Hydro-Electric Power

Hoover Dam (near Las Vegas, NV)
Arch-gravity dam. Black Canyon/Colorado River.
2.1 GW, (4-10)TWh, construction 1931-1936,
5y, $49M(1930)→$750M, >>100Ŧ

Agenda

- Hydroelectric energy resources,
  - Hydrological cycle,
  - Seasonal, climatic trends,
  - Schematics & types of hydroelectric plants,
- Operational principles of hydro power plants,
  - Ideal fluid dynamics laws,
  - Energy & momentum transfer,
  - Hydro turbine types,

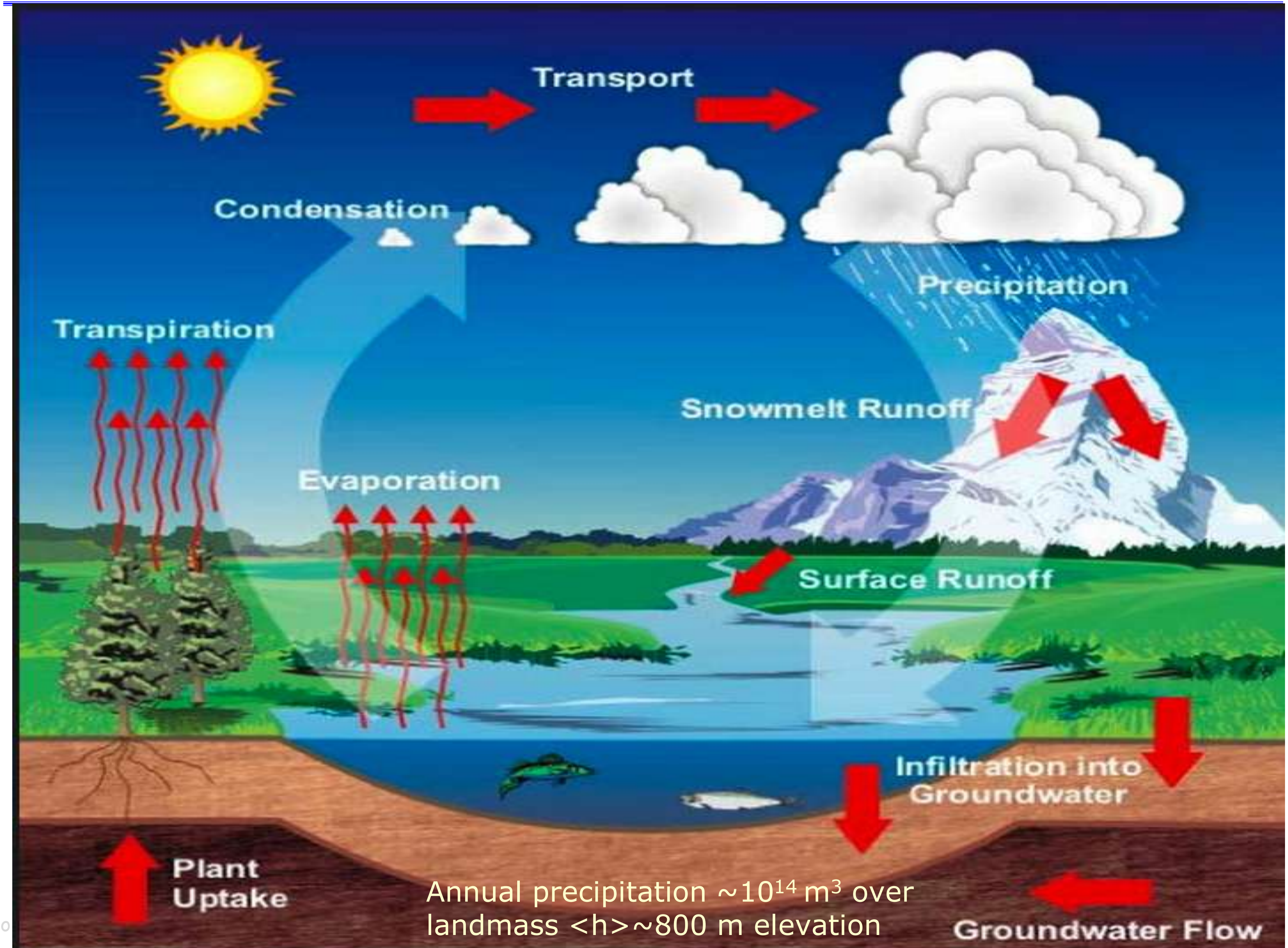
- World/US hydro-electricity generation,
  - Construction cost, electricity price,
  - Consumption,

- Major US and World hydro-electric dam projects,
- Strategic issues of hydro-electric energy production,
  - ecological impact, emissions,
  - non-renewable aspects.

- Next topic: Power from Biomass

Reading Assignments
A&J Ch. 4, 5.1-5.7
LN 3.2

The Hydrological Cycle



Annual precipitation ~10^{14} m^3 over landmass <h>~800 m elevation

Hydroelectric Plant Reservoir/Dam Architecture

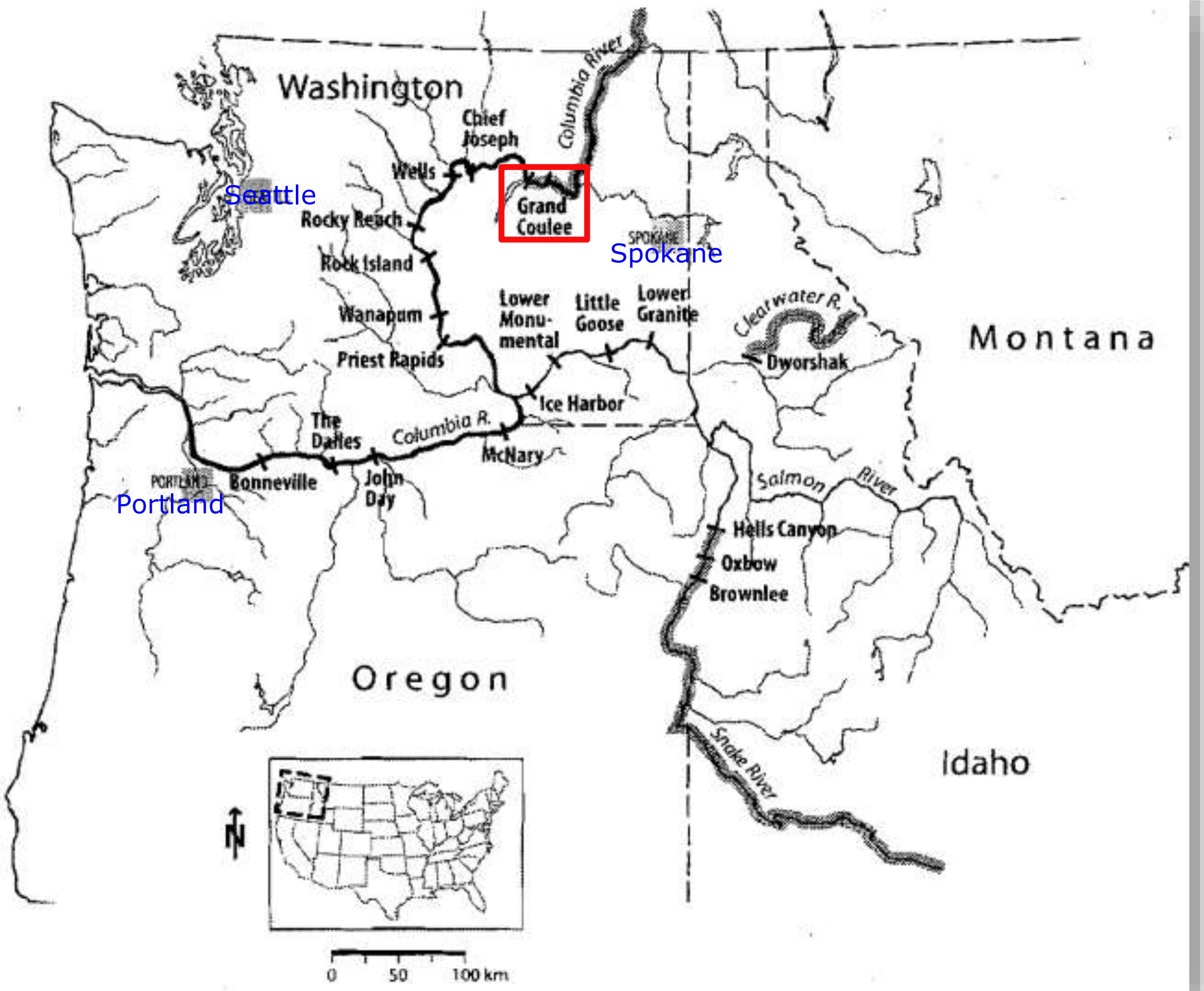
High-head dam:
Grande Dixence (Val des Dix, Switzerland), 285 m high dam, collects melt from Alpine glaciers

