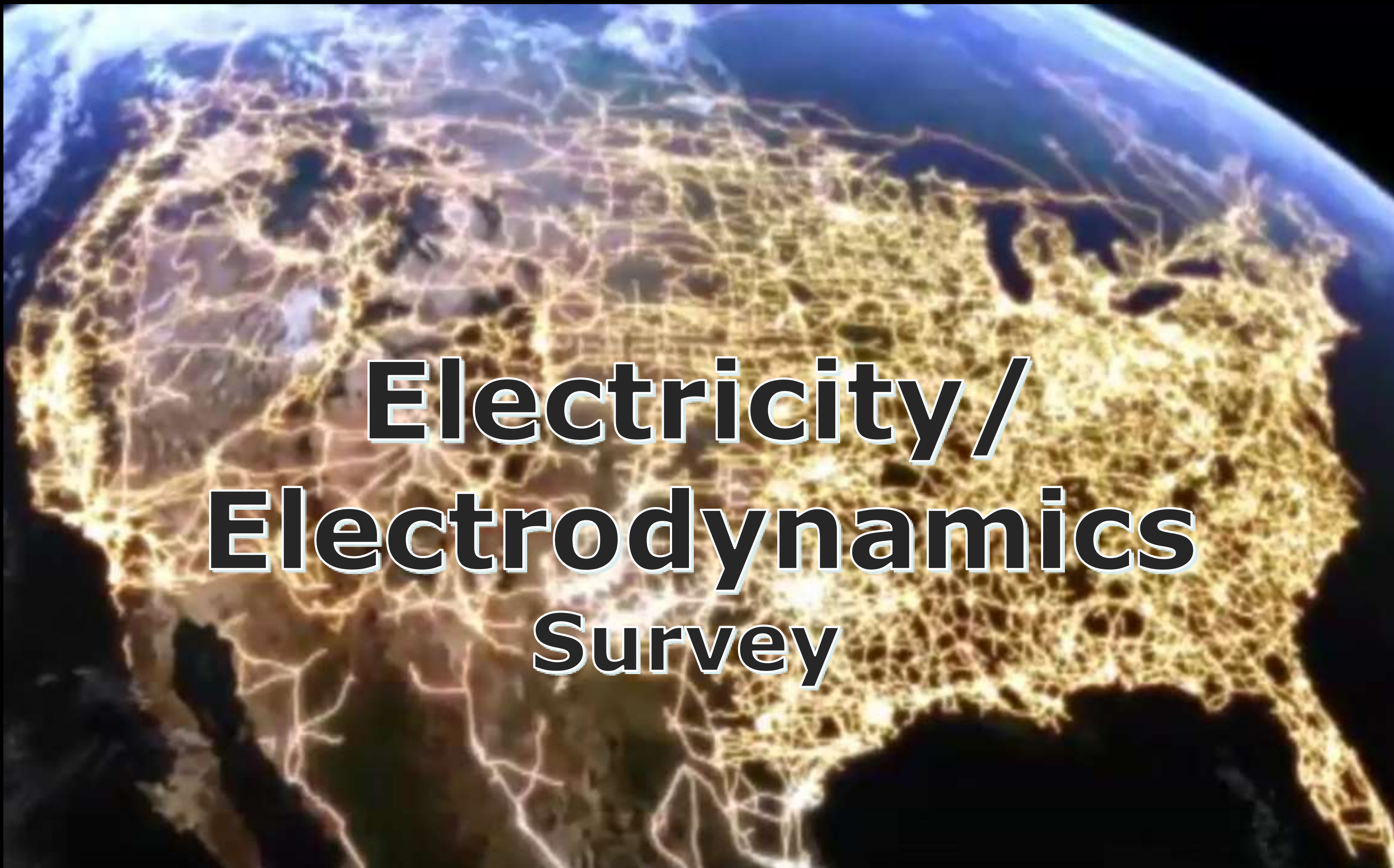


Electricity/ Electrodynamics

Survey

Agenda: Energy Conversion and Transformation

Work and other Energy Forms

Potential and kinetic energy,

Molecular binding and rearrangement energies,

pV work, kinetic energy equilibration, heat flow,
Mechanical equivalent of heat,
Basic fluid dynamics, laminar & turbulent flow, (→later)

