimates of private good weights from an alternative specification interacting committee type with partisan affiliation.

election spending to influence candidate selection or behavior, this may result in patterns of spending that appear as if party committees are in competition with their copartisans.

As an auxiliary analysis, we rerun the model allowing α_{ki} to vary by both committee type and partisan affiliation. Table 3 reports the results.[28] The auxiliary analysis gives additional context for two key features of the main model estimates. First, the apparent intra-coalition competition by party committees is much stronger for the Democratic Party than for its Republican counterpart. The estimated α_{ki} for the GOP is similar in magnitude to other non-candidate committees, while the estimate for Democrats is considerably larger. In fact, the lower bound of the confidence interval for the Democratic estimate exceeds the point estimate of α_{ki} for every other committee type. Second, the lack of intra-party competition among Super PACs may be specific to Democrats. For Republican Super PACs, our estimate of α_{ki} is positive, roughly the same as the estimate for dark money groups, and larger than that of traditional PACs.

[28]We use the same first-stage model as in the main specification, so the estimates for district bias and committee effectiveness are unchanged.

6 Post-Estimation Analysis

To further analyze strategic incentives and behavior in elections with outside spending, we plug our parameter estimates for specific observations back into the formal model. Throughout this section, we use the estimates from the main model reported in Table 2.

6.1 Strategic Interdependence

To quantify incentives for competition and free-riding in our set of House races, we calculate the effect of spending by other committees on each committee's incentive to spend. Specifically, we calculate cross-partials: the second derivative of each candidate's utility function with respect to its own spending and that of a committee on the same side or the opposing side, each evaluated at the observed levels of spending in the data. There are *strategic complements* when the cross-partial is positive, and *strategic substitutes* otherwise. When considering the cross-effects between committees with the same partisan alignment, strategic substitutes indicate the free-riding incentive.

First consider the effect of copartisan spending on a committee's marginal benefit from its own spending. Returning to the formal model, and again working with the normalized spending parameters, we have the following equation for the effect of an aligned group's spending:

$$\frac{\partial^2 U_{ki}(s)}{\partial s_{ki} \partial s_{kj}} = \frac{\alpha_{ki}(2s^*_{ki} - \Sigma^*) - 2\Sigma^*_{-k}}{(\Sigma^*)^3}.$$

For a committee that places no weight on the private good, so that $\alpha_{ki} = 0$, this cross-partial is guaranteed to be negative, indicating strategic substitutes. If $\alpha_{ki} > 0$, then it is possible for copartisan spending to heighten a committee's incentive to spend—meaning there are strategic complements—but only under stringent conditions. A necessary condition is that the committee's bias/effectiveness-weighted spending must equal at least half of the total among all committees (including those on the other side). Thus, we should expect strate-