Buttress dam, external structural support braces.

Non-rigid trapezoidal embankment dam, reinforced core.

Columbia River Basin

Map showing the Columbia River Basin with its major tributary the Snake River. Large mainstem dams are noted on the lower Snake and Columbia Rivers. Shaded areas indicate historic salmon spawning areas now blocked by dams



Columbia, Colorado and other rivers have multiple dams, hydro-electric power, flood control, and irrigation.

Infringement on local habitat, culture/way of living has been resisted.
Similar public issues in Europe, South America.

Lake Mead Water Level: Recent History

Hoover Dam with Lake Mead Reservoir

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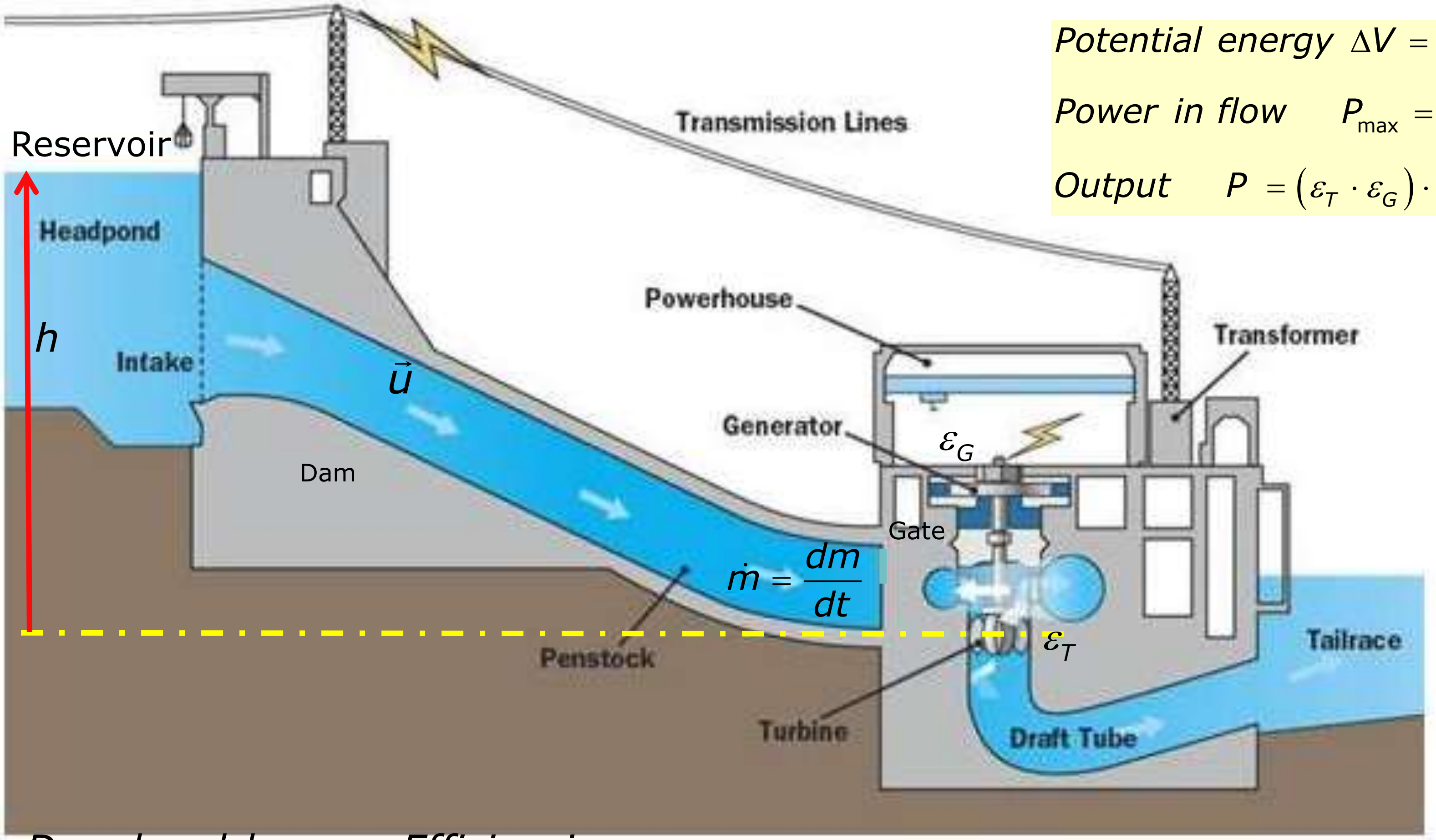
Schematics: Hydro-Electric Power Plant

Conversion gravitational potential → electric power



Potential energy $\Delta V = m \cdot g \cdot h$

Power in flow $P_{\max} = \dot{m} \cdot g \cdot h$

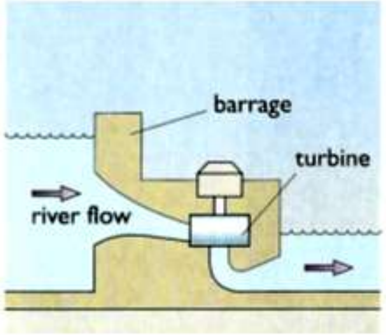
Output $P = (\varepsilon_T \cdot \varepsilon_G) \cdot \dot{m} \cdot g \cdot h$

Dam head h *Efficiencies* $\varepsilon_T, \varepsilon_G$

Mass of water m *of turbine, generator*

Flow $\dot{m} = dm/dt$

Types of Hydro-Electric Dams



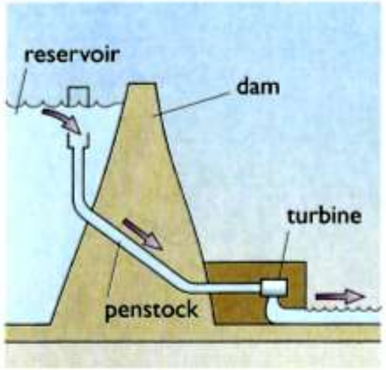
Hydro dams are classified according to head height or construction.

Low-head (10-25 m) dams have low water pressure

→ need large flow volumes (high kinetic energy)

Installed at slow moving rivers, "run-of-the-river"

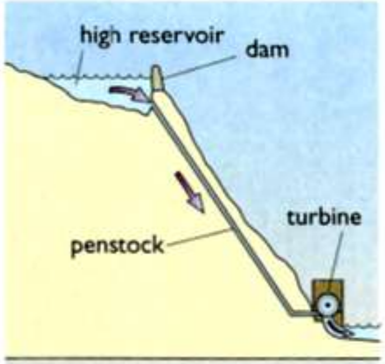
Construction: barrage/embankment type, locks, fish ladders.