Basic Electricity

Static electric, electro-magnetic phenomena,
Electromagnetic induction, generators, transformers,
Electrical current laws, AC/DC transmission,

Electronic circuits, reactance,

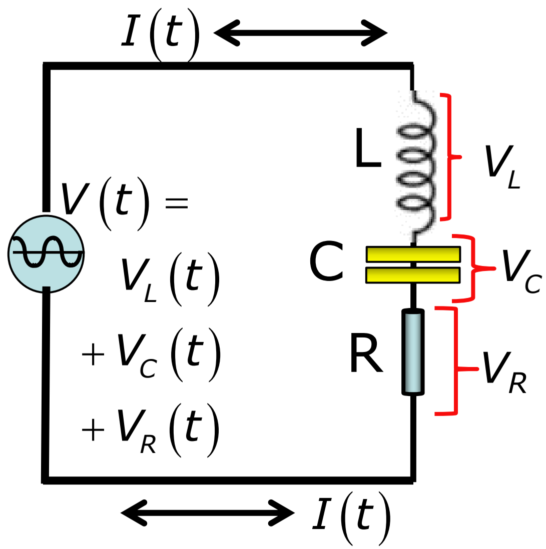
Principles of Thermodynamic Processes

Laws of Thermodynamics, state functions, reversible processes,

Carnot and other TD cycles, steam engines, gas turbines,

Electro-chemistry, batteries, hydrolyzers & fuel cells.

Basic Electrical Network Conservation Laws



$$\boxed{V(t) = V_L(t) + V_C(t) + V_R(t)}$$

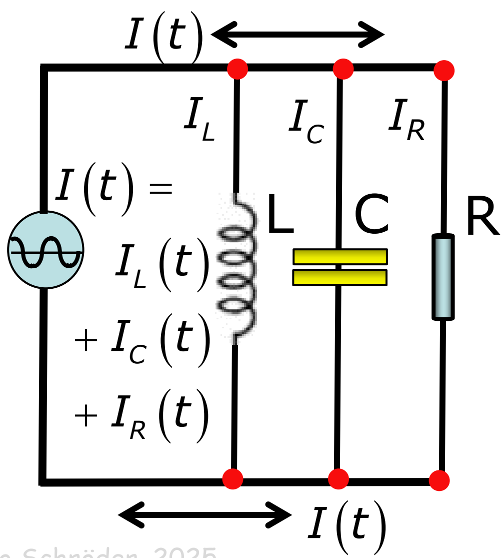
Kirchhoff's Loop (or mesh) Rule

The directed sum of the potential differences (voltages) over circuit elements is zero around any closed loop.

$$\boxed{\text{Ohm's Law } V(t) = Z \cdot I\left(t + \Delta t_{\phi}\right)}$$

$(\Delta t_{\phi} \leftarrow \Delta\phi_I = \textit{phase difference } I \textit{ rel } V)$

1. Ohm resistance Z = $\mathrm{Re}(Z)$ = R
2. Capacitive reactance $X_C = -1/\omega C$ ($\Delta\phi_I = +90^0$)
3. Inductive reactance $X_L = \omega L$ ($\Delta\phi_I = -90^0$)



$$\boxed{I(t) = I_L(t) + I_C(t) + I_R(t)}$$

Kirchhoff's Junction Rule for currents

(Parallel Loops)

The algebraic sum of currents in a network of conductors meeting at a point is zero.

Complex Notation

Phase differences are conveniently handled in complex notation *V(t), I(t)*

$$V(t)=|V(t)|\cdot e^{i\cdot(\varpi\cdot t+\phi_V)}; \quad I(t)=|I(t)|\cdot e^{i\cdot(\varpi\cdot t+\phi_I)}$$

