Electricity

Agenda: Energy Conversion and Transformation

Work and other Energy Forms

- Potential and kinetic energy,
- Molecular binding and rearrangement energies,
- pV work, kinetic energy equilibration, heat flow, Mechanical equivalent of heat, Basic fluid dynamics, laminar & turbulent flow, (→later)

Basic Electricity

- Static electric, electro-magnetic phenomena, Electromagnetic induction, generators, transformers, Electrical current laws, AC/DC transmission,
- Electronic circuits, reactance,

Principles of Thermodynamic Processes

- Laws of Thermodynamics, state functions, reversible processes,
- Carnot and other TD cycles, steam engines, gas turbines,
- Electro-chemistry, batteries, hydrolyzers & fuel cells.

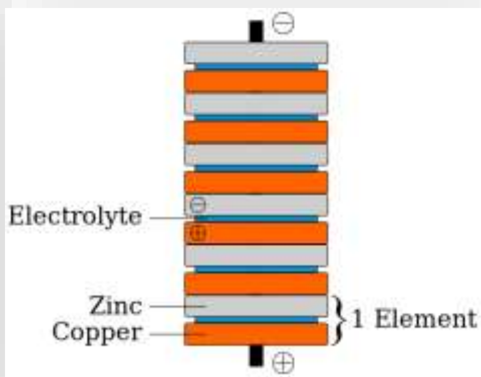
Electricity: Transformative Historical Power

Luigi Galvani
(1737-1798)

Alessandro Volta
(1745-1827)

Static electricity know since ancient times (Thales, 600 BCE). Created by rubbing of amber with animal fur, Galvani's physiological frog leg experiments.
Volta assisted Galvani, disagreed on nature of electricity.
Volta discovered battery ("Voltaic Pile"), announced March 20, 1800 to Royal Society, London.

Replica of Volta's first battery ("Voltaic Pile")
Museum Tempio Voltiano.

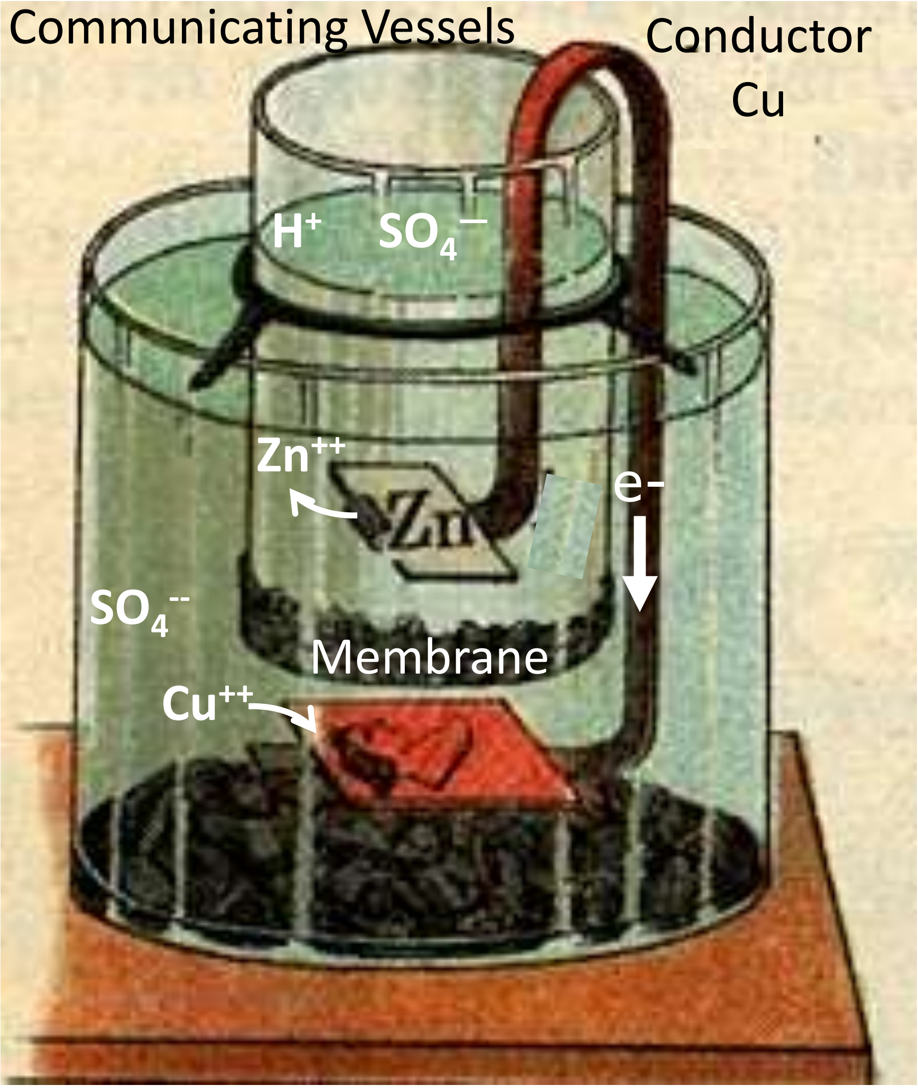


Schematics of Voltaic Pile

Stack of pairs of alternating metal disks, copper (or silver) and zinc. "Electrodes" separated by cloth or cardboard soaked in brine (=electrolyte).
Top and bottom contact wires produce spark (energy) when touching.
Electric current increases with height of the stack (number of elements). → Zn electrodes "disintegrate."

Electro-motive force (emf, unit=Volt) generated by chemical reaction between metals.

Electric Potentials & Electrolytic Solutions



Zn(s)| Zn^{2+}(aq)| Cu^{2+}(aq)|Cu(s)

Communicating vessels separated by membrane with H^+ ion permeability. Electrodes (Zn & C) immersed in dissociated electrolyte solution. Electrons have small free path before capture.

External metal conductor guides electrons.
→ electrostatic Redox potential $\Delta\Phi$

Cu^{2+} in aqueous $CuSO_4$ solution is deposited on ***C*** (graphite) image matrix,

→ Zn dissolves, Cu precipitates on C:

Zn(s)+Cu^{2+}(aq)→ Zn^{2+}(aq)+Cu(s)

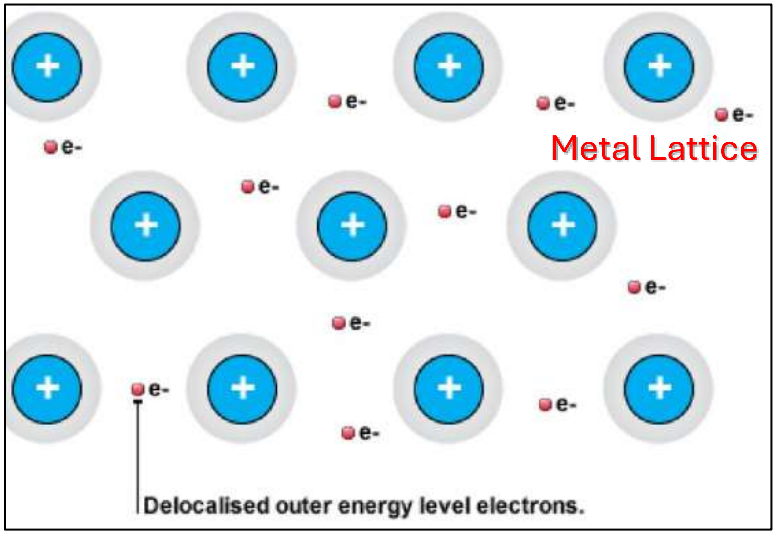
2 "half *redox* reactions"

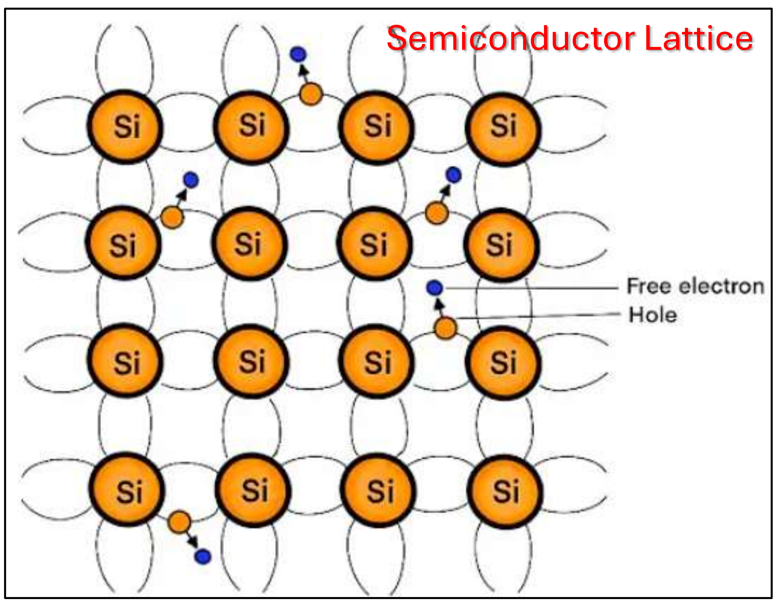
1) *Zn Oxidation:* Zn(s) → Zn^{2+}(aq)+2e^-

2) *Cu Reduction:* Cu^{2+}(aq)+2e^-→ Cu(s)

Free electrons can travel through metals, but are repelled by insulator materials, including liquids.