Intermediate-head dams have high water pressure

→ need smaller flow volumes.

Installed at river valleys/canyons, fed from very large artificial reservoirs created by flooding extensive areas, reliable power provider, if sufficient precipitation/snow & ice melting occurs.
Construction: arch, gravity, buttress types.



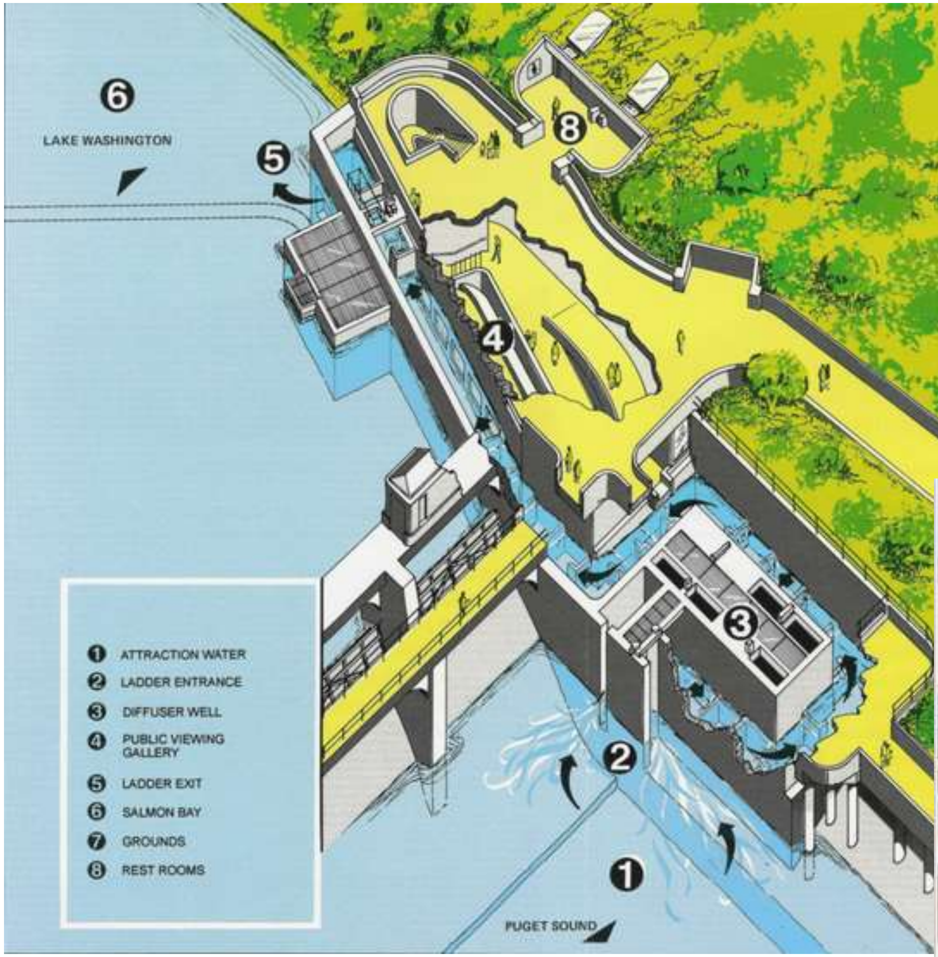
High-head dams have reservoirs located high above power plant,
→ work with small flow volumes but at high water pressures (high potential energy).

Installed in mountainous regions, long penstock tubes,

Construction: gravity/arch types.

G. Boyle, Renewable Energy, OUP 2004

Accommodating Wildlife: Fish By-Pass Ladders

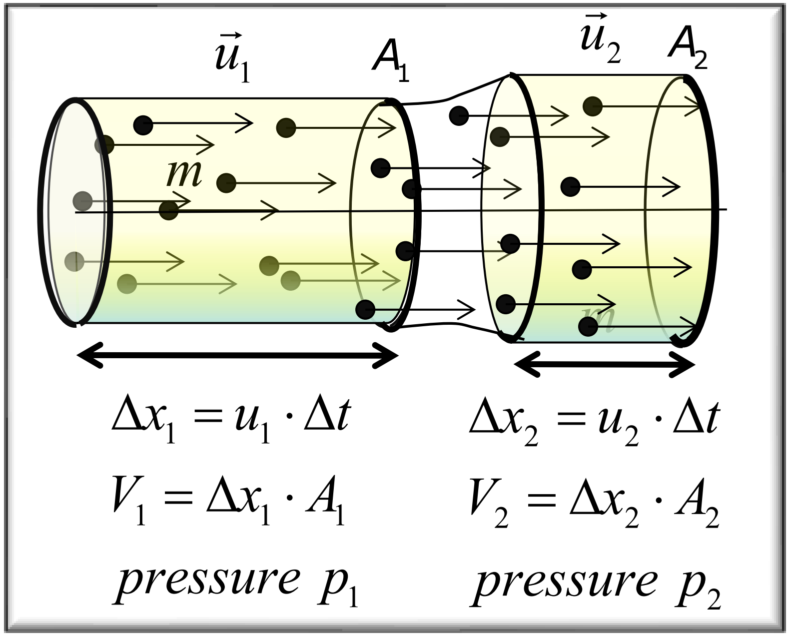


Some low-head dams have installations providing passage for fish to upstream spawning areas or fish hatcheries.
Not available at large dams → effect on habitat/fisheries.

For dams with high head (> 100 m):
Fish ladders not practical, or expensive, typical gradients < 20% (1′ elevation per 5′).

Fluid Dynamics
Turbine Technology

Energy Transport: Ideal Fluid Dynamics Laws



$$\Delta x_1 = u_1\cdot\Delta t \qquad \Delta x_2 = u_2\cdot\Delta t$$

$$V_1 = \Delta x_1\cdot A_1 \qquad V_2 = \Delta x_2\cdot A_2$$

$$pressure\ p_1 \qquad pressure\ p_2$$

Ideal incompressible, non-viscous liquid, "streamlines" (irrotational flow, no inertia)

Number density : $\rho = const.\ \left[\#N/cm^3\right]$

Mass density : $\rho_m = m\cdot\rho = const.\ \left[g/cm^3\right]$

Flux (# particles / time) through area $A = A_\perp$

$$\dot N = \frac{\Delta N}{\Delta t} = j\cdot A = \rho\cdot u\cdot A \rightarrow \frac{\Delta m}{\Delta t} = \rho_m\cdot\underbrace{(u\cdot A)}_{dV/dt}$$

Incompressibility ($\rho_1 = \rho_2$)→equal # of particles flow out of V_1 and into V_2.

$$|\dot N_1| = j_1\cdot A_1 = \rho\cdot u_1\cdot A_1 = \rho\cdot u_2\cdot A_2 = j_2\cdot A_2 = |\dot N_2|$$

Pushing particles from V_1 to V_2 requires **work**
$\Delta w = -p\cdot\Delta V = \Delta(E_{kin}+U_{pot})$ → **power=**

$$\frac{\Delta w_{1\to2}}{\Delta t} = \left[\overbrace{(p_2\cdot A_2)}^{Force}\cdot u_2 - (p_1\cdot A_1)\cdot u_1\right] = \overset{\Delta V/\Delta t}{\frac{\Delta m}{\rho_m\Delta t}}(p_2 - p_1)$$

$$Loss(?)\ \ \Delta E_{kin} = (1/2)\cdot\rho_m\cdot\left(u_2^2\cdot A_2 - u_1^2\cdot A_1\right)$$

$$Gain(?)\ \ \Delta V_{Pot} = \Delta m\left(\upsilon_{Pot,2} - \upsilon_{Pot,1}\right),\quad \Delta m = \rho_m\cdot A\cdot u\cdot\Delta t$$

$$\frac{\Delta m}{\rho_m\Delta t}(p_1 - p_2) = (1/2)\frac{\Delta m}{\Delta t}\left(u_2^2 - u_1^2\right) + \frac{\Delta m}{\Delta t}\left(\upsilon_{Pot,2} - \upsilon_{Pot,1}\right)$$