Amplitudes and phases are determined from initial conditions, *V(t=0), I(t=0).*

Example: $V(t=0)=V_0 \rightarrow V(t)=V_0\cdot\cos(\varpi\cdot t)$

Ohm's Law $\boxed{V(t)=Z\cdot I(t+\Delta t_\phi)}$ *in circuits at all times*

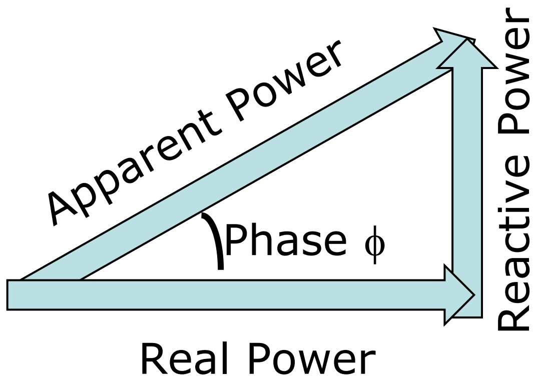
Impedance $Z(\omega) = R + \boldsymbol{i}\cdot[X_C(\omega) + X_L(\omega)]$ $(i := \sqrt{-1})$

1. Ohm resistance *R*
2. Capacitive reactance $X_C = -1/\omega C$ $(\Delta\phi_I = +90^0)$
3. Inductive reactance $X_L = \omega L$ $(\Delta\phi_I = -90^0)$

$$Euler's\ Formula: i = e^{i\cdot\pi/2} = \cos(\pi/2) + i\cdot\sin(\pi/2)$$

Real and Reactive Power

$Power\ \boxed{P(t) = V(t)\cdot I(t)}$ or, in complex notation $\boxed{P = V\cdot I^*}$ $(* = complex\ conjugate)$



$\color{blue}{Real}$ $(Ohm)\ resistive\ loads: P_R = V_R\cdot I = V^2/R = I^2\cdot R$

$\color{blue}{Real}$ $= apparent\ power\ P_R(t) = V_R(t)\cdot I(t)\quad [P_R] = W$

$Oscillating\ reactive\ power\ P_C(t) \rightleftarrows P_L(t)$

$Reactive\ power\ P_{LC}(t) = |P_A(t)|\cdot \sin\phi$

$\color{red}{"var"\ voltage - ampere - reactive}$

$Complex:$ $\color{red}{Real\ and\ reactive\ power\ are\ "out\ of\ phase"}$

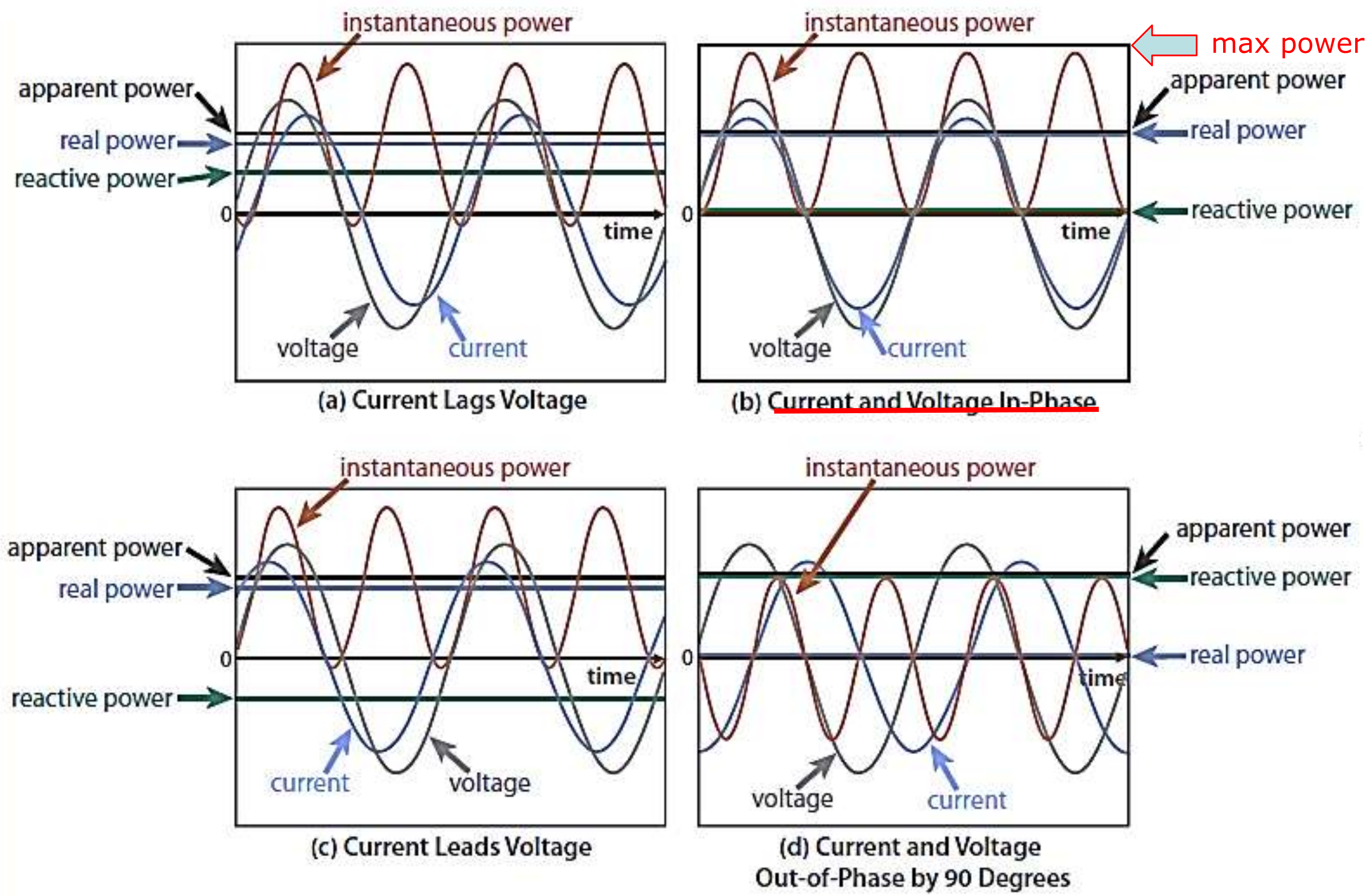
$\color{red}{Apparent\ power: P_A^2(t) = P_R^2(t) + \left[P_C^2(t) + P_L^2(t)\right]}$