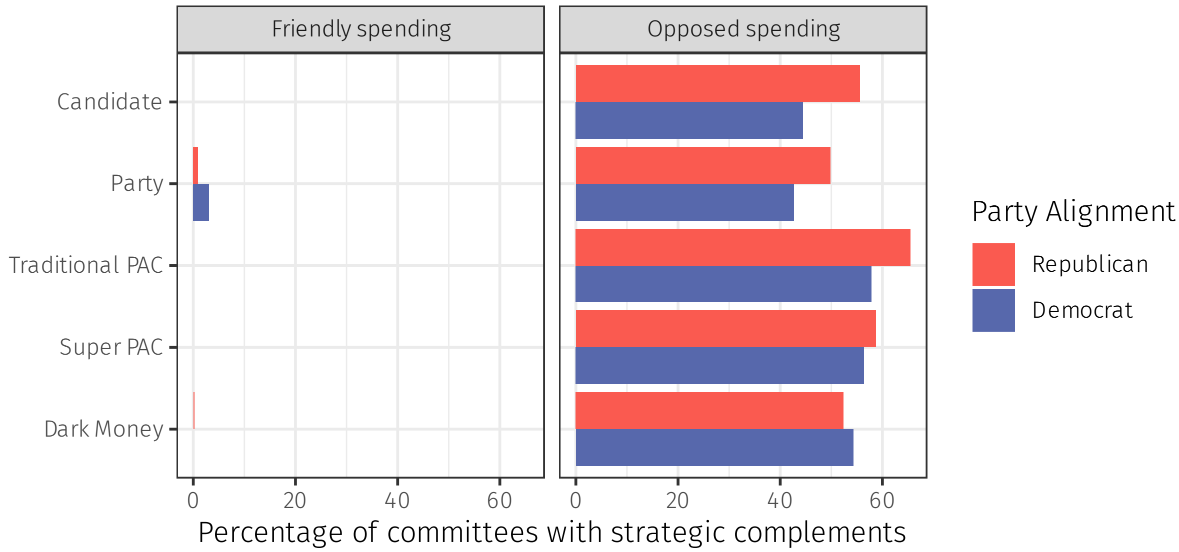

Figure 1. Frequency of strategic complements by party affiliation and committee type among races in our data, calculated at observed levels of spending.

gic substitutes among copartisans except in the special case of (1) private good–motivated committees and (2) highly imbalanced spending allocations.

We have a similar mathematical condition for the effect of the other side's spending on a committee's marginal benefit from its own spending:

$$\frac{\partial^2 U_{ki}(s)}{\partial s_{ki} \partial s_{-kj}} = \frac{\alpha_{ki}(2s^*_{ki} - \Sigma^*) + \Sigma^*_k - \Sigma^*_{-k}}{(\Sigma^*)^3}.$$

For a purely office-motivated committee, the effect of opposed spending depends entirely on relative spending. Opposed spending increases the incentive to spend for a coalition that is ahead (a competition effect), but decreases it for a coalition that is behind (a discouragement effect). Private good motivation may exacerbate or counteract these effects, depending on how the committee's own spending compares to that of all other groups.

We apply these formulas to each committee in our data to determine how their incentive to spend would be affected by marginal increases or decreases in other committees' allocations. Figure 1 plots the proportion of committees of each type and partisan alignment that have strategic complements with friendly and opposed committees. The results are stark. We see the overwhelming presence of a free-riding incentive within coalitions. Even for party committees, which have the strongest private good motivation per our estimates, only a small percentage have strategic complements with copartisan spending. And outside of party committees, we only see a hint of intra-coalition strategic complementarity for a small number of Republican dark money groups.

We see more variation in the strategic effects of opposed sides' spending. Depending on committees type and partisanship, about 40–60% of committees' marginal utility of spending increases with total spending by the opposed side. These strategic complements are more prevalent among outside groups than among candidate and party committees. Additionally, with the minor exception of dark money groups, these strategic complements are more common for Republican-aligned committees than for Democratic-aligned ones. In part this may reflect the Republican fundraising advantage in individual races in our sample—remember that the higher-spending coalition has strategic complements with opposed spending when α_{ki} is close to 0, as is the case for most committees in our sample.

6.2 Counterfactual Vote Shares

A major benefit of the structural modeling approach is that we can combine the parameter estimates with the underlying equilibrium model to simulate answers to counterfactual questions. Here we address a simple one: how would electoral outcomes change, if at all, if candidates did not expect any outside spending to take place? To answer this, we run a counterfactual experiment on the set of races we observe with outside spending. We take every player besides the candidate committees out of the game, and calculate equilibrium spending in the resulting two-player contests.[29] We then plug equilibrium spending into the voter's utility function to calculate counterfactual election outcomes in each race.

Figure 2 graphs the results of the counterfactual experiment, comparing actual vote shares to counterfactual win probabilities. When we account for adjustments in strategic behavior,

[29] This particular experiment allows us to sidestep the multiplicity issue: with one committee per side, the spending game reduces to an asymmetric Tullock contest, which has a unique equilibrium.

