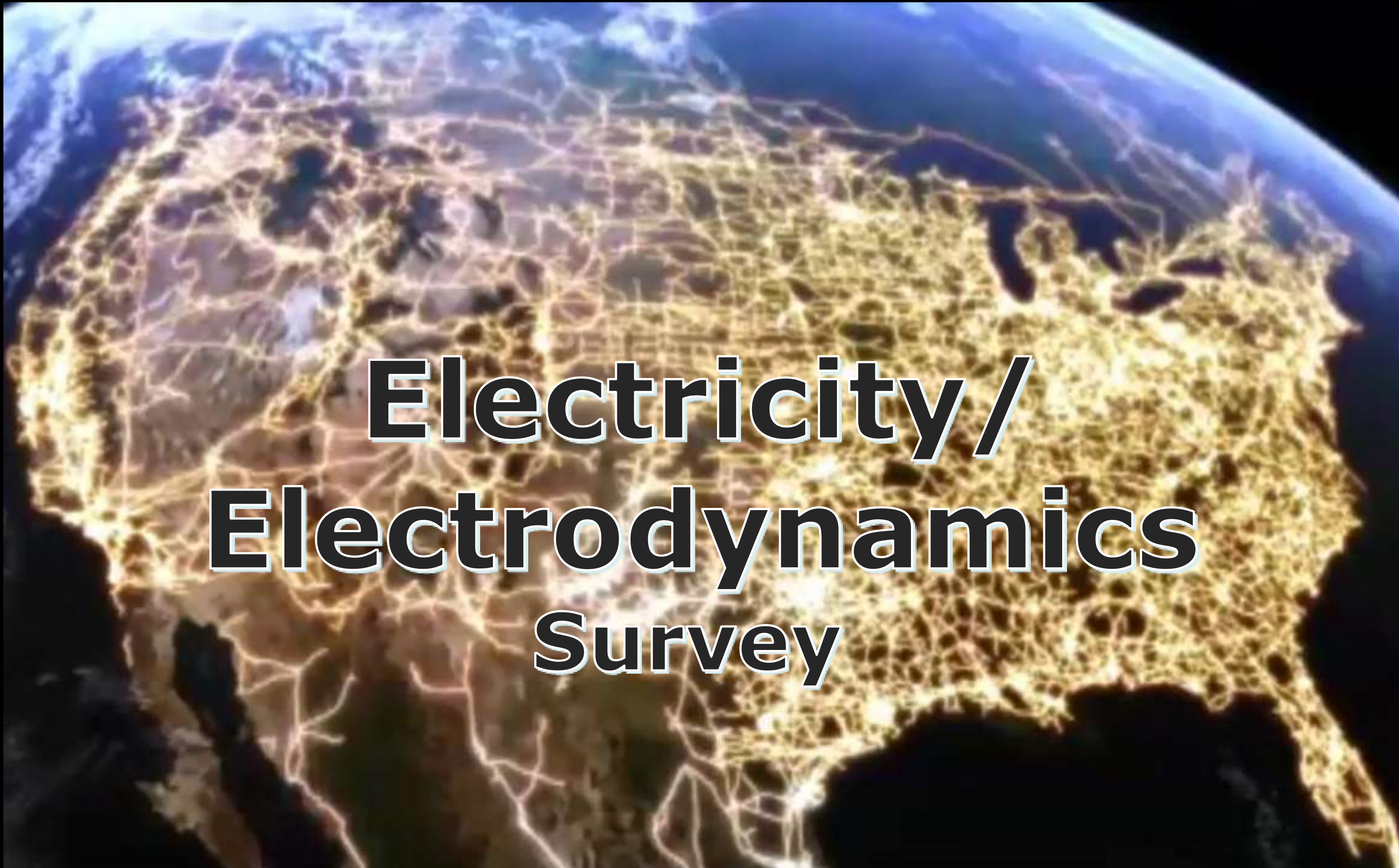
Electricity/ Electrodynamics

Survey



Agenda: Energy Conversion and Transformation

Work and other Energy Forms

- Potential and kinetic energy,
- Molecular binding and rearrangement energies,
- pV work, kinetic energy equilibration, heat flow,
- Mechanical equivalent of heat,
- Basic fluid dynamics, laminar & turbulent flow, (→later)