$In\ General:\ Apparent\ power\ P_A(t) = \sqrt{P_R^2(t) + P_{LC}^2(t)}$

$Useful\ real\ power\ \boxed{P_R(t) = |P_A(t)|\cdot\cos\phi} \rightarrow$ $\color{blue}{Power\ Factor = P_R/|P_A|}$

Actual loads on the power supply (e-grid) like an e-motor are always complex (Ohm + capacitive + inductive) → have feedback effect on supply → Affect power factor (available power) and frequency.
→ General effect on stability of grid.

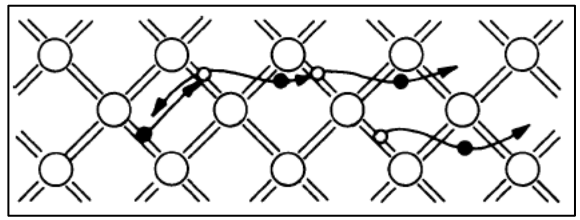
Effect of Reactance in AC System



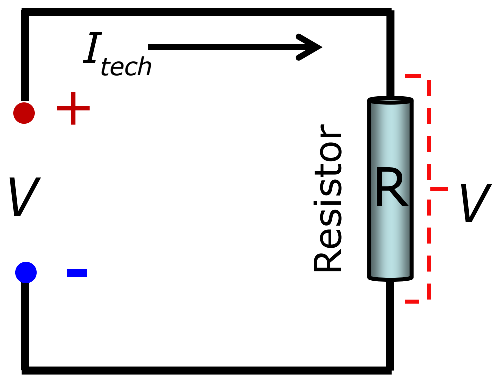
Components of Electrical DC Circuits: Resistors

Electric current I unit=[1 A(mpere)= 1C/s]:
e^- stream [dq/dt=#e^-/sec] transfers power through metallic wires, dissipates e^- energy→ heat. All metallic wires have intrinsic (distributed) resistivity ≠0. Always finite electric conductivity σ = inverse of resistivity $\rho \neq 0$!!

$$[\sigma] = \left(A/m^2\right)\Big/\left(V/m\right)$$
$$= \left(A/V\right)/m = 1/\Omega \cdot m$$
$$[\rho] = \Omega \cdot m$$