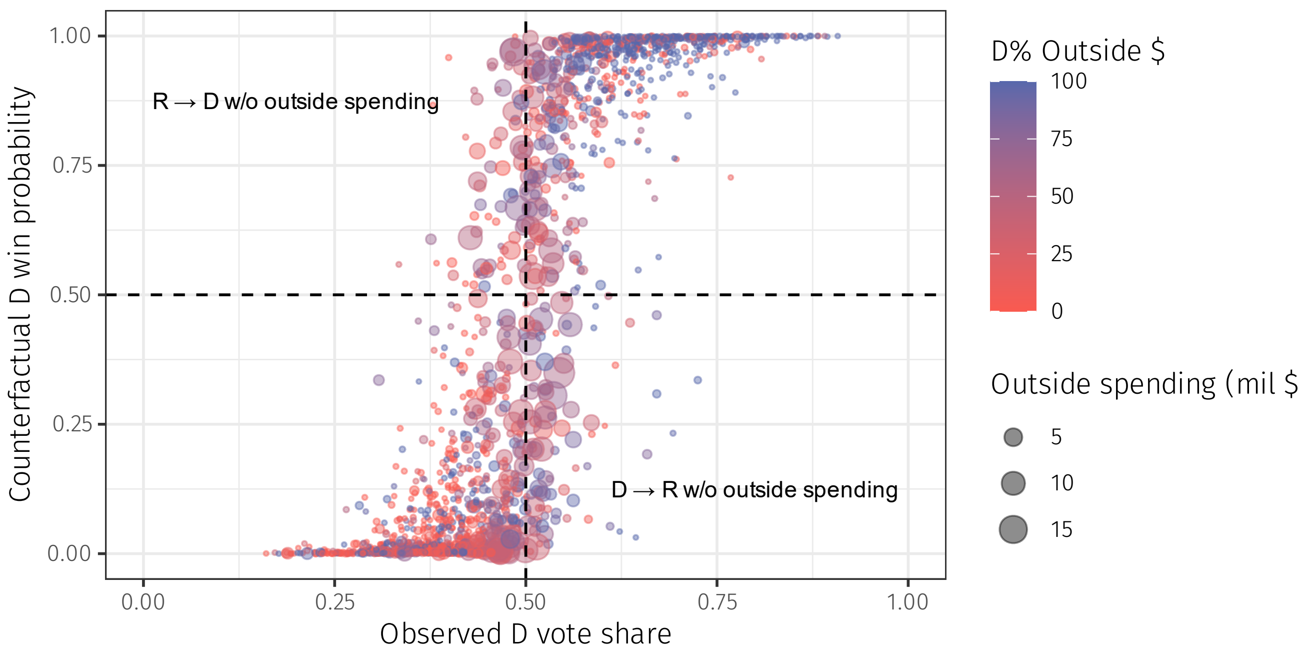

Figure 2. Electoral results in the counterfactual experiment removing non-candidate committees from each electoral contest in the data.

we estimate that electoral outcomes would not shift much in the absence of outside spending. Removing outside spending would slightly disadvantage Democrats, who win 44.3% of races in the actual data but are favored in only 43.0% of races in the counterfactual experiment. But the effect is not uniform, as is evident from the northwest and southeast quadrants of Figure 2. We estimate that eliminating outside spending would make Democrats favored in 99 races that Republicans won, but would make Republicans favored in 72 races that Democrats won.

These results highlight the importance of equilibrium reasoning when evaluating the effects of campaign spending on political outcomes. The electoral consequences of lowering spending might appear more drastic in a partial equilibrium analysis, without considering how the removal of one actor would affect others' incentives to raise and spend campaign funds. Once we approach this question through the lens of a full equilibrium model, we see that high absolute levels of outside spending may not translate into large effects on final electoral outcomes.