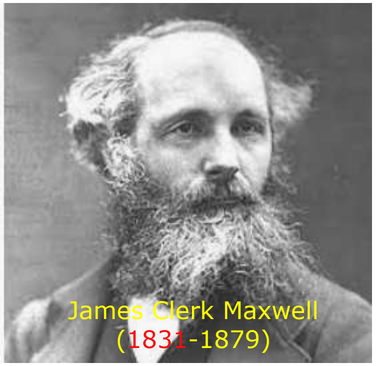
Basic Electricity

- Static electric, electro-magnetic phenomena,
- Electromagnetic induction, generators, transformers,
- Electrical current laws, AC/DC transmission,
- Electronic circuits, reactance,

Principles of Thermodynamic Processes

- Laws of Thermodynamics, state functions, reversible processes,
- Carnot and other TD cycles, steam engines, gas turbines,
- Electro-chemistry, batteries, hydrolyzers & fuel cells.

Electromagnetic Field Theory: Maxwell Equations



Combination of individual laws of electric and magnetic interactions into one theoretical framework:
MEq describe an electric vector field $\vec{E}(\vec{r},t)$ and a magnetic (pseudo) vector field, $\vec{B}(\vec{r},t)$, as well as their interactions.
The sources are the total electric charge density (total charge per unit volume), $\boldsymbol{\rho}$, and the total electric current density (total current per unit area), $\boldsymbol{J}$.

Name	Integral equations	Differential equations
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_\Omega \rho\, dV$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot d\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d}{dt}\iint_\Sigma \mathbf{B}\cdot d\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0\left(\iint_\Sigma \mathbf{J}\cdot d\mathbf{S} + \varepsilon_0\frac{d}{dt}\iint_\Sigma \mathbf{E}\cdot d\mathbf{S}\right)$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

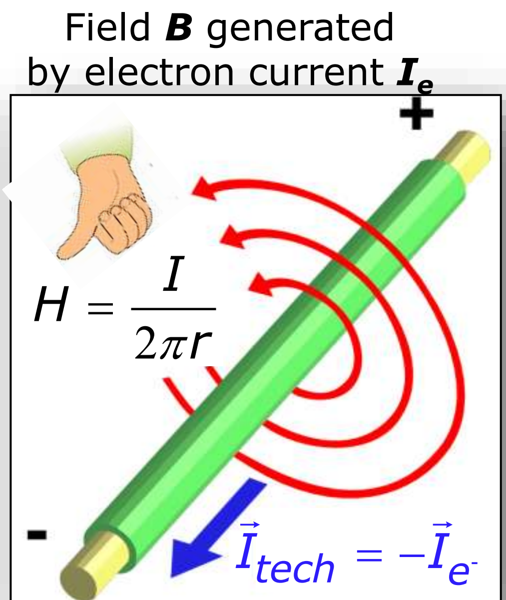
$\vec{B} := magnetic\ \ flux\ \vec{\Phi}/area\, S$

the permittivity of free space, ε_0, and
the permeability of free space, μ_0, and
the speed of light, $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$

Magnetic flux through plane surface $\vec{S}$

$$\vec{\Phi} := \vec{B}\cdot\vec{S} := B_\perp \cdot S\cdot \vec{n}_S \quad Unit\,[\Phi] = Wb\,(Weber) = volt - s$$

Magnetic Observables, Definitions & Units



Magnetic flux $\vec{\Phi}$ *through surface* $\vec{S}$,

$$\boxed{\vec{\Phi} := \int_{\vec{S}} \vec{B} \cdot d\vec{s}} \rightarrow \vec{B} \cdot \vec{S} := B_\perp \cdot S \cdot \vec{n}_S; \; [\Phi] = Wb\,(Weber) = V \cdot s$$