Multiple scattering of e^- in lattice of ***R***



Ohm's Law, ***R*** real (no ω dependence)

$$\boxed{I = \frac{V}{R} \rightarrow V = I \cdot R}$$

$$[R] = 1\Omega\ (Ohm) = \frac{1V}{1A}$$

Power dissipated in resistor (Temp ↗)

$$\boxed{P = V \cdot I = I^2 \cdot R} \qquad [P] = W\ (Watt) = J/s$$

Commercial resistors are made of materials with high electric resistivity ρ.
Ref. Cu: ρ =1.68·10^{-8}Ω·m

Components of Electrical AC Networks: Resistors

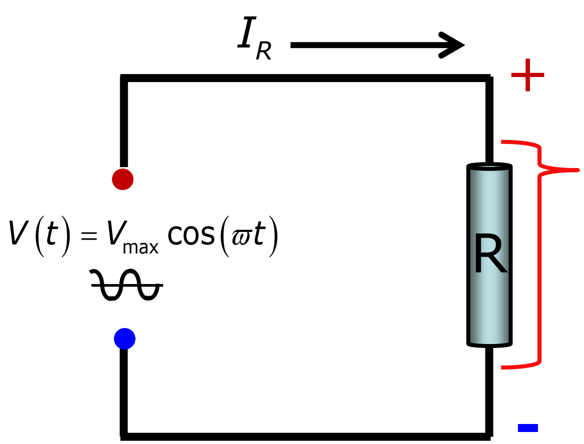
Transfer of electrical power through metallic wires → Electric conductivity σ → Electrons dissipate kinetic energy through scattering
→ Ohm resistance $R \sim 1/\sigma \quad [R] = \Omega\ (Ohm)$ Extension: R=generic workload

$Applied\ AC\ voltage\ [V] = V\ (Volt)$

$\boxed{V(t) = V_{\max}\cos(\varpi t)} \rightarrow effective\ V = \langle V(t)\rangle = V_{\max}/\sqrt{2}$

$effective\ I = \langle I(t)\rangle = I_{\max}/\sqrt{2}$

Averaged over 1 period



$I_R(t) = I_{\max}\cos(\varpi t)$; $\boxed{I_{\max} = \frac{V_{\max}}{R}}$ Ohm's Law

$Current\ I_R(t)\ \{[I] = A\ (Ampere)\}$ *in phase with* $V(t)$

Power dissipated in resistor via scattering

$$P_R(t) = V(t)\cdot I_R(t) = V_{\max}I_{\max}\cos^2(\varpi t)$$

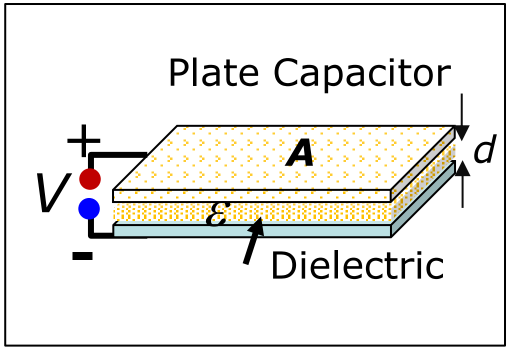
$$= \frac{1}{2}V_{\max}I_{\max}\{1+\cos(2\varpi t)\} = V\cdot I_R\{1+\cos(2\varpi t)\}$$

effective

Dissipated power $\boxed{\langle P_R(t)\rangle = V\cdot I_R = \frac{V^2}{R} = I_R^2\cdot R}$

This is real power loss, dissipated into heat (no w).

Components of Electrical DC Circuits: Capacitors

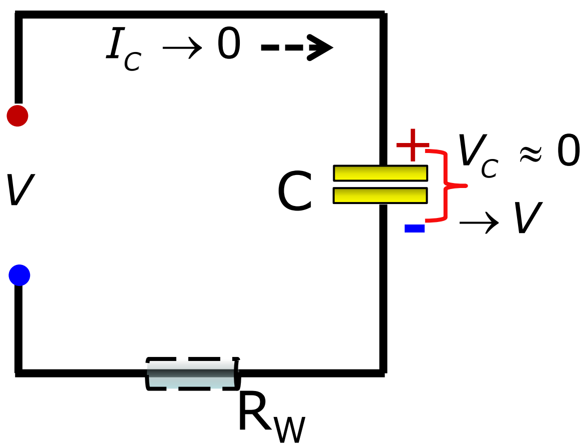


Metal plates (area *A*) separated by dielectric medium (ε) of thickness *d* form a capacitor w. capacitance