Continuity Equation

$$j_m\cdot A = \rho_m\cdot u\cdot A = \frac{\Delta m}{\Delta t} = const.$$

Bernoulli Equation

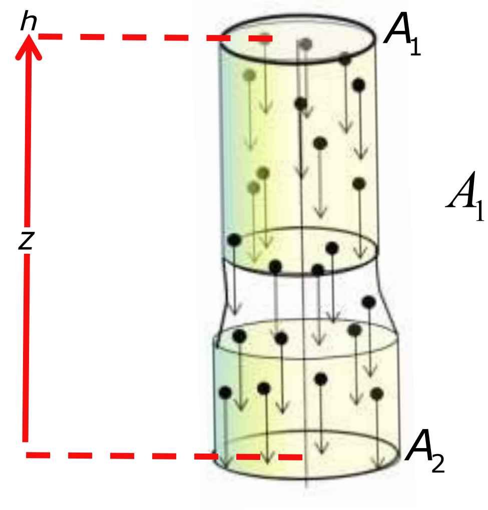
$$p + (1/2)\rho_m u^2 + \rho_m\upsilon_{Pot} = \frac{E}{V} = const.$$

Gravitational Flow Energy Conversion

Apply *Bernoulli Equation* to flow of water with gravitational energy $\Delta V_{pot}=V_{pot}=m\cdot g\cdot h$

At $z=\boldsymbol{h}$: $p=0,\ u=0,\ V_{pot}=m\cdot g\cdot h$

$$\cancel{p} + \frac{1}{2}\rho_m \cancel{u^2} + \rho_m \upsilon_{Pot} = \rho_m \cdot g \cdot h$$



$$A_1 = A_2 = A$$

At bottom, $z=0$: $V_{pot}:=0$
$p \le \rho_m\cdot g\cdot h \quad (= \text{for } u=0)$

$$p + \frac{1}{2}\rho_m u^2 + \rho_m \cancel{\upsilon_{Pot}} = \rho_m \cdot g \cdot h$$

Static backup pressure

In free fall through potential difference $\Delta V_{pot}=m\cdot g\cdot h$, no static "backup" pressure differential $(p=0)$ → ***"jet"***

$$E_{kin} \text{ per } \Delta V \rightarrow (1/2)\,\rho_m u^2 = \rho_m \cdot g \cdot h \rightarrow \boxed{u = \sqrt{2g\cdot h}}$$

If stream with velocity u exits through area A, →
Volume flow rate $\dot{Q} := \dot{V} = dV/dt$ and power $P =$

$$\dot{Q} = \frac{dV}{dt} = A\cdot u = A\cdot\sqrt{2\cdot g\cdot h} \qquad [\,] = Volume/Time$$

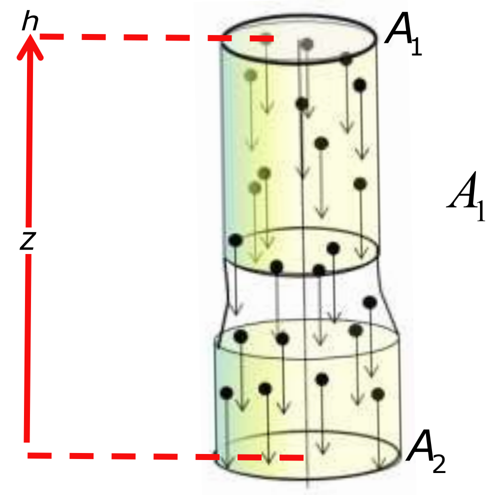
$$P = \underbrace{\dot{Q}\cdot(\rho_m}_{dm/dt}\cdot g\cdot h) \approx 45\cdot A\cdot h^{3/2}\ kW$$

Gravitational Flow Energy Conversion

Apply *Bernoulli Equation* to flow of water with gravitational energy $\Delta V_{pot}=V_{pot}=m\cdot g\cdot h$

At $z=\boldsymbol{h}$: $p=0$, $u=0$, $V_{pot}=m\cdot g\cdot h$

$$\cancel{p} + \frac{1}{2}\rho_m \cancel{u^2} + \rho_m \upsilon_{Pot} = \rho_m \cdot g \cdot h$$



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$$\cancel{p} + \frac{1}{2}\rho_m u^2 + \rho_m \cancel{\upsilon_{Pot}} = \rho_m \cdot g \cdot h$$

Static backup pressure, set=0

In free fall through potential difference $\Delta V_{pot}=m\cdot g\cdot h$, no static "backup" pressure differential $(p=0)$ → ***"jet"***

$$E_{kin} \text{ per } \Delta V \rightarrow (1/2)\,\rho_m u^2 = \rho_m\cdot g\cdot h \rightarrow \boxed{u=\sqrt{2g\cdot h}}$$

If stream with velocity u exits through area A, →
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$$P = \underbrace{\dot{Q}\cdot(\rho_m}_{dm/dt}\cdot g\cdot h) \approx 45\cdot A\cdot h^{3/2}\ kW \qquad \text{Rule of Thumb}$$

Example: Head at $h=175\ m$, diameter of penstock $d=3\ m$ ($A = \pi\cdot(d/2)^2=2.41\text{m}^2$)

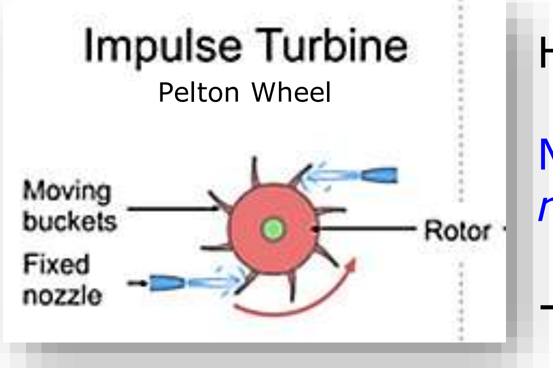
$$u = \sqrt{2\cdot g\cdot h} = \sqrt{2\cdot 9.81\cdot 175}\,\frac{m}{s} = 58.6\,\frac{m}{s}$$

$$\dot{Q} = A\cdot u = 2.41\,m^2\cdot 58.6\,\frac{m}{s} = 141.3\,\frac{m^3}{s} = 1.4\cdot 10^5\ L/s$$

$$\rightarrow \boxed{P = 251\ MW\ (= P_{max})\quad \text{contained in flow}}$$

Power generation in turbines

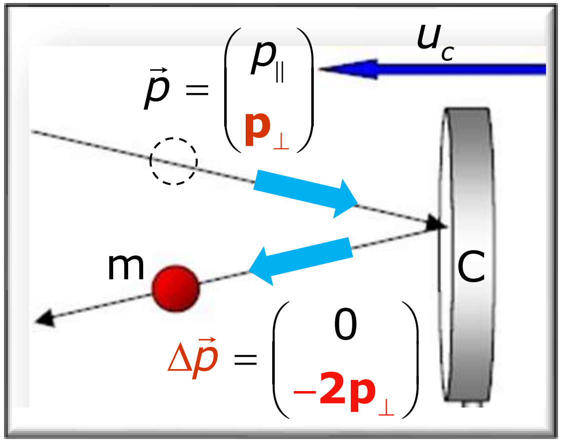
Power Generation in Impulse Turbines



Heads > 300m @ atm. *P*

Momentum transfer $\Delta p_\perp$ by *m* colliding with bucket (C),

→ *L*-transfer $\Delta\vec{L} = \vec{r} \times \Delta\vec{p}_\perp$



$$\textit{Mass } (H_2O) \textit{ flow density}: \; j_m(u) = \rho_m \cdot u \left[\frac{g}{cm^2 \cdot \Delta t}\right]$$