Magnetic flux density $\boxed{B = \Phi / area\, S}$ $(B_\perp)$; $[B] = \frac{Wb}{m^2} = \frac{V \cdot s}{m^2}$

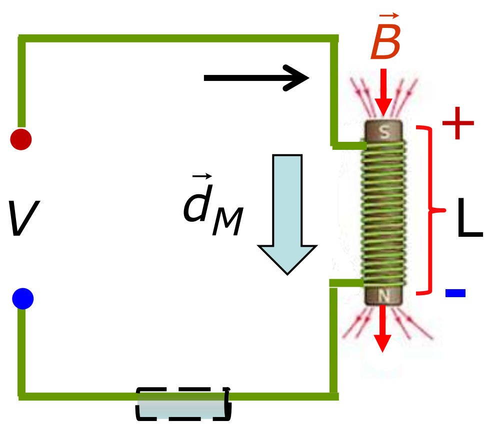
Magnetic field intensity $\boxed{H = B/\mu_0}$ *applied ext. field*

$[H] = Oersted\,(Oe) = \frac{A}{m}$; $\quad \boxed{\mu_0 = 1.256 \cdot 10^{-6}\, V \cdot s / A \cdot m}$

Flux density in material $\boxed{B = \mu \cdot (H + M)}$ *(aligned)*

Magnetization M from intrinsic material magnetism

e.g. magnetic moment, magnetic domains in iron (Fe)



Magnetic (M1) interaction energy $U_{M1} = -\vec{d}_M \cdot \vec{B}$

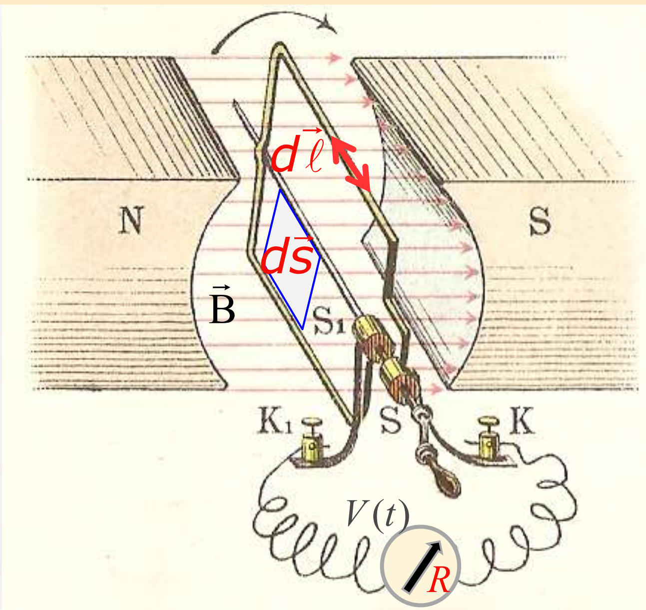
magnetic dipole moment $\vec{d}_M = M \cdot \vec{L}$; $[\,] = A \cdot m^2$

Force : $\vec{F} = \vec{\nabla}\left(\vec{d}_M \cdot \vec{B}\right) \rightarrow \vec{F} \approx d_M \cdot \vec{\nabla}\left|\vec{B}\right|$ (*aligned* $\vec{d}_M$)

Torque $T = \vec{d}_M \times \vec{B}$ Inhom. B

Principle of Generator (Dynamo)

Voltage induced on wire loop rotated ⊥B field



Direction of flow of electricity (electrons e⁻) in a wire-conductor loop → induced electro-motive force *emf* (Faraday's Law of Induction)

$$\frac{\partial}{\partial t}\Phi_{Loop} = \frac{\partial}{\partial t}\int_{Loop\ Area} \vec{B}(t)\cdot d\vec{s} = -\oint_{Loop\ Rim} \vec{E}\cdot d\vec{\ell} \propto work\ (on\ q)$$