$$\boxed{C = \frac{\varepsilon \cdot A}{d}} \quad [C] = F\,(Farad)$$

→Carries charge $\boxed{Q(t) = C \cdot V(t)}$

As "load" element :



$$Switch-on\ voltage\ V \to brief\ \ current\ I_C(t)$$

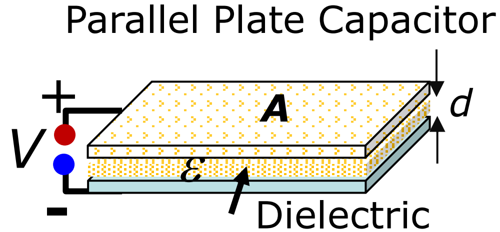
$$V_C(t) = \frac{1}{C}Q(t) = \frac{1}{C}\int^{t} I_C(t')dt' \to V_C(t) = V \cdot \left(1 - e^{-t/\tau}\right)$$

Energy content W_C of capacitor C

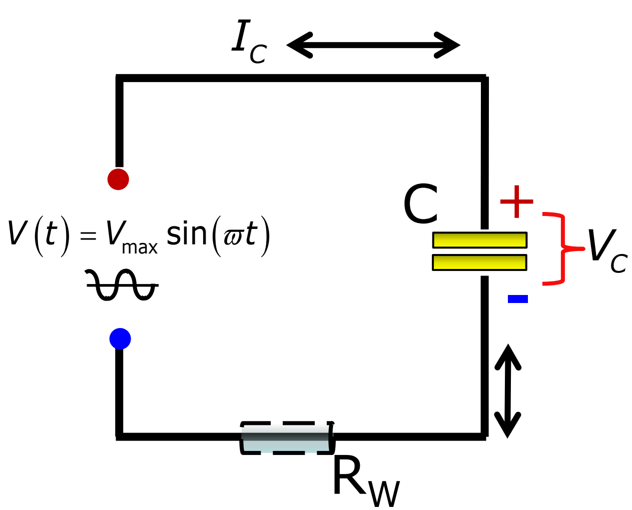
$$\boxed{W_C = \frac{1}{2}Q \cdot V = \frac{C}{2}V^2 = \frac{Q^2}{2C}}$$

A purely capacitive load in an electrical circuit does not lose charge or energy.
But circuit wire conductors have always an additional $R_W \neq 0$.
→ Slow dissipation of energy content of source, *t-const.* $\tau = R_W \cdot C$

Components of Electrical AC Networks: Capacitors



$$C = \frac{\varepsilon \cdot A}{d};\ [C] = F\,(Farad)$$



Metal plates (area A) separated by dielectric medium (ε) of thickness d form a capacitor w. capacitance

As "load" element in AC (frequency ω) circuit:

→Carries electric charge $\boxed{Q(t) = C \cdot V(t)}$

$$V_C(t) = \frac{1}{C}Q(t) = \frac{1}{C}\int^{t} I_C(t')dt' = \frac{I_{max}}{\varpi C}\cos(\varpi t)$$

$$= V_{max}\cos(\varpi t) \rightarrow \boxed{V_{max} = I_{max} \cdot X_C} \quad \text{Ohm's Law}$$

I_C

Reactance $|X_C| = \dfrac{1}{\varpi C}$

Current leads voltage : phase difference $\Delta\phi = +\dfrac{\pi}{2}$

Power in capacitor

$$P_C(t) = V(t) \cdot I_R(t) = V_{max} I_{max} \sin(\varpi t) \cdot \cos(\varpi t)$$

$$= \frac{1}{2} V_{max} I_{max} \{\sin(2\varpi t)\} \rightarrow \boxed{\langle P_C(t) \rangle = 0}$$