Bucket area A_c *is hit per* Δt *by (velocity jet* u*, bucket* u_c*)*

$\Delta m \approx j_m(u - u_c) \cdot A_c \cdot \Delta t$, *rel. momentum* $p = \Delta m \cdot (u - u_c)$,

$(u - u_c)$ = *speed relative to bucket*

initally $u_c \ll u$, *increases in time.*

$\rightarrow \boxed{\textit{transfer } |\Delta p| = 2p \textit{ to cup in } \Delta t} \rightarrow \boxed{\text{Force F} = \Delta p/\Delta t}$

$$\rightarrow F = 2p/\Delta t \approx 2 \cdot \left[j_m\left(u - u_c\right) \cdot A_c\right] \cdot \left(u - u_c\right) = 2\rho_m A_c \left(u - u_c\right)^2$$

Example of dissipative force

$$\textit{Energy (work) transfer to bucket}: \Delta E = F \cdot \Delta x = F \cdot u_c \cdot \Delta t$$

$$\textit{Power transferred to bucket}: \boxed{P = F \cdot u_c = \left[2\rho_m A_c\right]\left(u - u_c\right)^2 \cdot u_c}$$

Speed of bucket increases, *maximum power* *transfer* $P_{\max} \rightarrow \boxed{u_c = \frac{1}{3}u}$

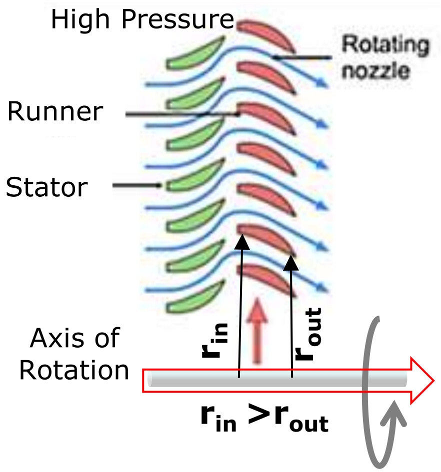
$$P_{\max} = \frac{2}{27}\left[\rho_m \cdot A_c\right] \cdot u^3 = \frac{4}{27}\left[\frac{1}{2}\rho_m \cdot u^2\right] \cdot \underbrace{\left(A_c \cdot u\right)}_{\dot{Q} = dV/dt} \rightarrow \boxed{P_{\max} = \frac{4}{27}\frac{d}{dt}\left(\frac{M}{2} \cdot u^2\right)}$$

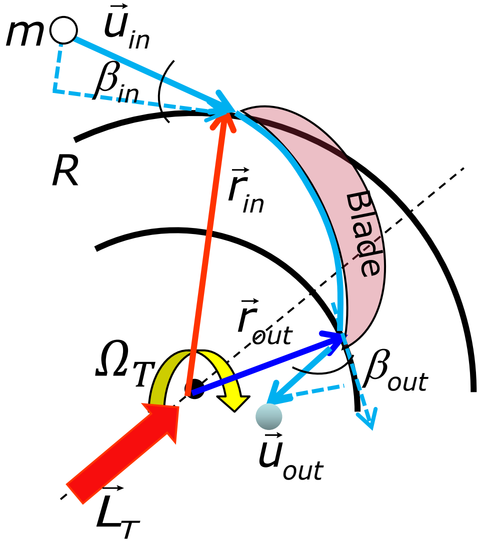
$$\boxed{\varepsilon \lesssim 16\%}$$

Other Turbines

Angular Momentum Transfer in Reaction Turbines

Gen Reaction Turbine



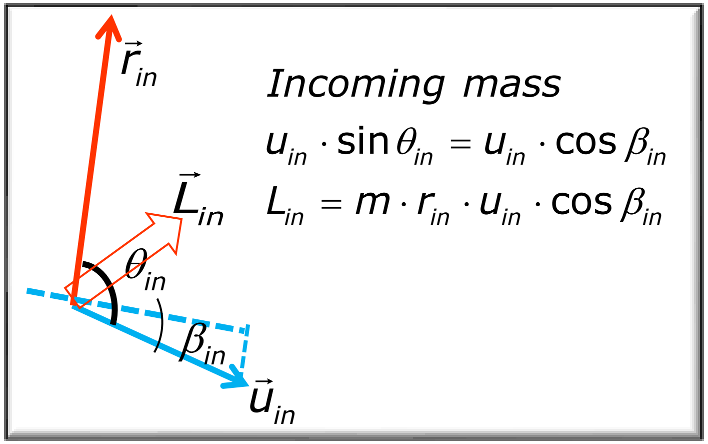


Energy transfer leads to speed up of turbine rotation = increased angular momentum $\vec{L}_T$ by $\Delta\vec{L}_T$.

Fluid has lost this angular momentum.

$$\boxed{\vec{L}_{out} = \vec{L}_{in} - \Delta\vec{L}_T}$$

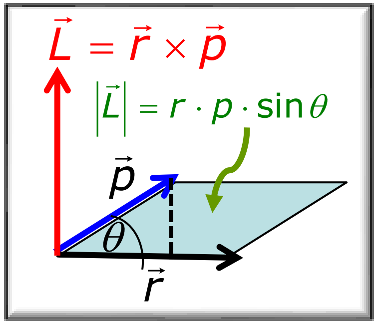
all parallel



Incoming mass

$$u_{in} \cdot \sin\theta_{in} = u_{in} \cdot \cos\beta_{in}$$

$$L_{in} = m \cdot r_{in} \cdot u_{in} \cdot \cos\beta_{in}$$



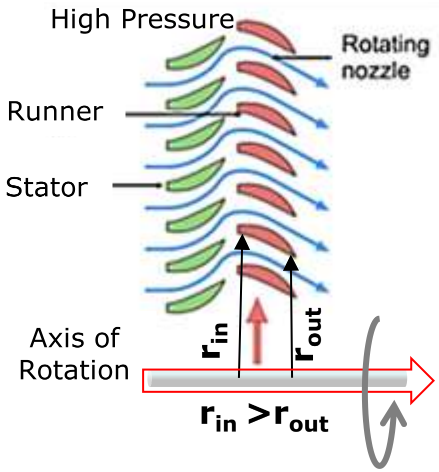
$$\vec{L} = \vec{r} \times \vec{p}$$

$$\left|\vec{L}\right| = r \cdot p \cdot \sin\theta$$

Outer vector product of two vectors $\vec{r}$ and $\vec{p}$

Angular Momentum Transfer in Turbines

Gen Reaction Turbine



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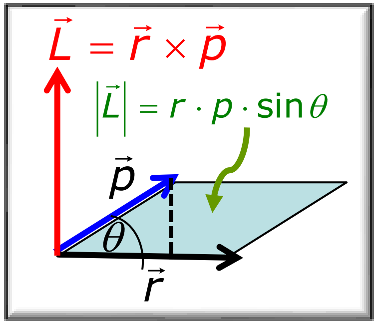
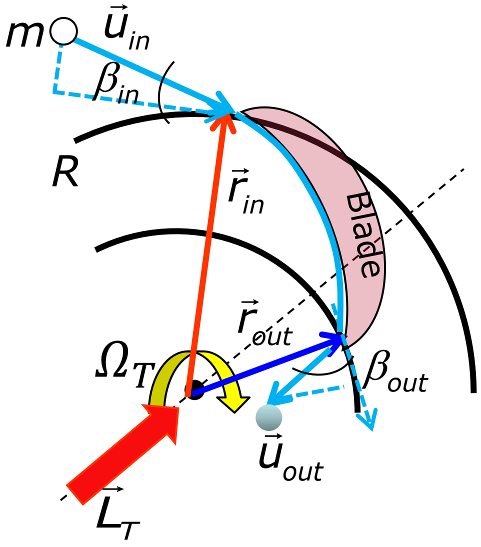
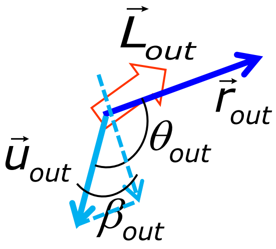
all parallel