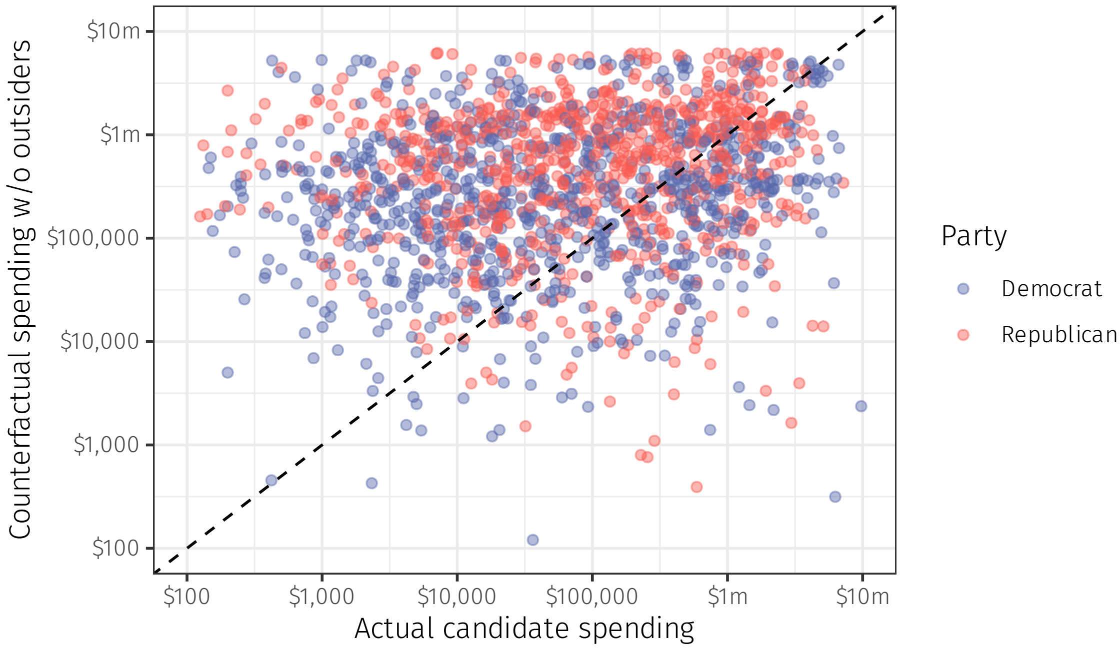

Figure 3. Spending in the counterfactual experiment removing non-candidate committees from each electoral contest in the data. This plot only shows candidates with positive predicted spending in the counterfactual (45% of observations).

6.3 Counterfactual Spending

In a final counterfactual experiment, we analyze how the amount that candidates raise would change in the absence of all outside spending. As in the previous experiment, we restrict to the subset of observations with positive outside spending on at least one side of the election.

Compared to actual fundraising by candidates in the sample, we find a kind of polarization of predicted fundraising in the counterfactual analysis. We estimate that 68% of candidates would spend *less* in the absence of outside spending. On its face, this may seem to contradict our earlier finding of a substantial free-riding incentive for candidates, given our estimate of $\alpha_{ki} = 0$ for them. However, candidate spending in this counterfactual is responding not only to the reduction of outside support on one's own side, but also the lack of outside spending for the opposed side. In fact, much of the predicted reduction in outside spending is driven by candidates who would not have an incentive to fundraise at all in equilibrium, according to the counterfactual model. We estimate that 55% of these candidates would not spend in equilibrium in the absence of all outside spending.

For the 45% of candidates who would spend, however, we find that the lack of outside support would lead to greater expenditures by the candidate. Figure 3 illustrates the results for these candidates. Within this subset, we estimate that 71% would raise more money in the absence of all outside spending than they did in the actual data. On average, these candidates raise about $400,000 more in the counterfactual than in the observed data.