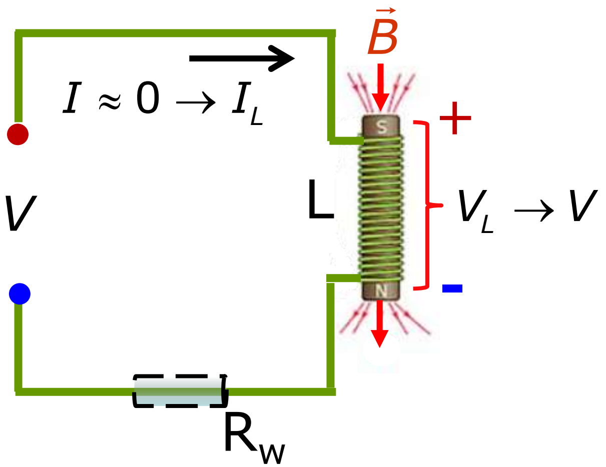
A pure capacitive load in an AC circuit does not dissipate (lose) power.
But circuit wire conductors have always an additional $R_W \neq 0$.
→ Slow dissipation of energy content of source, *time-const.* $\tau = R_W \cdot C$

Components of Electrical DC/AC Circuits: Inductors

Helical coil area A: *N* windings insolated Cu wire around core → plastic ("solenoid") or ferrite/carbon-iron. Applied electric *V* → *current I*, → static axial *B* field ("electro-magnet").

Inductance $\boxed{L = \mu_0 K\left(N^2 A/\ell\right)_{coil}}$ $\quad [L] = H = \frac{V \cdot s}{A} \; (Henry)$

As "load" element in circuit:



$$\boxed{V_L(t) = -L\frac{dI}{dt}} \to Work \;\; \frac{dW_L}{dt} = -L \cdot I \cdot \frac{dI}{dt}$$

Energy content W_L stored in inductor L

$$\boxed{W_L = \frac{1}{2} L \cdot I_L^2}$$

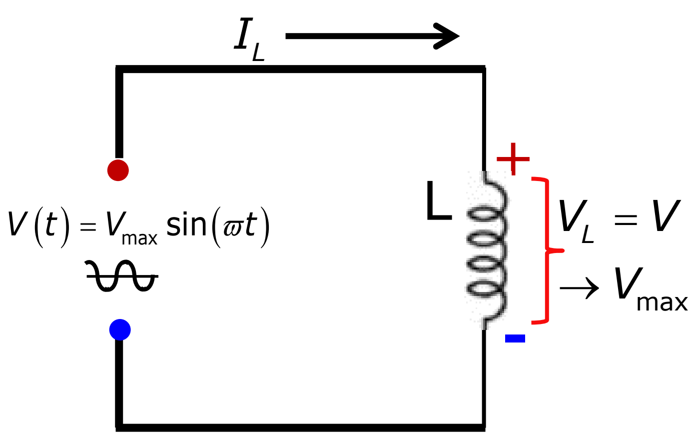
A purely inductive load in an electric circuit does not dissipate electric energy.
But circuit wire conductors have always an additional $R_W \neq 0$.
→ Slow dissipation of energy content of source, *time-const.* $\tau = L/R_w$

Components of Electrical AC Networks: Inductors

Helical coil insolated Cu wire wound around plastic ("solenoid") or ferrite/carbon-iron core. → Large resistance ($|\omega \cdot L|$) to high-frequency AC spikes.

Inductance L, $L_{solenoid} = \mu_0 K \left(N^2 A/\ell \right)_{coil}$ $[L]$=H=Vs/A (Henry)

As "load" element in AC (frequency ω) circuit:



$$V_L(t) = -L\frac{dI}{dt} = -\varpi L \cdot I_{max} \cos(\varpi t)$$
$$= V_{max} \cos(\varpi t) \rightarrow \boxed{V_{max} = I_{max} \cdot X_L}$$