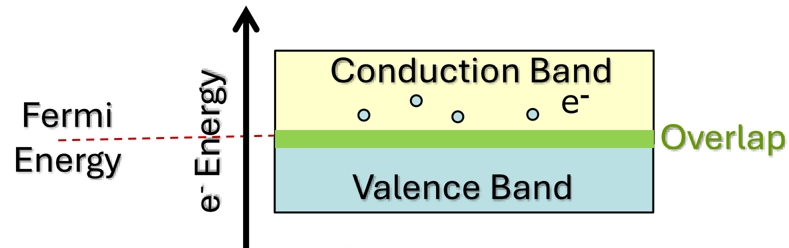
Metals and Semiconductors





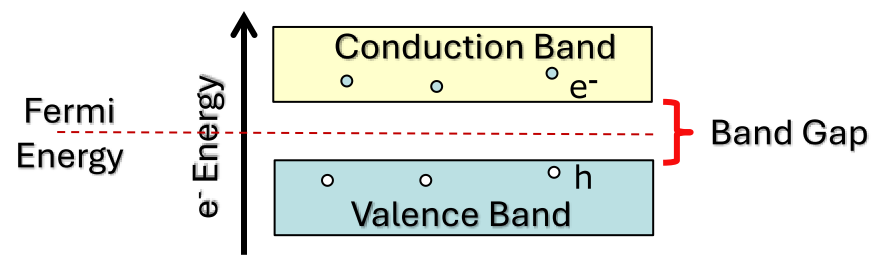
Many-body (~10^{23}) electron states in macroscopic lattices have overlapping energy states → 2 energy bands, separated by gap.

Fermi energy = reference ε_F(material)



Conductor-Metal

Ground State: Filled valence band + free electrons in conduction band (=Fermi gas).

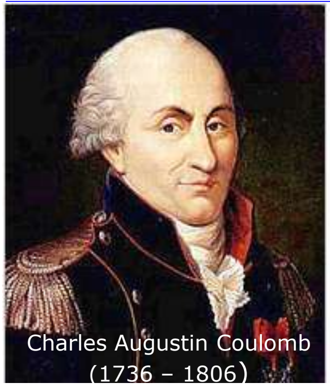


Semiconductor

Insulator materials have tightly bound electrons → very large band gaps (>few eV),

Different materials have different e^- mobilities, different Fermi energies ε_F(material) → different electrostatic potential energies $\Delta\Phi$.

Electrostatic and Magnetic Fields



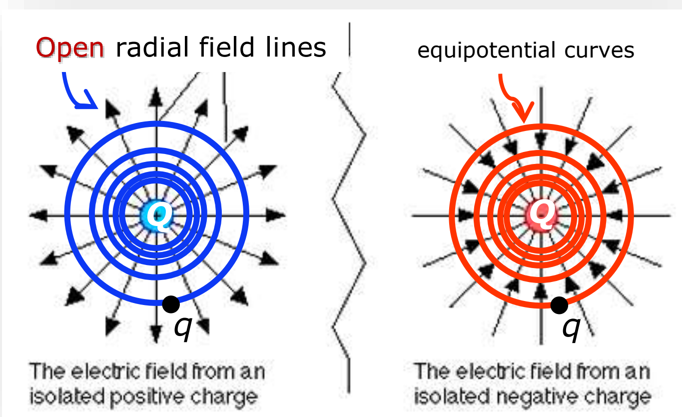
Coulomb Law: Electric potential (energy) of a point charge *q*, at a distance r from the charge Q (relative to potential at infinity)

$$U(r) = q \cdot \frac{Q}{4\pi\,\varepsilon_0\, r_{qQ}}; \quad \text{vacuum permittivity} \quad \varepsilon_0 = 8.854 \cdot 10^{-12}\, F/m \;\; (Farad \propto C^2/N \cdot m)$$

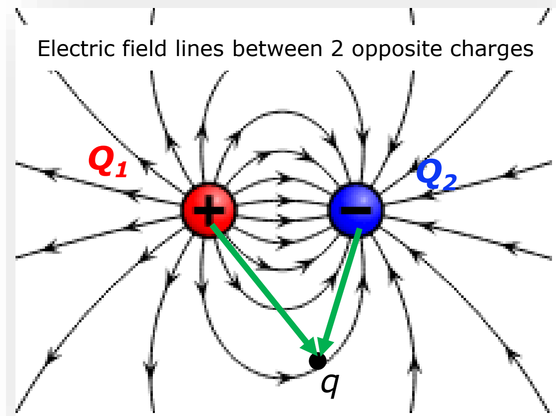
Linear superposition of potentials generated by Q_1 and Q_2 at any position $\vec{r}$

$$U_{12}(\vec{r}\,) = \frac{q}{4\pi\,\varepsilon_0} \cdot \left(\frac{Q_1}{|\vec{r} - \vec{r}_1|} + \frac{Q_2}{|\vec{r} - \vec{r}_2|} \right) \quad \text{Equipotential curves } U_{12}(\vec{r}\,) = const.$$

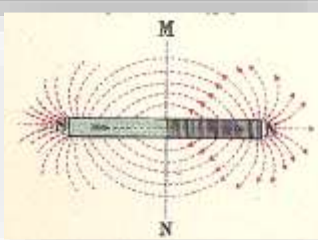
$$\text{Coulomb force (field)} \propto \text{potential gradient} \qquad \vec{F}(\vec{r}\,) = -\vec{\nabla} U(\vec{r}\,) := q \cdot \vec{E}$$



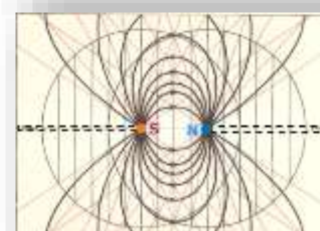
Electrostatic Interaction



Permanent Dipole Magnets: N and S (no monopoles !)

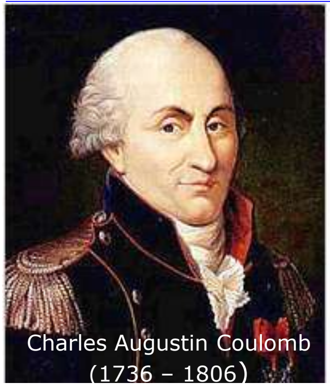


Magnetostatic Interaction



Closed magnetic field lines between N and S poles

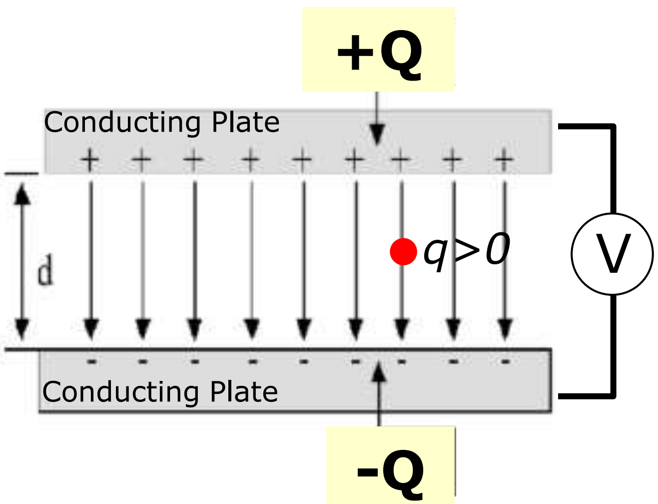
Electrostatic Fields



Coulomb Law: Electric potential (energy) of a point charge q at distance r from the charge Q (relative to r=∞) is defined as

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q\cdot Q}{r}; \quad permittivity\ \varepsilon_0 = 8.854\cdot 10^{-12}\, F/m\ \ (Farad \propto C^2/N\cdot m)$$

$$Coulomb\ \ force\ (field)\ \boxed{\vec{F}(\vec{r}) = -\vec{\nabla}U(\vec{r}) := q\cdot\vec{E}} \rightarrow electric\ \ field\ \vec{E}(\vec{r})$$



Uniform, Homogeneous Electric Field $\vec{E}$
between two parallel conducting plates, with vacuum or dielectric medium, permittivity ε.

$$\boxed{\left|\vec{E}\right| = \frac{V}{d} \quad unit\ [E] = \frac{Volts}{m}}$$

Particle of charge *q*, unit [*q*]= e = 1.602·10^{-19}C.