Outer vector product of two vectors $\vec{r}$ and $\vec{p}$





Outgoing mass

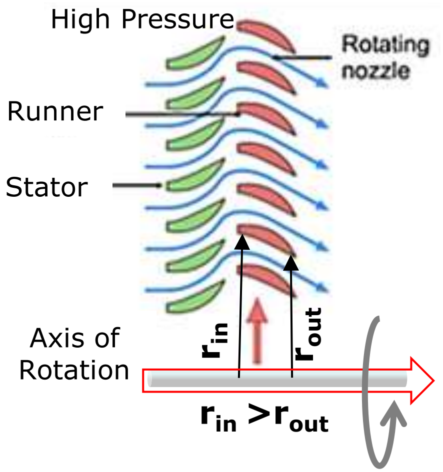
$$u_{out} \cdot \sin\theta_{out} = u_{out} \cdot \cos\beta_{out}$$

$$L_{out} = m \cdot r_{out} \cdot u_{out} \cdot \cos\beta_{out}$$

$$\Delta L_T = L_{in} - L_{out}$$

Angular Momentum Transfer in Hydro Turbines

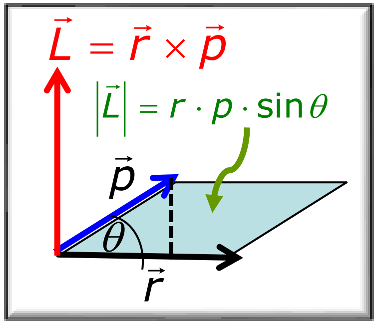
Gen Reaction Turbine



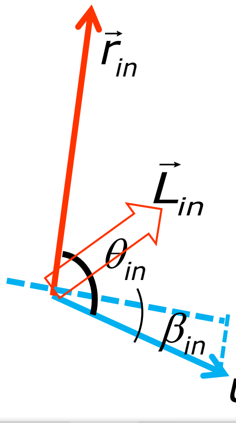
Energy transfer leads to speed up of turbine rotation = increased angular momentum $\vec{L}_T$ by $\Delta\vec{L}_T$.

Fluid has lost this angular momentum.

$\boxed{\vec{L}_{out} = \vec{L}_{in} - \Delta\vec{L}_T}$ all parallel



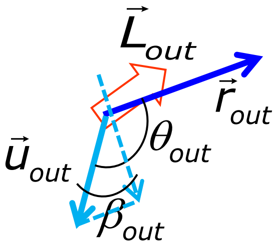
Outer vector product of two vectors $\vec{r}$ and $\vec{p}$



Incoming mass

$$u_{in} \cdot \sin\theta_{in} = u_{in} \cdot \cos\beta_{in}$$

$$L_{in} = r_{in} \cdot \underbrace{\left(m \cdot u_{in}\right)}_{p_{in}} \cdot \cos\beta_{in}$$

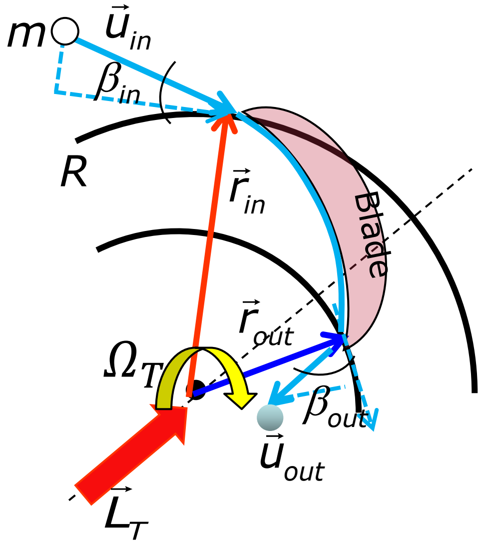


Outgoing mass

$$u_{out} \cdot \sin\theta_{out} = u_{out} \cdot \cos\beta_{out}$$

$$L_{out} = r_{out} \cdot \underbrace{\left(m \cdot u_{out}\right)}_{p_{out}} \cdot \cos\beta_{out}$$

Transferred : $\Delta L_T = L_{in} - L_{out}$



$$\rightarrow \textit{Torque}\ M = \Delta L_T / \Delta t = \dot{m} \cdot \left(r_{in} \cdot u_{in} \cdot \cos\beta_{in} - r_{out} \cdot u_{out} \cdot \cos\beta_{out}\right)$$

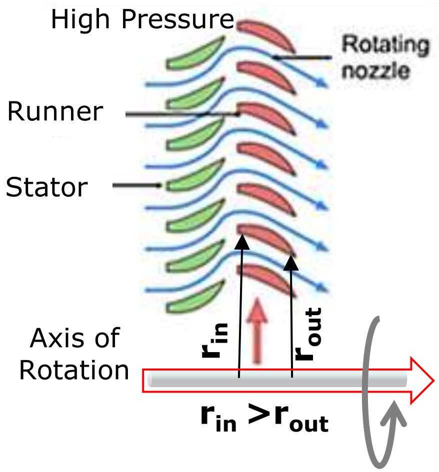
Power $P = M \cdot \Omega_T$; *Mass flow* $\dot{m} = \rho_m \cdot \dot{Q}$

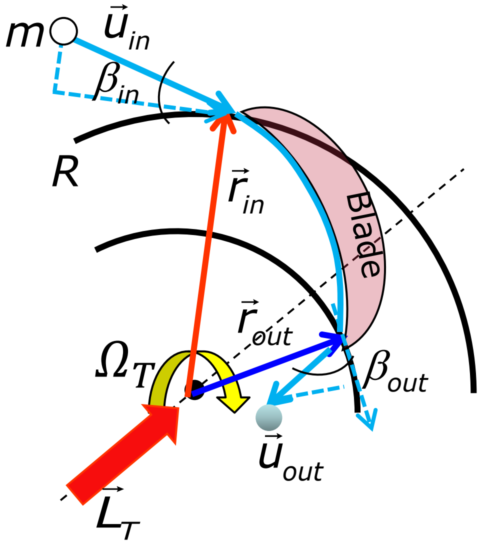
Euler's Turbine Equation

$$\boxed{P = \Omega_T \cdot \rho_m \cdot \dot{Q} \cdot \left(r_{in} \cdot u_{in} \cdot \cos\beta_{in} - r_{out} \cdot u_{out} \cdot \cos\beta_{out}\right)}$$

Angular Momentum Transfer in Rxn Turbines

Gen Reaction Turbine





Angular momentum to turbine (runner) by driving fluid (water)

$$\boxed{\Delta\vec{L}_T = \vec{L}_{in} - \vec{L}_{out};\ Torque\ M = \Delta L_T/\Delta t;\ Power\ P = M\cdot\Omega_T}$$

Turbine power does not depend on many construction details but blade geometry → maximize angular momentum transfer!

$$\boxed{P = \Omega_T\cdot\rho_m\cdot\dot{Q}\cdot\left[\left(\vec{r}\times\vec{u}\right)_{in} - \left(\vec{r}\times\vec{u}\right)_{out}\right]}$$

Euler's Turbine Equation

Power is maximized if fluid brings in maximum angular momentum ($\beta_{in}=0^0$) and carries no angular momentum on the way out ($\beta_{out}=90^0$) → tangential inflow & radial outflow.

$$P_{\max}\left(r_{in}\right) = \Omega_T\cdot\underbrace{\rho_m\cdot\dot{Q}}_{=dm/dt}\cdot r_{in}\cdot u_{in}\cdot\cos\beta_{in} \rightarrow$$

$$\boxed{P_{\max}\le\dot{m}\cdot R\cdot\Omega_T\cdot\left(u_{in}\right)_{tang}}$$