Ohm's Law

Reactance $|X_L| = \varpi L$

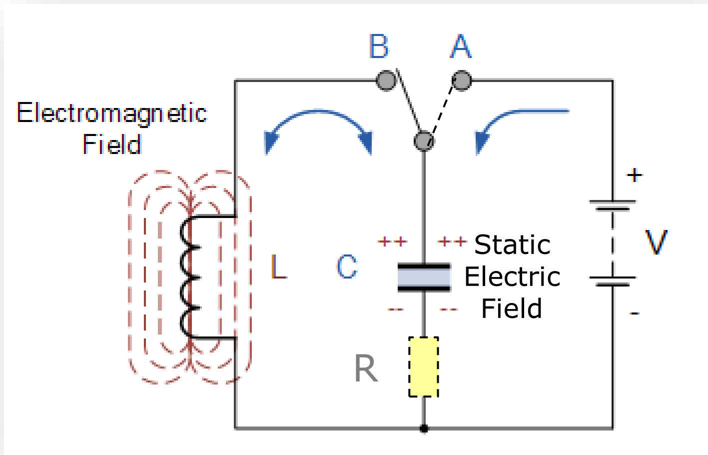
Current lags voltage : phase difference $\Delta\phi = -\frac{\pi}{2}$

Power in inductor

$$P_L(t) = V_L(t) \cdot I(t) = -V_{max} I_{max} \sin(\varpi t) \cdot \cos(\varpi t)$$
$$= -\frac{1}{2} V_{max} I_{max} \left\{ \sin(2\varpi t) \right\} \rightarrow \boxed{\langle P_L(t) \rangle = 0}$$

A pure inductive load in an AC circuit does not dissipate (lose) electric power, it moves energy between electron currents and magnetic field and changes relative phase of current vs. voltage → reactive power

Electronic LC Oscillator (E-Store)



$Kirchoff's\ Laws$: $Closed\ circuit$:

$$I_C = I_L = I$$

$$V_C + V_L = 0,\ V_C = C \cdot \frac{dI_C}{dt},\ V_L = L \cdot \frac{dI_L}{dt}$$

$$Charge\ Q(t),\ I(t) = dQ/dt$$

$$I_C = \frac{dQ_C}{dt} = C \cdot \frac{dV_C}{dt} = I \quad (Neglect\ R\ at\ first)$$

$$\rightarrow I(t) = C \cdot \frac{dV_C}{dt} = -C \cdot \frac{dV_L}{dt} = -L \cdot C \cdot \frac{d^2 I}{dt^2}$$

$$Define\ \omega_0 := \frac{1}{\sqrt{L \cdot C}}$$

Differential Equation for **Oscillator**

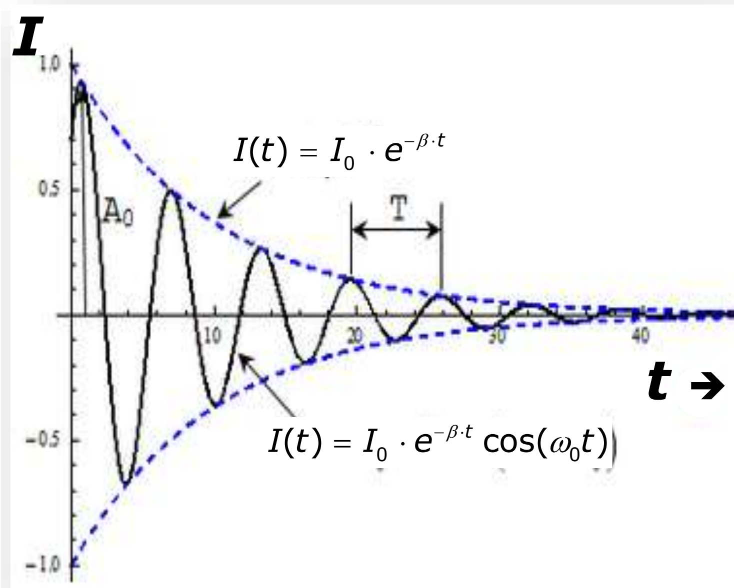
$$\rightarrow \boxed{\frac{d^2 I(t)}{dt^2} + \omega_0^2 I(t) = 0} \quad \rightarrow I(t) = a \cdot e^{i(\omega_0 \cdot t + \varphi)} + b \cdot e^{-i(\omega_0 \cdot t + \varphi)}$$

Constants a, b defined by initial conditions $(t = 0)$

$$a e^{i\varphi} := \frac{I_0}{2},\quad b e^{-i\varphi} = a^* = \frac{I_0}{2} \rightarrow I(t) = I_0 \cdot \cos(\omega_0 \cdot t)$$

Including Ohm resistance $\rightarrow$ damped oscillations

Including Ohm Resistance R



Fin Electricity