Force on particle: $\boxed{\vec{F} = q\cdot\vec{E}}$ $\ unit\,[F] = N$

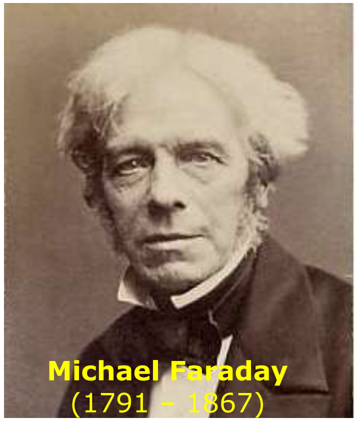
Energy gained between plates $W = F\cdot d = q\cdot V$

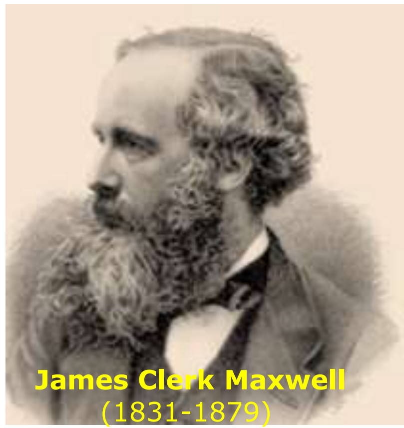
Power gained $P = \frac{\Delta W}{\Delta t} = \frac{q}{\Delta t}\cdot E\cdot d \rightarrow \boxed{P = I\cdot V}$

Electric current I, $[I] = A\ (Ampere)$

Particle of charge *q=e* falling through 1V differential gains energy *E=1eV.* 1eV=1.602·10^{-19}*J.*

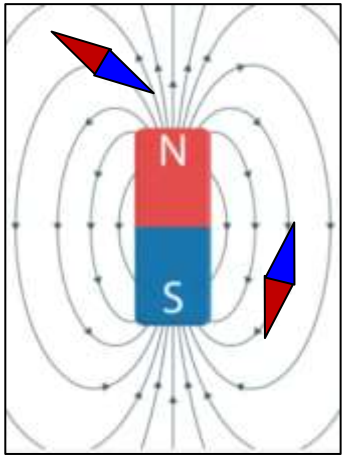
Static Electro-Magnetic Field





Faraday: English experimenter → magnetic fields around electrical wires. J.C. Maxwell: Unified theory of electromagnetism.

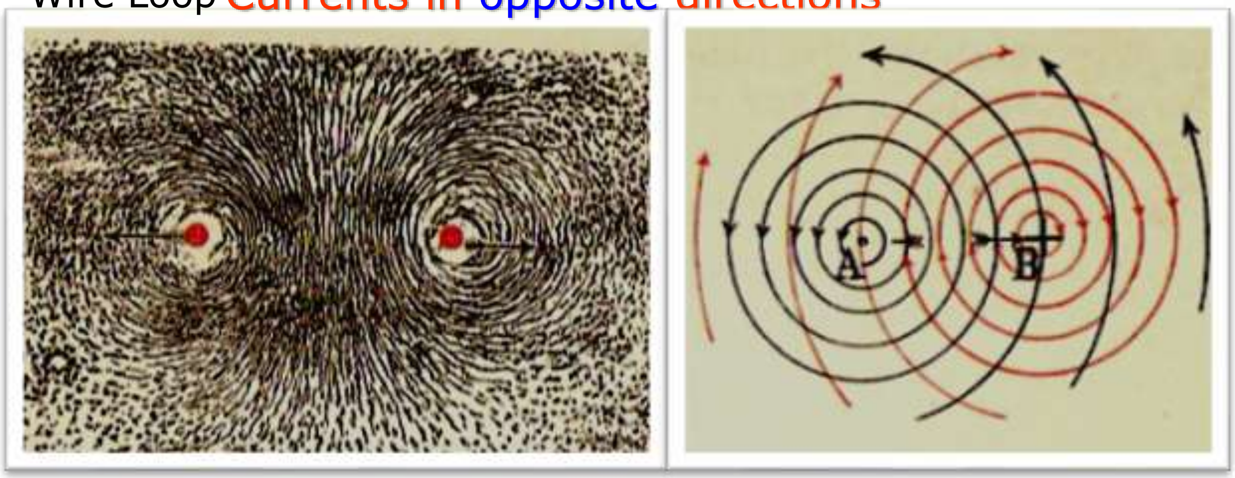
Bar Magnet with fields lines



Direction of magnetic field lines = direction of force that a *North* pole experiences. Form always closed loops N→S: (iron filings).

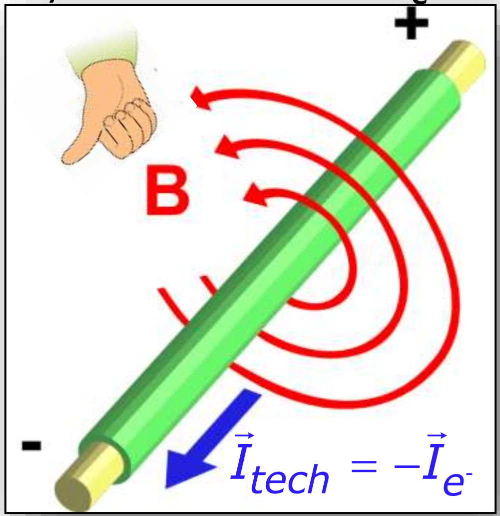
No magn. Monopoles!

Wire Loop Currents in opposite directions



Magnetic fields add (in the inside region)

Field ***B*** generated by electron current $\boldsymbol{I_e}$

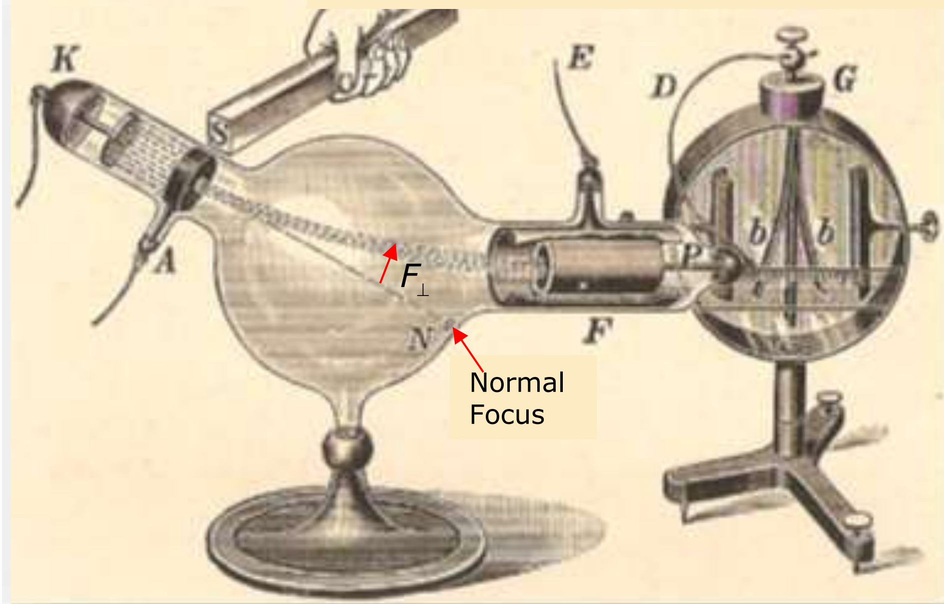


Electrodynamics of Moving Charges

James Clerk Maxwell
(1831-1879)

Developed a unified understanding of electric and magnetic phenomena → Maxwell's equations = set of partial differential equations that, together with the Lorentz force law, form the foundation of classical electrodynamics, classical optics, and electric circuits, much of today’s technology.