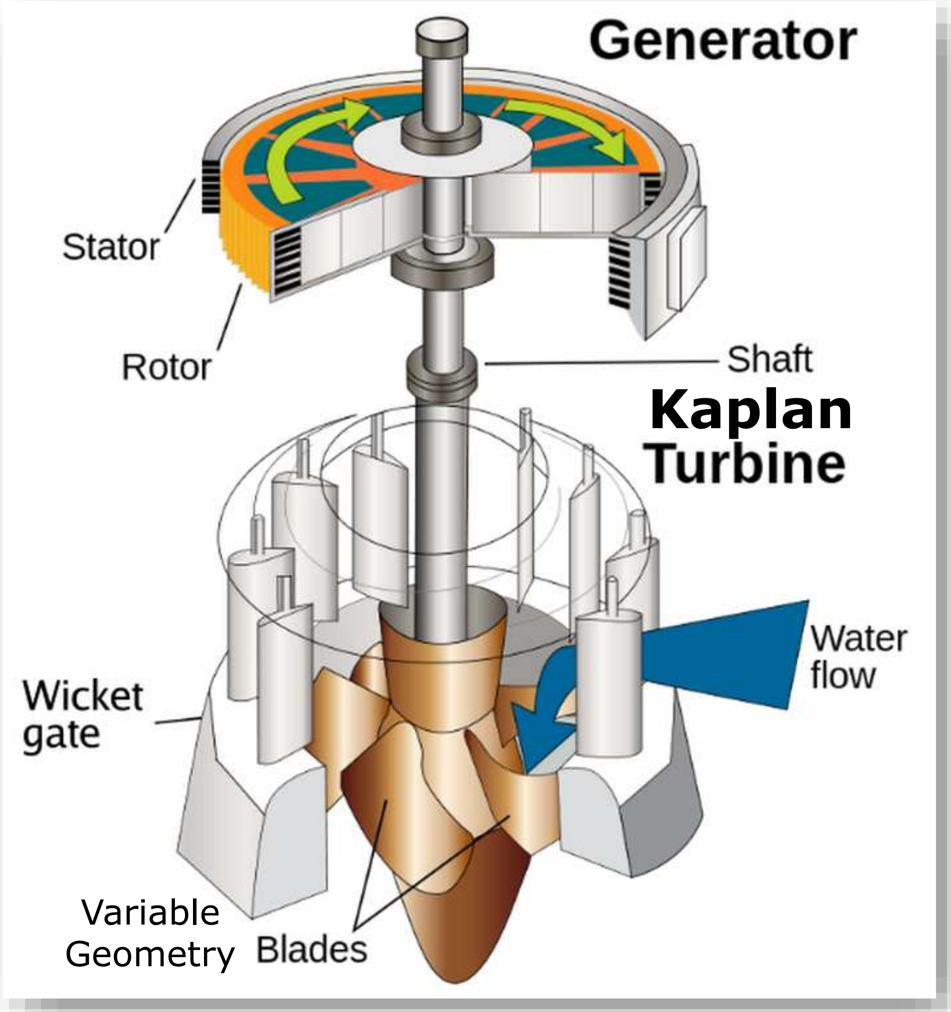
$R =$ *injection radius for turbine,*

$\left(u_{in}\right)_{tang} =$ *tangential jet velocity*

High power produced by large turbines: high water inflow + tangential injection ($\beta_{in}=0^0$) + radial outflow ($\beta_{out}=90^0$).

Synchronized el. power output steered by governor circuitry controlling gate position.

Turbine Blade Arrangements

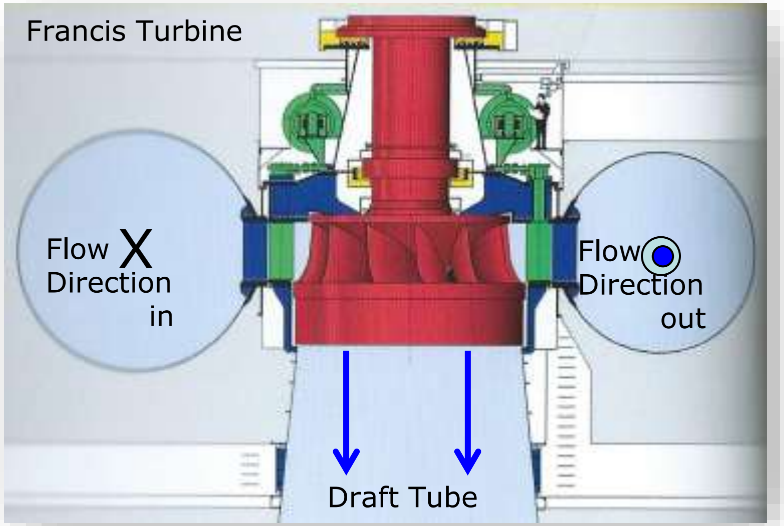


Wikipedia

Propeller turbines for low heads. Fixed blades or variable pitch. Schematics of power generation with a Kaplan turbine = high efficiency @all loads.

Francis turbine: Heads< 360 m.
Guide vanes→ tangential injection → radial out flow
"Runner"

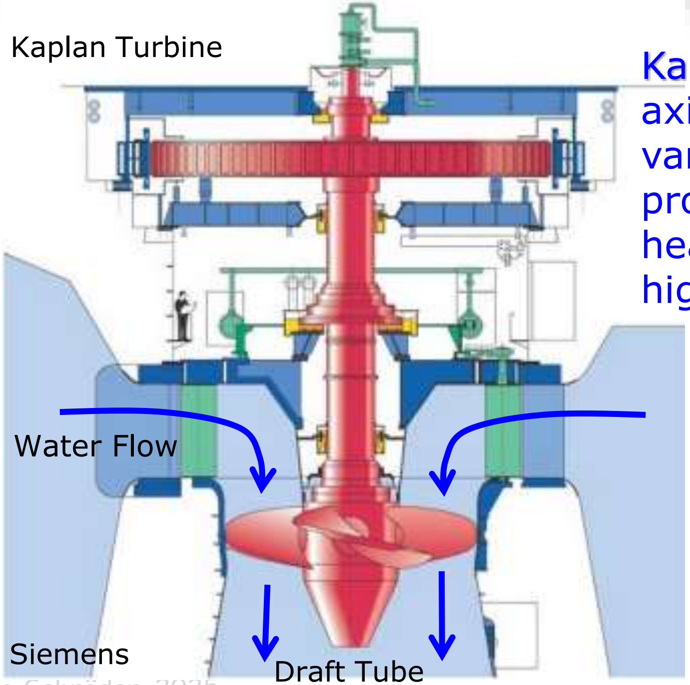
Turbine Types



Hydro turbines are impact or reaction turbines.

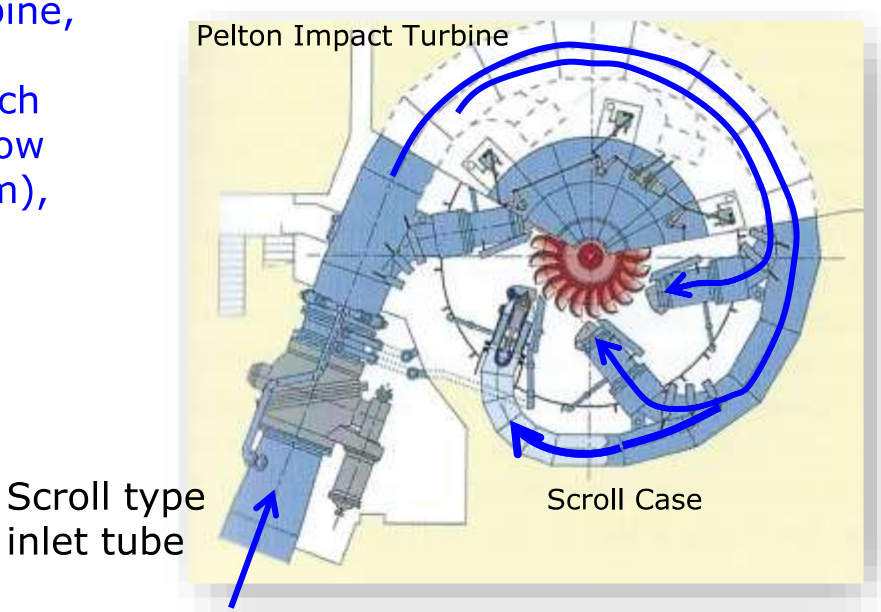
Francis Turbine, radial flow, dia 0.5- 6 m
Fully submerged, horizontal or vertical modes.
Axial outflow.