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Appendix

Contents

A Summary of Notation

A.1 Formal Model

Symbol	Meaning
D	Democratic candidate
R	Republican candidate
$k, l \in \{D, R\}$	Generic candidate
$N_k \geq 1$	Number of committees supporting candidate k
ki	Generic spending committee
$\mathcal{K}$	Set of all spending committees
$h \in [0, 1]$	Generic voter
$\mathcal{H}$	Set of all voters
$S_{ki} \geq 0$	Spending by committee ki
$S \in \mathbb{R}_+^{\lvert\mathcal{K}\rvert}$	Vector of spending decisions
$V_h \in \{D, R\}$	Vote choice by voter h
$Y_k \in [0, 1]$	Vote share of candidate k
$b_k \in \mathbb{R}$	Systematic electorate-level bias toward/against k
$B_k \in \mathbb{R}_+$	$\exp(b_k)$
$\eta_k \in \mathbb{R}$	Stochastic electorate-level bias toward/against k
$\epsilon_{kh} \in \mathbb{R}$	Stochastic voter h bias toward/against k
$\theta_{ki} > 0$	Spending effectiveness of committee ki
$\bar{C}_{ki} > 0$	ki's expected marginal cost of fundraising
$\nu_{ki} \in (-\bar{C}_{ki}, \infty)$	Stochastic shock to ki's marginal cost
$\sigma^2 > 0$	Variance of distribution of ν_{ki}
$C_{ki} > 0$	ki's marginal cost of fundraising ($C_{ki} = \bar{C}_{ki} + \nu_{ki}$)
$U_h^k(S)$	Voter h's utility from voting for k, given spending

$W_k(S)$	Probability of victory by k, given spending
$\alpha_{ki} > 0$	Value of private good to committee ki
$Q_{ki}(S)$	ki's share of private good, conditional on k winning
$U_{ki}(S)$	ki's expected utility
$s_{ki} \geq 0$	Normalized spending $s_{ki} = B_k(\frac{1}{N_k} + \theta_{ki}S_{ki})$
$\Sigma > 0$	Total normalized spending: $\Sigma = \sum_{ki \in \mathcal{K}} s_{ki}$
$\Sigma_k > 0$	Total normalized spending by k: $\Sigma = \sum_{i=1}^{N_k} s_{ki}$
$c_{ki} > 0$	Normalized costs: $c_{ki} = \frac{C_{ki}}{B_k \theta_{ki}}$

A.2 Structural Model

Symbol	Meaning
$m \in \{1, \ldots, M\}$	Observation index
M	Total districts in data
$\mathcal{M}$	Total committees in data
$x^m \in R^{\mathcal{X}}$	District covariates
$z_{ki}^m \in \mathbb{R}^{\mathcal{Z}}$	Committee covariates
$\omega_b \in \mathbb{R}^{\mathcal{X}}$	District bias coefficients
$\omega_\theta \in \mathbb{R}^{\mathcal{Z}}$	Committee effectiveness coefficients
$\gamma_\alpha \in \mathbb{R}^{\mathcal{Z}}$	Private good weight coefficients
$\gamma_C \in \mathbb{R}^{\mathcal{Z}}$	Cost function coefficients
DU_{ki}^m	Fixed component of committee's FOC
$\mathbf{H} \in \mathbb{R}^{\mathcal{M} \times 2\mathcal{Z}}$	Jacobian of committee FOCs
$\mathbf{a} \in \mathbb{R}^{\mathcal{M}}$	Constant in committee FOCs
π_{ki}^m	Committee's relative spending effectiveness: $\pi_{ki}^m \propto B_k^m \theta_{ki}^m$, $\sum_{ki \in \mathcal{K}^m} \pi_{ki}^m = 1$
$\mathbf{P} \in \mathbb{R}^{\mathcal{M} \times 2\mathcal{Z}}$	Instrumented FOC Jacobian