Diverting an electron beam with a magnet



Particle el. charge q, velocity $\vec{\upsilon}$

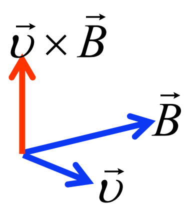
Electric $(\vec{E})$, magnetic $(\vec{B})$ fields

→ Lorentz Force:

$$\vec{F} = q \cdot \left(\vec{E} + \vec{\upsilon} \times \vec{B}\right)$$

$$E = 0 \rightarrow F_{\perp} = q\upsilon B = i \cdot B$$

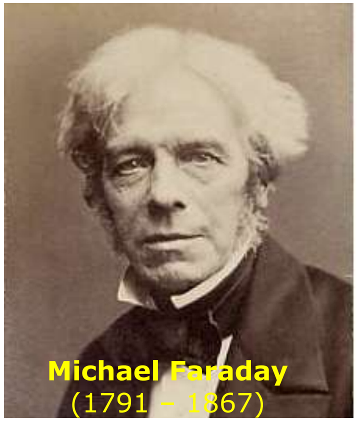
Electric current $i = q \cdot \upsilon$



Vector cross product

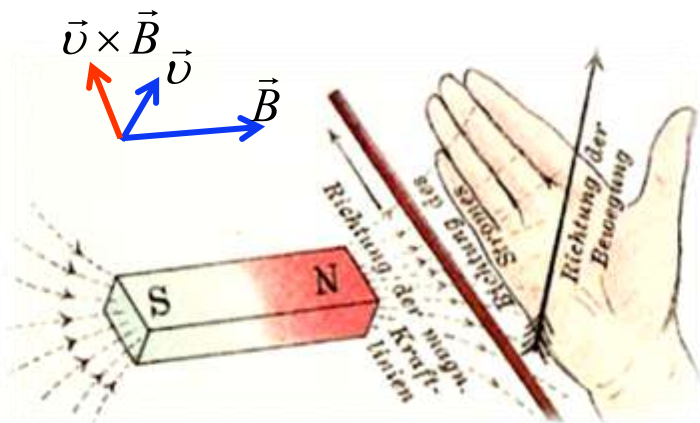
Moving electric charges q across magnetic field direction ($\vec{\upsilon} \perp \vec{B}$) produces a force $F_{\perp}$ on the charges, perpendicular to velocity direction → accelerates electric charges → produces charge movement = **electrical current $I(t)$ perpendicular to $B(t)$ field.**

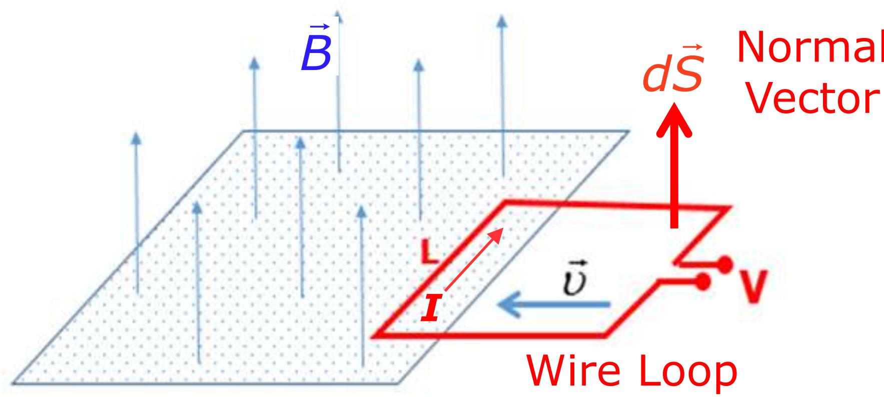
Electromagnetic Induction



Induction:
Moving electric conductor (Cu wire) across static magnetic field *B* interacts *like* *B*(*t*) with electrons in wire → induces current (electron flow) in the wire (attempts to cancel effect of external *B* field → generates charge ΔQ and potential difference ΔU ("voltage") between wire ends).

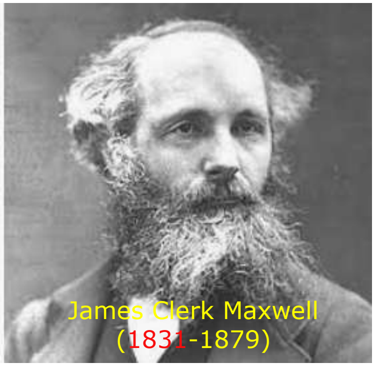
Right-hand rule (Lenz's Rule):
Moving wire in direction of thumb through B forces induces electron current in direction of fingers (opposing the B field).





Moving electric charges = current $I = dq/dt \triangleq q \cdot \upsilon$
←→ magnetic field *B*,
Changing magnetic field Δ*B*(*t*)←→electron current Δ*I*(*t*).

Electromagnetic Field Theory: Maxwell Equations



Combination of individual laws of electric and magnetic interactions into one theoretical framework:
MEq describe an electric vector field $\vec{E}(\vec{r},t)$ and a magnetic (pseudo) vector field, $\vec{B}(\vec{r},t)$, as well as their interactions.
The sources are the total electric charge density (total charge per unit volume), ***ρ***, and the total electric current density (total current per unit area), ***J***.

Name	Integral equations	Differential equations
Gauss's law	$\oint\!\!\!\oint_{\partial\Omega} \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_\Omega \rho\, dV$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oint\!\!\!\oint_{\partial\Omega} \mathbf{B}\cdot d\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d}{dt}\iint_\Sigma \mathbf{B}\cdot d\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0\left(\iint_\Sigma \mathbf{J}\cdot d\mathbf{S} + \varepsilon_0\frac{d}{dt}\iint_\Sigma \mathbf{E}\cdot d\mathbf{S}\right)$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

$Here: \vec{B} := magnetic\ \ flux\ \vec{\Phi}$

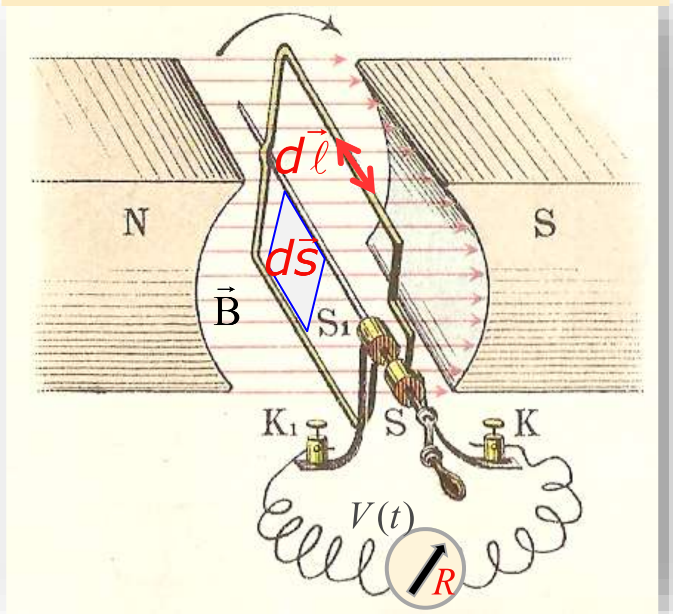
the permittivity of free space, ε_0, and
the permeability of free space, μ_0, and
the speed of light, $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$

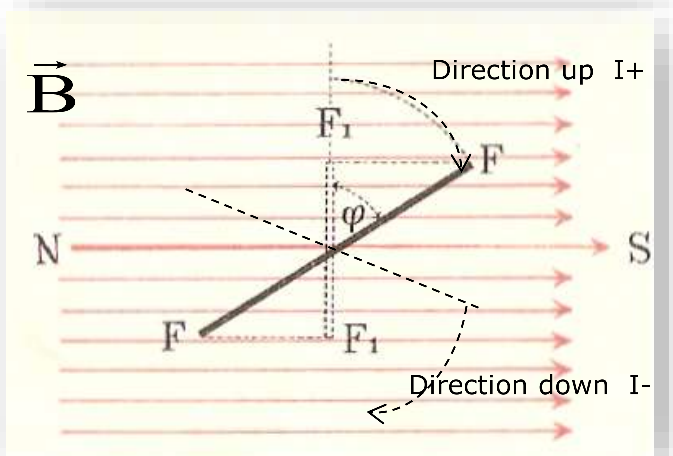
Magnetic flux through plane surface $\vec{S}$

$$\vec{\Phi} := \vec{B}\cdot\vec{S} := B_\perp \cdot S \cdot \vec{n}_S \quad Unit\,[\Phi] = Wb\,(Weber) = volt - s$$

Principle of Generator (Dynamo)

Voltage induced on wire loop turning in B field





Direction of flow of electricity (electrons e$^-$) in a wire-conductor loop → induced electro-motive force *emf* (Faraday's Law of Induction)

$$\frac{\partial}{\partial t}\int_{Loop\ Area}\vec{B}(t)\cdot d\vec{s} = -\oint_{Loop\ Rim}\vec{E}\cdot d\vec{\ell} \propto work\ (on\ q)$$

Time dependent A/C Voltage V(t) and Current I(t)