Popular design, versatile & useful for very different effective heads

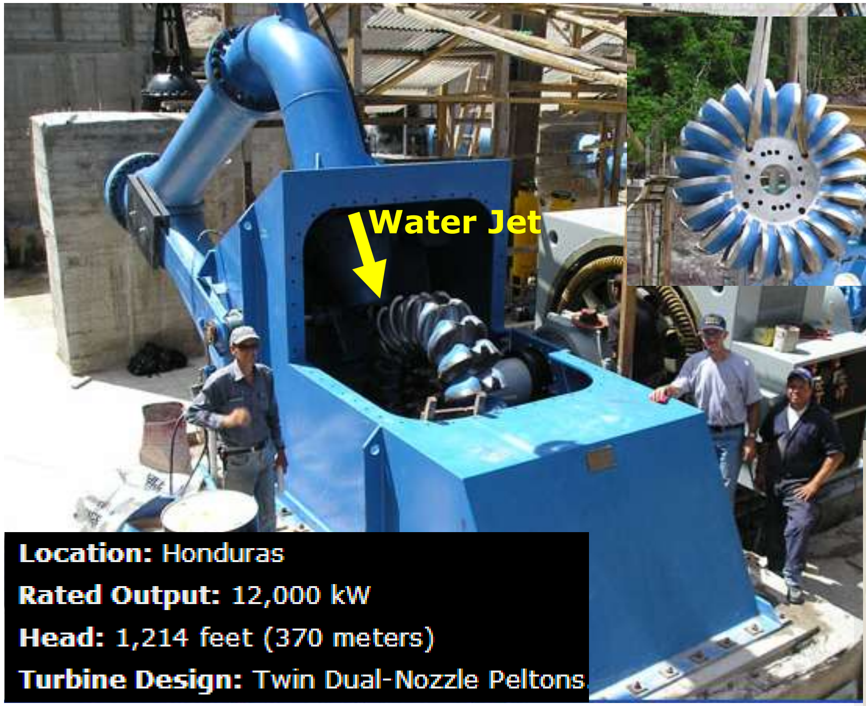


Kaplan Turbine, axial flow, variable-pitch propeller. Low head (<50m), high flow.

Pelton impact/impulse turbine, tangential flow, fixed buckets, low head, low/medium flow.



Francis & Pelton Hydro Turbines



Location: Honduras
Rated Output: 12,000 kW
Head: 1,214 feet (370 meters)
Turbine Design: Twin Dual-Nozzle Peltons

Canyon Hydro

Upper Swift Creek
Location: Wyoming, USA
Rated Output: 910 kW
Head: 140 feet (42.7 meters)
Turbine Design: Francis

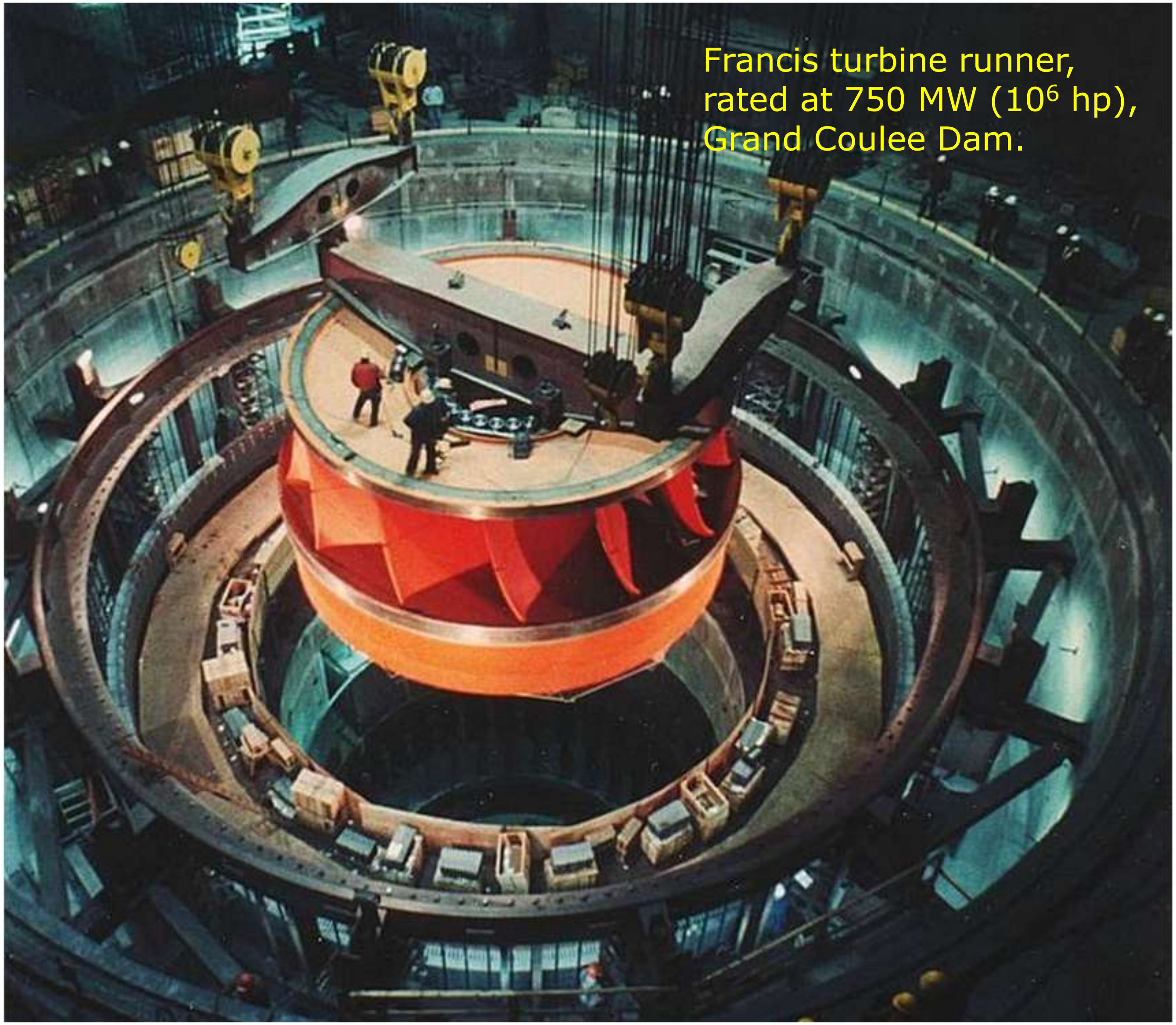
Kaplan & Francis Hydro Turbine Rotors

Public, Wikipedia

A Kaplan turbine after 61 years of service @ Bonneville Dam (313m long, head 23m), Columbia River (US), 8 generators, Total capacity of 558.2 MW.

A Three-Gorges Dam (China) Francis turbine runner before installation.

Grand Coulee Power Plant



Francis turbine runner, rated at 750 MW (10^6 hp), Grand Coulee Dam.