B Formal Model Analysis

B.1 Proof of Lemma 1

Lemma 1. *The* ex ante *probability of victory by candidate k as a function of all committees' spending is*

$$W_k(S) = \frac{B_k(1 + \sum_{i=1}^{N_k} \theta_{ki} S_{ki})}{\sum_{l \in \{D,R\}} B_l(1 + \sum_{i=1}^{N_l} \theta_{li} S_{li})}. \tag{4}$$

Proof. Each voter $h \in \mathcal{H}$ votes for the Democrat if and only if

$$\epsilon_{Rh} - \epsilon_{Dh} \leq \log\left(\frac{1 + \sum_{i=1}^{N_D} \theta_{Di} S_{Di}}{1 + \sum_{i=1}^{N_R} \theta_{Ri} S_{Ri}}\right) + b_D - b_R + \eta_D - \eta_R. \tag{A.1}$$

Because ϵ_{Dh} and ϵ_{Rh} are independent and symmetric about 0, their difference is symmetric about 0 as well. Consequently, the Democrat receives the majority of votes if and only if the RHS of Equation A.1 is non-negative. To characterize the *ex ante* probability of Democratic victory, note that $\eta_R - \eta_D$ has a standard logistic distribution. Therefore, we have

$$\begin{aligned} W_D(S) &= \Lambda\left(\log\left(\frac{1 + \sum_{i=1}^{N_D} \theta_{Di} S_{Di}}{1 + \sum_{i=1}^{N_R} \theta_{Ri} S_{Ri}}\right) + b_D - b_R\right) \\ &= \Lambda\left(\log\left(\frac{B_D(1 + \sum_{i=1}^{N_D} \theta_{Di} S_{Di})}{B_R(1 + \sum_{i=1}^{N_R} \theta_{Ri} S_{Ri})}\right)\right) \\ &= \frac{B_D(1 + \sum_{i=1}^{N_D} \theta_{Di} S_{Di})}{B_D(1 + \sum_{i=1}^{N_D} \theta_{Di} S_{Di}) + B_R(1 + \sum_{i=1}^{N_R} \theta_{Ri} S_{Ri})}. \end{aligned}$$

□

B.2 Proof of Proposition 1

Proposition 1. *s^* is an equilibrium of the spending stage if and only if*

$$\frac{\Sigma^*_{-k} + \alpha_{ki}(\Sigma^* - s^*_{ki})}{(\Sigma^*)^2} = c_{ki} \tag{5}$$

for each ki such that $s^*_{ki} > 0$, and

$$\frac{\Sigma^*_{-k} + \alpha_{ki}(\Sigma^* - \frac{B_k}{N_k})}{(\Sigma^*)^2} \leq c_{ki} \tag{6}$$

for each ki such that $s^*_{ki} = 0$, where Σ^* denotes total overall effective spending ($\Sigma^* = \sum_{lj \in \mathcal{K}} s^*_{lj}$) and Σ^*_{-k} denotes total effective spending by the side opposite k ($\Sigma^*_{-k} = \sum_{lj:l \neq k} s^*_{lj}$).

Proof. The marginal utility of effective spending for committee ki is

$$\begin{aligned} \frac{\partial U_{ki}(s)}{\partial s_{ki}} &= \frac{(\alpha_{ki} + 1)\sum_{lj \in \mathcal{K}} s_{lj} - \alpha_{ki} s_{ki} - \sum_{j=1}^{N_k} s_{kj}}{(\sum_{lj \in \mathcal{K}} s_{lj})^2} - c_{ki} \\ &= \frac{\sum_{lj:l \neq k} s_{lj} + \alpha_{ki} \sum_{lj:lj \neq ki} s_{lj}}{(\sum_{lj \in \mathcal{K}} s_{lj})^2} - c_{ki}. \end{aligned}$$

This expression is strictly decreasing in s_{ki}, so the committee's expected utility is strictly concave in its own effective spending and first-order conditions are necessary and sufficient for best responses. The claim of the proposition then follows immediately. □

C Full Estimation Results

See Table 6. (Though some cost terms round to 0.00 at two digits, none are numerically zero in estimation.)