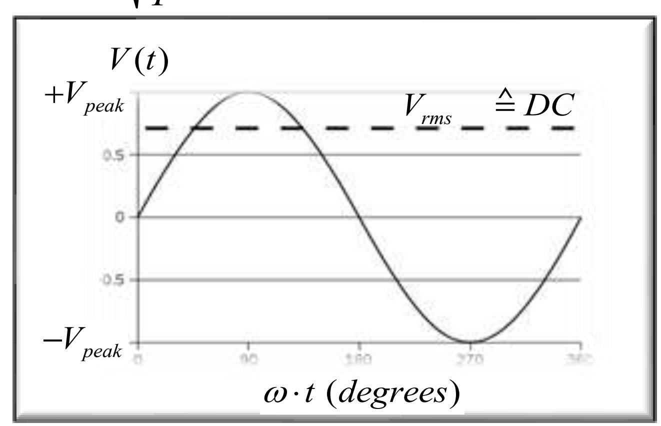
$$V(t) = V_{peak}\cdot\sin(\omega\cdot t) \qquad I(t) = \left(V_{peak}/R\right)\cdot\sin(\omega\cdot t)$$

Amplitude Phase

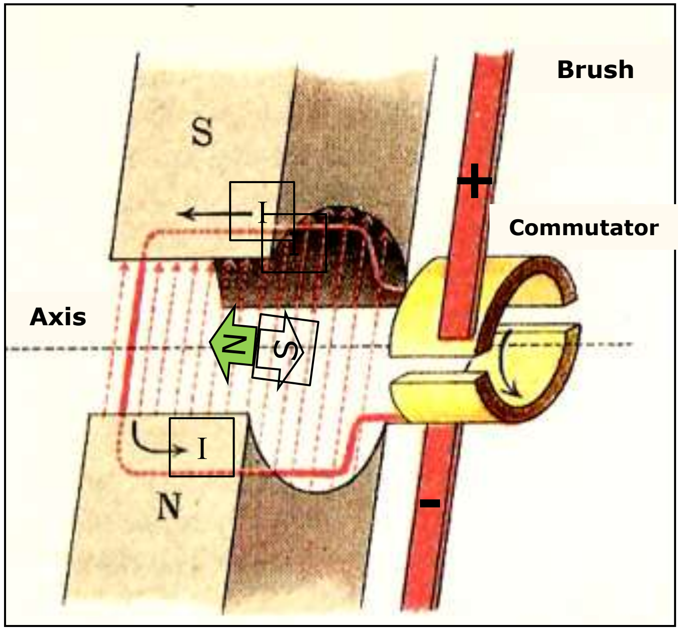
$$Amplitude\ V_{peak},\ V_{peak-to-peak} = 2V_{peak}$$

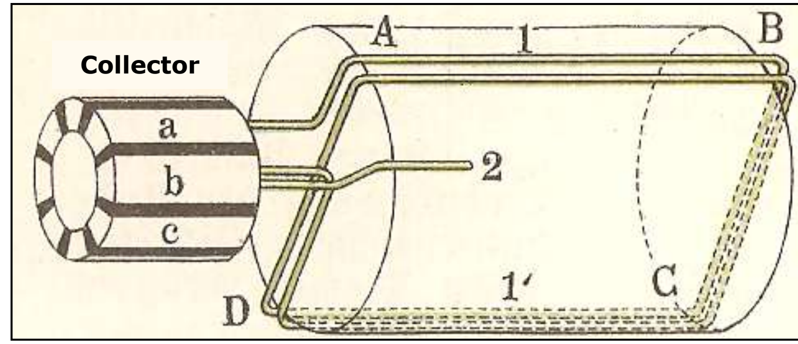
$$Angular\ \ frequency\ \omega = \partial\varphi/\partial t = 2\cdot\pi\cdot f = 2\pi/T$$

$$Effective\ voltage:\ \langle V(t)\rangle = V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt} = V_{peak}/\sqrt{2}$$

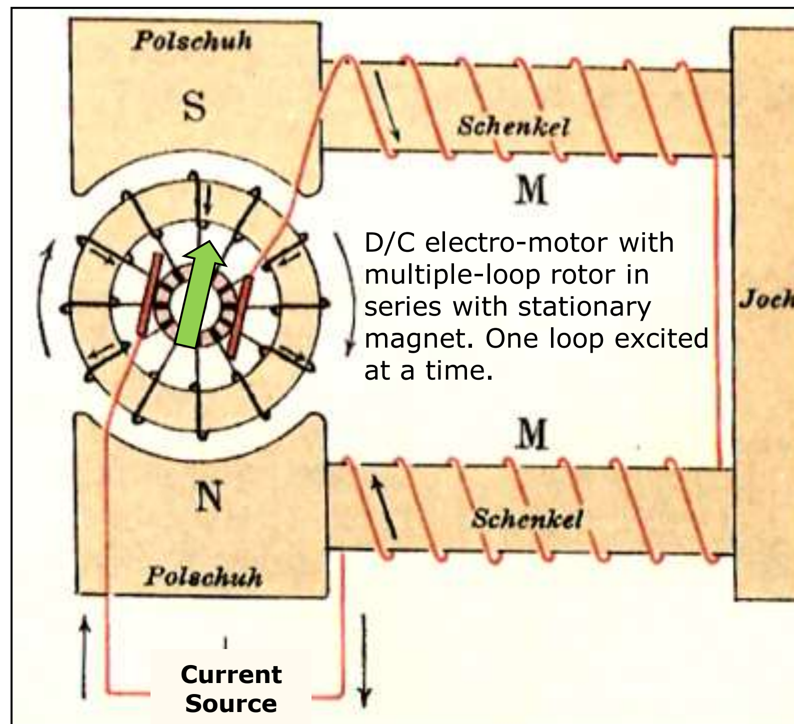


Principle of D/C Electro-Motor





Current (I) loop creates alternating *N-S* electro-magnet, which "feels" a torque and tends to align parallel to the field of permanent magnets and turns loop. Polarity reverses magnet polarity at max. alignment.



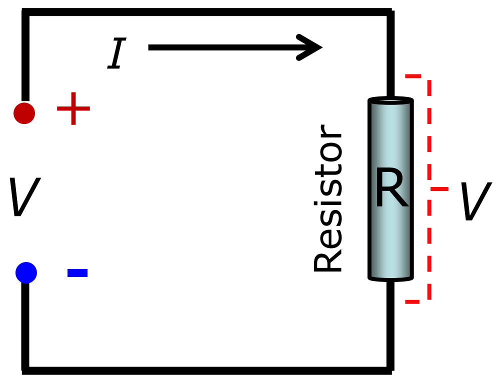
Mechanical rotation of wire loop in field of permanent magnets generates voltage at commutator/collector → **Dynamo/Alternator**
Direction of current depends on orientation of loop in magnetic field → AC or DC currents

Components of Electrical DC Circuits: Resistors

Electric current I unit=[1 A(mpere)= 1C/s]:
e^- stream [dq/dt=#e^-/sec] transfers power through metallic wires, dissipates e^- energy→ heat. All metallic wires have intrinsic (distributed) resistivity.

→ Unidirectional (DC) electrical (e^-) current sustained by applied electric potential = Voltage differential V=constant in time.

Always finite electric conductivity σ → 1/Ohm resistance $R \neq 0$!!



Ohm's Law

$$I = \frac{V}{R} \rightarrow V = I \cdot R$$

$$[R] = 1\Omega\ (Ohm) = \frac{1V}{1A}$$

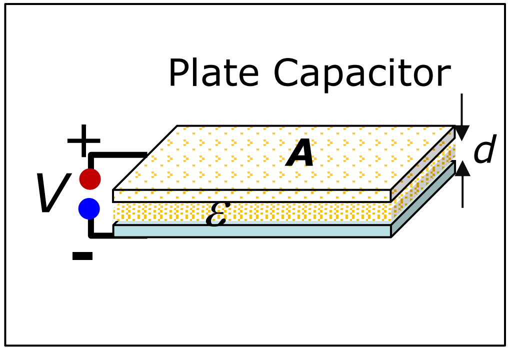
Power dissipated in resistor

$$p = V \cdot I = I^2 \cdot R$$

$$[p] = W\ (Watt) = J/s$$

Commercial resistor elements are made of materials with low electric conductivity.

Components of Electrical DC Circuits: Capacitors

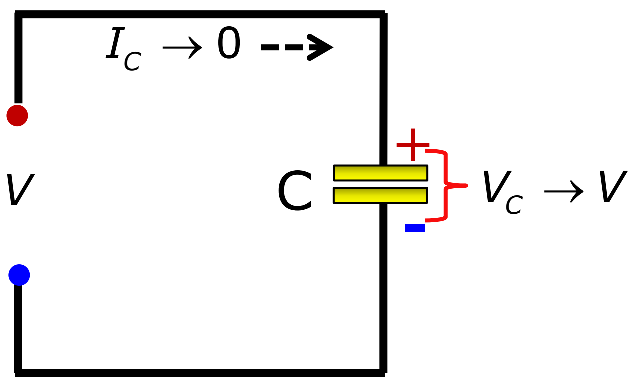


Metal plates (area *A*) separated by dielectric medium (ε) of thickness *d* form a capacitor w. capacitance

$$C = \frac{\varepsilon \cdot A}{d} \qquad [C] = F\left(Farad\right)$$

→Carries static charge $Q = C \cdot V$

As "load" element :



$$Switch - on\ voltage\ V \to brief\ \ current\ I_C\left(t\right)$$

$$V_C\left(t\right) = \frac{1}{C}Q\left(t\right) = \frac{1}{C}\int^{t} I_C\left(t'\right)dt' \to V$$

Energy content W_C of capacitor C

$$W_C = \frac{1}{2}Q \cdot V = \frac{C}{2}V^2 = \frac{Q^2}{2C}$$

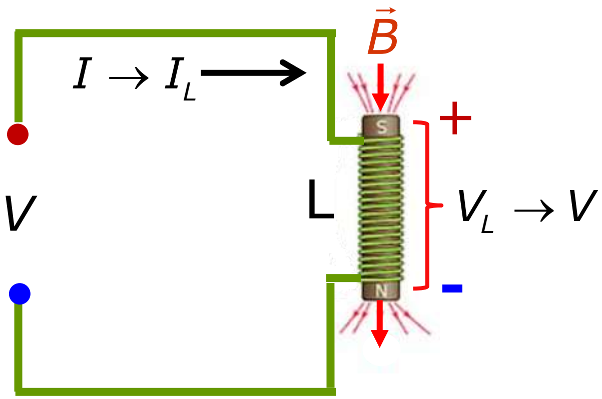
A purely capacitive load in an electrical circuit does not dissipate (lose) power.

Components of Electrical DC Circuits: Inductors

Helical coil of insolated Cu wire wound around plastic ("solenoid") or ferrite/carbon-iron core. Connected to electric battery it produces static axial magnetic field ("electro-magnet")

$$\text{Inductance}\quad L = \mu_0 K\left(N^2 A/\ell\right)_{coil} \qquad [L] = H = Vs/A \ \ (Henry)$$

As "load" element in circuit:



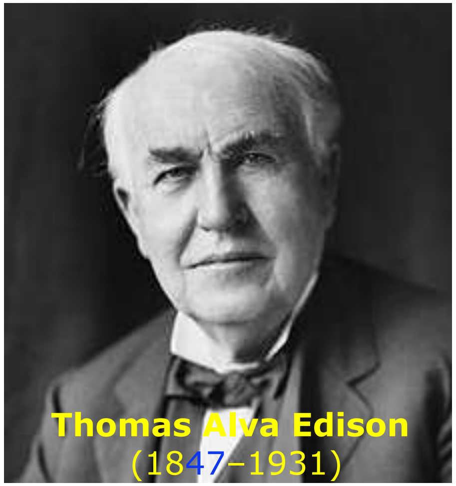
$$V_L(t) = -L\frac{dI}{dt} \rightarrow Work\ \frac{dW_L}{dt} = -L \cdot I \cdot \frac{dI}{dt}$$

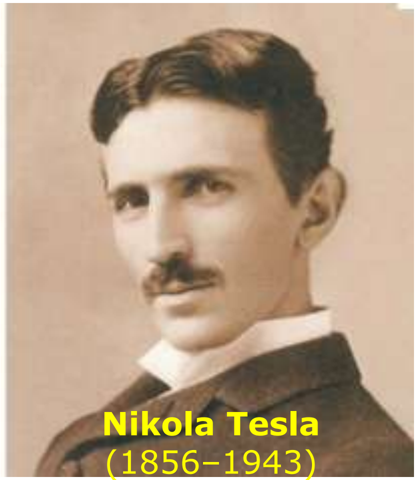
Energy content W_L stored in inductor L

$$W_L = \frac{1}{2} L \cdot I_L^2$$

A purely inductive load in an electric circuit does not dissipate electric energy.

Advent of Hydroelectric Power

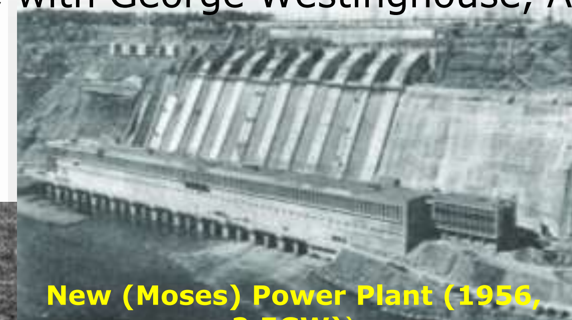


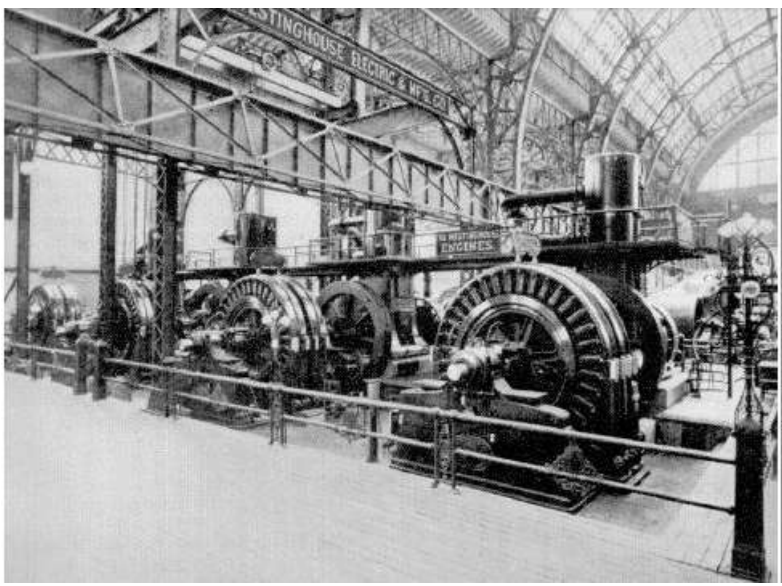


Influential inventors of DC (Edison) and AC (Tesla) electrical power transmission over large distances.

Electrical lighting, wireless radio,….
Power wars (→ J.P. Morgan).

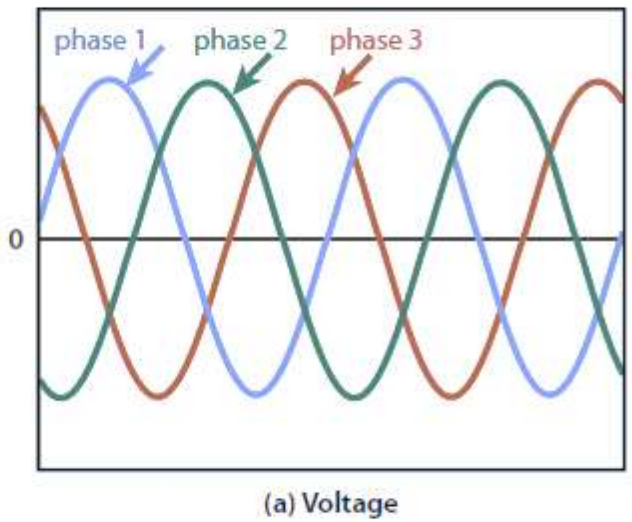
1895: Built 1. hydro electric power station (Niagara, with George Westinghouse, AC)