Term	Estimate	Std. Err	95% Interval
bias:Intercept	2.32	0.48	[1.35, 3.29]
bias:race_type[T.Open]	−3.47	0.31	[−4.18, −2.96]
bias:race_type[T.Rincumbent]	−5.92	0.34	[−6.82, −5.49]
bias:f_year[T.2006]	1.35	0.57	[0.13, 2.46]
bias:f_year[T.2008]	0.96	0.60	[−0.14, 2.20]
bias:f_year[T.2010]	−1.47	0.55	[−2.64, −0.43]
bias:f_year[T.2012]	1.34	0.54	[0.32, 2.49]
bias:f_year[T.2014]	−1.30	0.92	[−3.13, 0.49]
bias:f_year[T.2016]	0.84	0.87	[−1.44, 2.00]
bias:f_year[T.2018]	2.50	0.60	[1.19, 3.61]
bias:f_year[T.2020]	−0.81	0.95	[−3.14, 0.67]
bias:ideology	−8.63	0.65	[−10.27, −7.74]
theta:Intercept	0.60	20.59	[−18.11, 2.45]
theta:f_year[T.2006]	0.21	19.86	[−13.27, 10.88]
theta:f_year[T.2008]	0.54	5.72	[−2.35, 11.53]
theta:f_year[T.2010]	−10.17	29.19	[−68.52, 3.72]
theta:f_year[T.2012]	0.28	4.59	[−3.03, 10.37]
theta:f_year[T.2014]	1.68	25.26	[−35.75, 11.25]
theta:f_year[T.2016]	1.68	4.53	[−2.61, 12.73]
theta:f_year[T.2018]	−1.23	4.90	[−3.25, 10.22]
theta:f_year[T.2020]	3.00	31.40	[−35.12, 30.70]
theta:D_indicator	1.27	20.05	[−0.56, 10.84]
alpha:type[Candidate]	0.00	0.00	[0.00, 0.00]
alpha:type[Dark]	0.02	0.05	[0.00, 0.18]
alpha:type[PAC]	0.01	0.03	[0.00, 0.09]
alpha:type[Party]	0.22	0.45	[0.05, 1.05]
alpha:type[Super]	0.00	0.07	[0.00, 0.21]
cost:D_year[2004R]	0.08	0.07	[0.00, 0.26]
cost:D_year[2004D]	0.16	0.17	[0.01, 0.64]
cost:D_year[2006R]	0.10	0.19	[0.00, 0.67]
cost:D_year[2006D]	0.19	0.16	[0.00, 0.68]
cost:D_year[2008R]	0.13	0.27	[0.00, 0.52]
cost:D_year[2008D]	0.25	0.24	[0.08, 0.83]
cost:D_year[2010R]	0.00	0.08	[0.00, 0.18]
cost:D_year[2010D]	0.00	0.37	[0.00, 0.45]
cost:D_year[2012R]	0.07	0.06	[0.00, 0.21]
cost:D_year[2012D]	0.15	0.07	[0.05, 0.32]
cost:D_year[2014R]	0.13	0.28	[0.00, 0.87]
cost:D_year[2014D]	0.40	1.10	[0.00, 3.98]
cost:D_year[2016R]	0.13	0.17	[0.00, 0.44]
cost:D_year[2016D]	0.26	0.46	[0.02, 1.60]
cost:D_year[2018R]	0.03	0.05	[0.00, 0.13]
cost:D_year[2018D]	0.04	0.03	[0.03, 0.11]
cost:D_year[2020R]	0.16	0.23	[0.00, 0.83]
cost:D_year[2020D]	0.65	0.80	[0.00, 2.71]
Races	2276		
Committees	15776		

Table 6. Full structural model results.