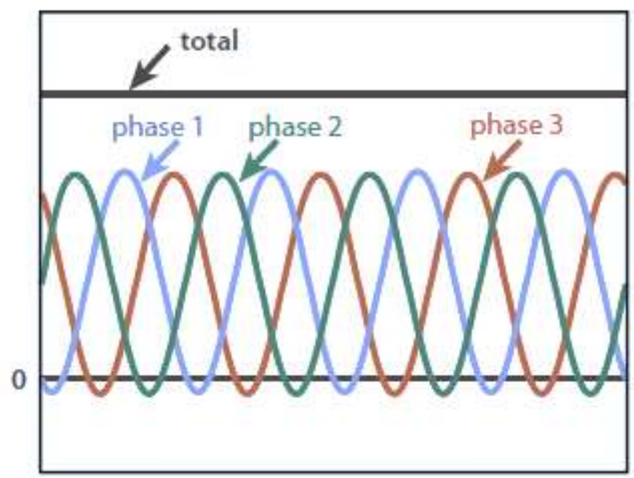


Start of new chapter in hydropower
→ many hydro-electric power plants

3-Phase Current



(a) Voltage



(b) Instantaneous Power

Stator

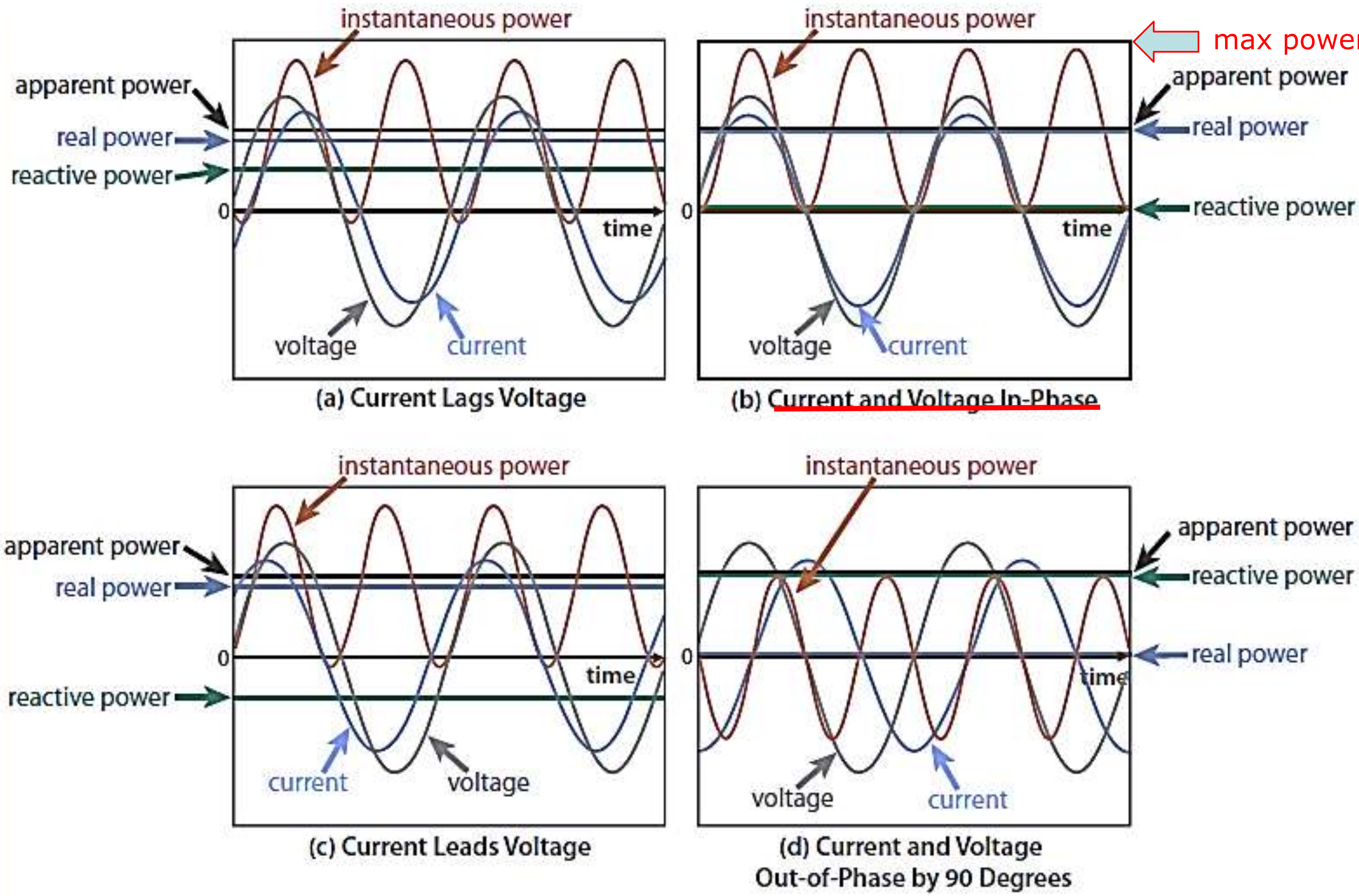
Invented by Nikola Tesla.

Stator has 3 segments, $\Delta\Theta=120^0$ rotation.
Combined power always > 0.
At Ohm load resistor *R* (no reactances)

→ P(t) = const.

See HW problems.

Effect of Reactance in AC System



Components of Electrical AC Networks: Resistors

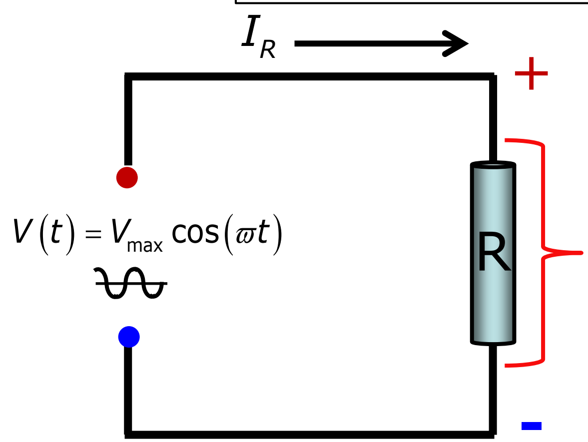
Transfer of electrical power through metallic wires → Electric conductivity σ → Electrons dissipate kinetic energy through scattering
→ Ohm resistance $R \sim 1/\sigma \quad [R] = \Omega\ (Ohm)$ Extension: R=generic workload

$$\text{Applied AC voltage } [V] = V\ (Volt)$$

$$V(t) = V_{\max}\cos(\varpi t) \rightarrow \text{effective } V = \langle V(t)\rangle = V_{\max}/\sqrt{2}$$

$$\text{effective } I = \langle I(t)\rangle = I_{\max}/\sqrt{2}$$

Averaged over 1 period



$$I_R(t) = I_{\max}\cos(\varpi t); \qquad I_{\max} = \frac{V_{\max}}{R}$$ Ohm's Law

$$\text{Current } I_R(t)\ \{[I] = A\ (Ampere)\} \text{ in phase with } V(t)$$

Power dissipated in resistor $[p] = W\ (Watt)$

$$p_R(t) = V(t)\cdot I_R(t) = V_{\max} I_{\max}\cos^2(\varpi t)$$

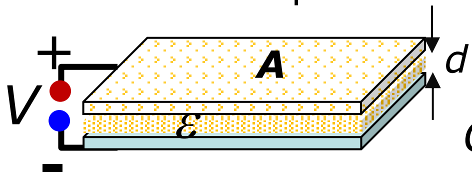
$$= \frac{1}{2} V_{\max} I_{\max}\{1+\cos(2\varpi t)\} = V \cdot I_R\{1+\cos(2\varpi t)\}$$

effective

$$\text{Effective dissipated power } \langle p_R(t)\rangle = V\cdot I_R = \frac{V^2}{R} = I_R^2\cdot R$$ This is real power loss.

Components of Electrical AC Networks: Capacitors

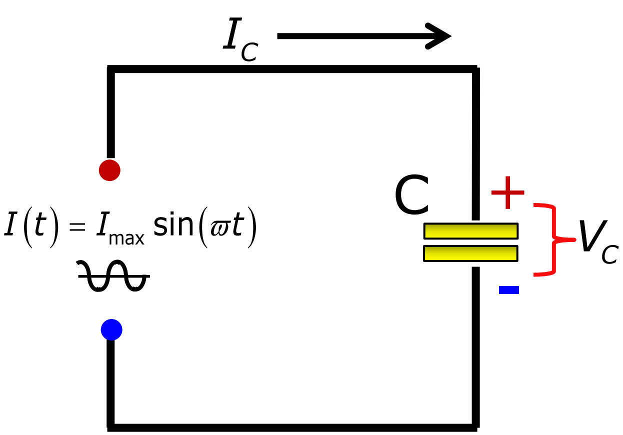
Plate Capacitor



Metal plates (area A) separated by dielectric medium (ε) of thickness d form a capacitor w. capacitance

$$C = \frac{\varepsilon \cdot A}{d};\ [C] = F\,(Farad)$$ → Carries static charge $Q = C \cdot V$

As "load" element in AC (frequency ω) circuit:



$$V_C(t) = \frac{1}{C}Q(t) = \frac{1}{C}\int^{t} I_C(t')dt' = \frac{I_{max}}{\varpi C}\cos(\varpi t)$$

$$= V_{max}\cos(\varpi t) \rightarrow \boxed{V_{max} = I_{max}\cdot X_C}$$ Ohm's Law

Reactance $|X_C| = \dfrac{1}{\varpi C}$

Current leads voltage: *phase difference* $+\dfrac{\pi}{2}$

Power in capacitor

$$p_C(t) = V(t)\cdot I_R(t) = V_{max}I_{max}\sin(\varpi t)\cdot\cos(\varpi t)$$

$$= \frac{1}{2}V_{max}I_{max}\{\sin(2\varpi t)\} \rightarrow \boxed{\langle p_C(t)\rangle = 0}$$

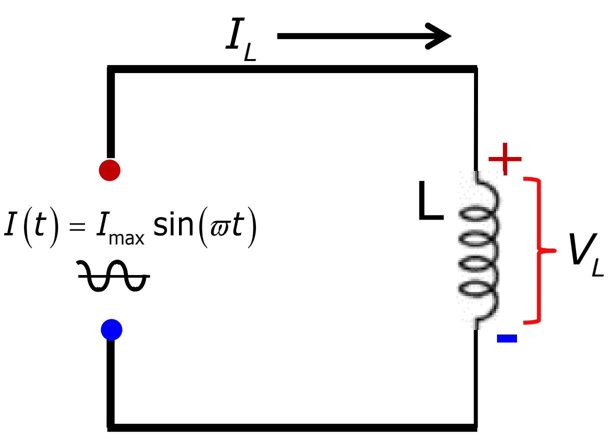
A pure capacitive load in an AC circuit does not dissipate (lose) power, it moves energy between electron currents and electric field and changes relative phase of current vs. voltage → reactive power

Components of Electrical AC Networks: Inductors

Helical coil insolated Cu wire wound around plastic ("solenoid") or ferrite/carbon-iron core. Connected to electric battery it produces static axial magnetic field ("electro-magnet")

Inductance L, $L_{solenoid} = \mu_0 K \left(N^2 A / \ell \right)_{coil}$ [L]=H=Vs/A (Henry)

As "load" element in AC (frequency ω) circuit:



$$V_L(t) = -L \frac{dI}{dt} = -\varpi L \cdot I_{\max} \cos(\varpi t)$$

$$= V_{\max} \cos(\varpi t) \rightarrow \boxed{V_{\max} = I_{\max} \cdot X_L} \quad \text{Ohm's Law}$$

Reactance $|X_L| = \varpi L$

$$\textit{Current lags voltage: phase difference} -\frac{\pi}{2}$$

Power in inductor

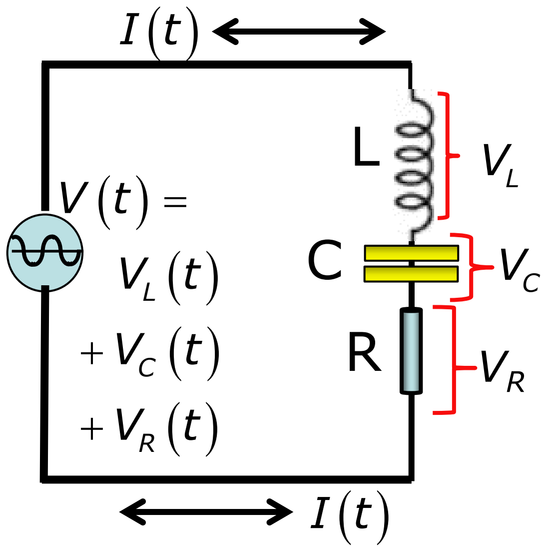
$$p_L(t) = V_L(t) \cdot I(t) = -V_{\max} I_{\max} \sin(\varpi t) \cdot \cos(\varpi t)$$

$$= -\frac{1}{2} V_{\max} I_{\max} \left\{ \sin(2\varpi t) \right\} \rightarrow \boxed{\left\langle p_L(t) \right\rangle = 0}$$

A pure inductive load in an AC circuit does not dissipate (lose) electric power, it moves energy between electron currents and magnetic field and changes relative phase of current vs. voltage → reactive power

Basic Electrical Circuit Laws

Ohm's Law $V(t) = Z \cdot I(t)$ $(\Delta\phi_I = phase\ difference\ I\ rel\ V)$

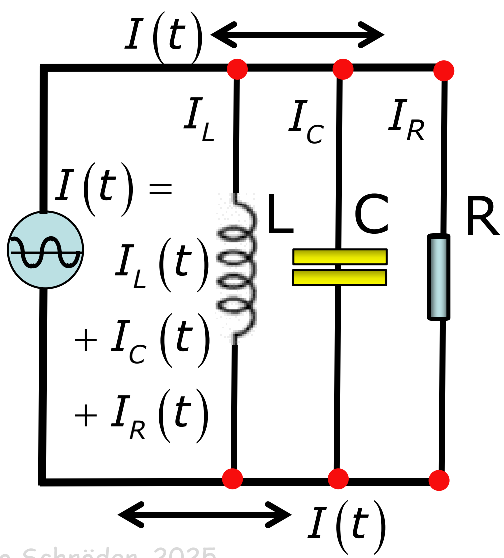


1.Ohm resistance R
1.Capacitive reactance $X_C = -1/\omega C$ ($\Delta\phi_I = +90^0$)
2.Inductive reactance $X_L = \omega L$ ($\Delta\phi_I = -90^0$)

Kirchhoff's Loop (or mesh) Rule

The directed sum of the potential differences (voltages) around any closed loop is zero.

$$V(t) = V_L(t) + V_C(t) + V_R(t)$$



Kirchhoff's Junction Rule (Parallel Loops)

The algebraic sum of currents in a network of conductors meeting at a point is zero.

$$I(t) = I_L(t) + I_C(t) + I_R(t)$$

Complex Notation

Phase differences are conveniently handled in complex notation *V(t), I(t)*

$$V(t) = |V(t)| \cdot e^{i\cdot(\varpi \cdot t + \phi_V)}; \quad I(t) = |I(t)| \cdot e^{i\cdot(\varpi \cdot t + \phi_I)}$$

Amplitudes and phases are determined from initial conditions, *V(t=0), I(t=0).*
Example:

$$V(t=0) = V_0 \rightarrow V(t) = V_0 \cdot \cos(\varpi \cdot t)$$

Ohm's Law $V(t) = Z \cdot I(t)$ *at all times*

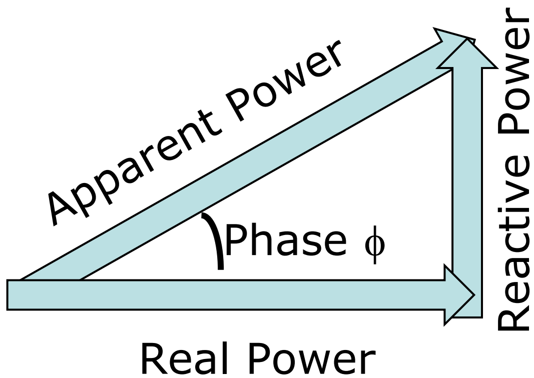
Impedance $Z = R + i \cdot [X_C(\omega) + X_L(\omega)]$ $(i := \sqrt{-1})$

1. Ohm resistance *R*
1. Capacitive reactance $X_C = -1/\omega C$ $(\Delta\phi_I = +90^0)$
2. Inductive reactance $X_L = \omega L$ $(\Delta\phi_I = -90^0)$

$$Euler's\ Formula: i = e^{i\cdot\pi/2} = \cos(\pi/2) + i \cdot \sin(\pi/2)$$

Real and Reactive Power

$Power\ P(t) = V(t) \cdot I(t)$ or, in complex notation $P = V \cdot I^{*}$ ($* =$ complex conjugate)



$Purely\ resistive\ loads: P_R = V_R \cdot I = V^2/R = I^2 \cdot R$

$Real\ power\ P_R(t) = V_R(t) \cdot I(t)$

$Apparent\ power: P_A(t) = \sqrt{P_R^2(t) + P_{LC}^2(t)}$

$Real\ power\ P_R(t) = P_A(t) \cdot \cos\phi$

$Power\ Factor = P_R / P_A$

Real and reactive power are "out of phase"

$Apparent\ power: P_A^2(t) = P_R^2(t) + \left[P_C^2(t) + P_L^2(t)\right]$

$Oscillating\ reactive\ power\ P_C(t) \rightleftarrows P_L(t)$

$Reactive\ power\ P_{LC}(t) = P_A(t) \cdot \sin\phi$ "var" *voltage – ampere – ractive*

Actual loads on the power supply (e-grid) like an e-motor are always complex (Ohm + capacitive + inductive) → have feedback effect on supply → Affect power factor (available power) and frequency.
→ General effect on stability of grid.