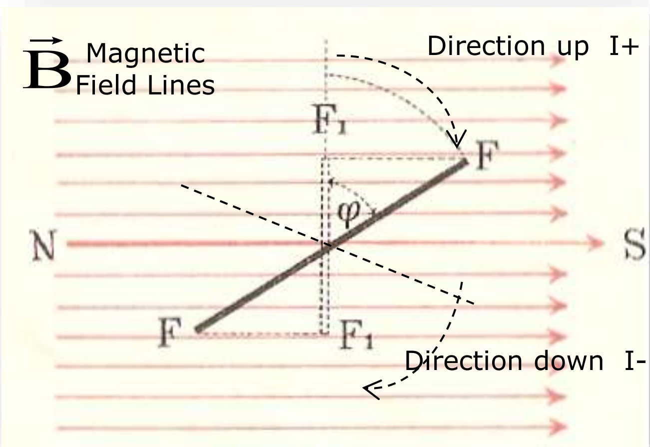
Time dependent A/C Voltage $V(t)$ and Current $I(t)$

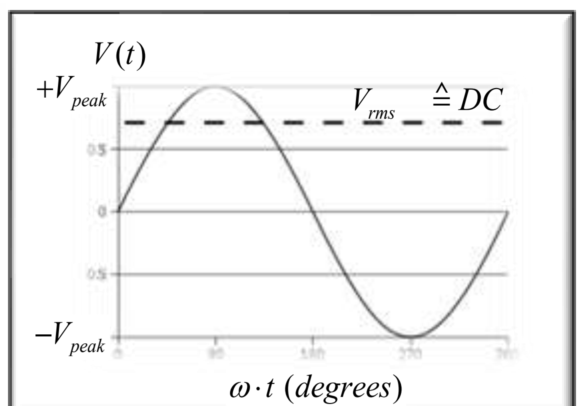
$$V(t) = V_{peak}\cdot\sin\left(\omega\cdot t+\varphi_V\right);\ I(t) = \left(V_{peak}/R\right)\cdot\sin\left(\omega\cdot t+\varphi_I\right)$$

Amplitude Phase

$$Amplitude\ V_{peak},\ V_{peak-to-peak} = 2V_{peak}$$

$$Angular\ frequency\ \omega = \frac{\partial\varphi}{\partial t} = 2\cdot\pi\cdot f = \frac{2\pi}{T}$$



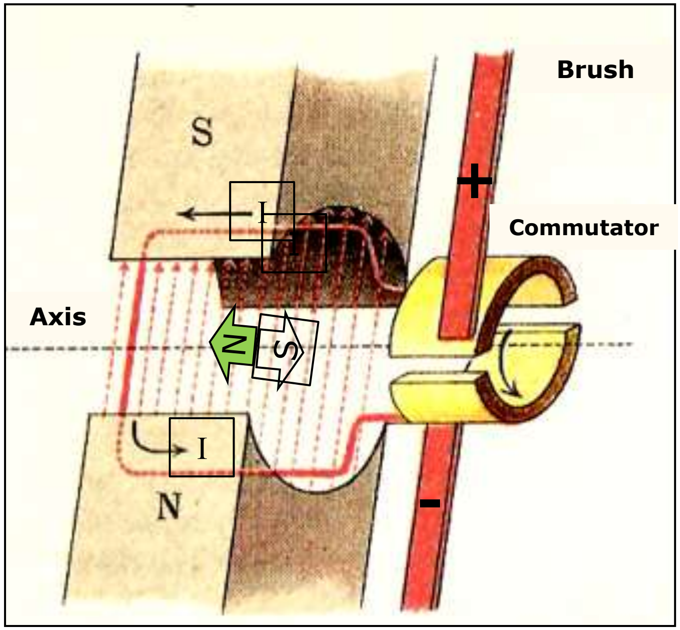


Effective voltage:

$$\langle V(t)\rangle = \frac{V_{peak}}{\sqrt{2}} = V_{rms}$$

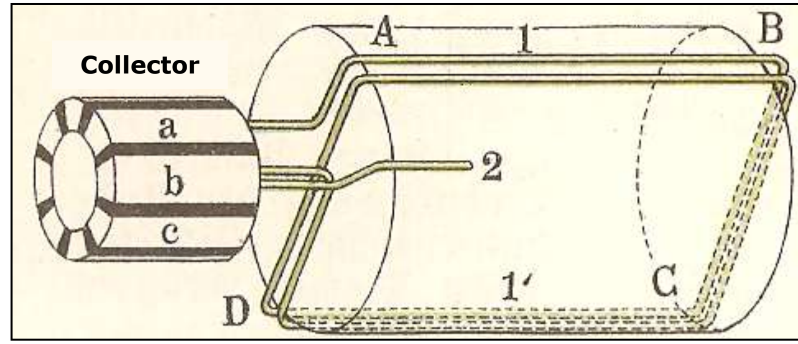
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt} =$$

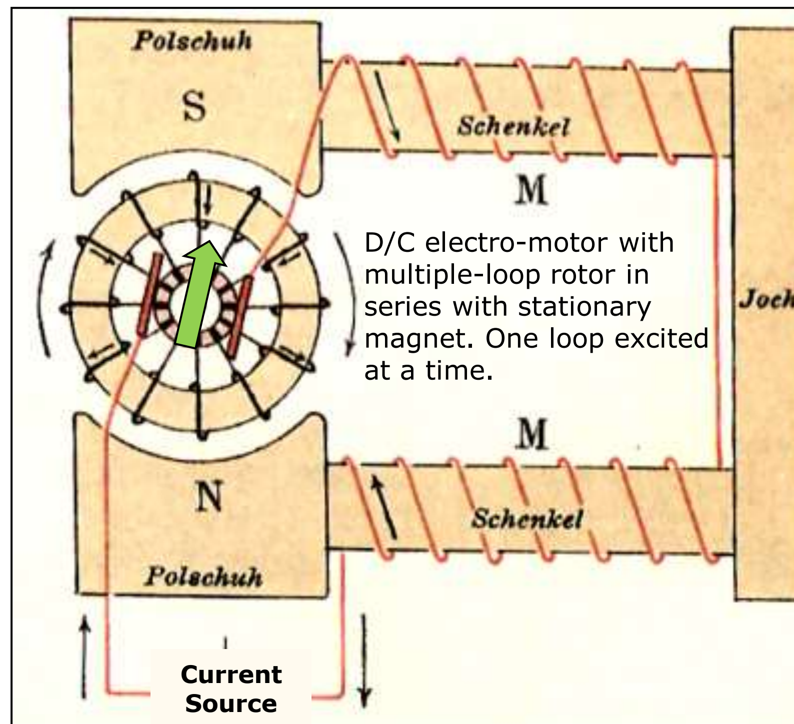
Principle of D/C Electromotor



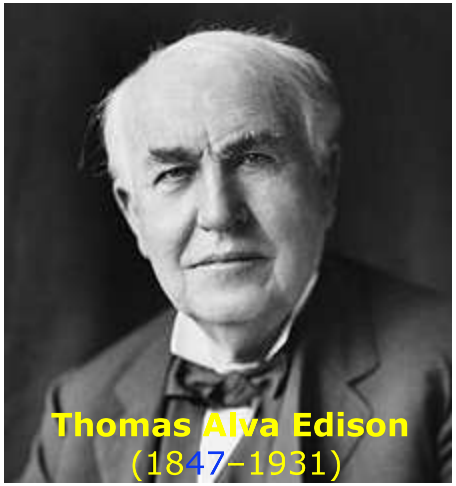
Current (I) loop creates alternating *N-S* electro-magnet, which "feels" a torque and tends to align parallel to the field of permanent magnets and turns loop. Polarity reverses magnet polarity at max. alignment.

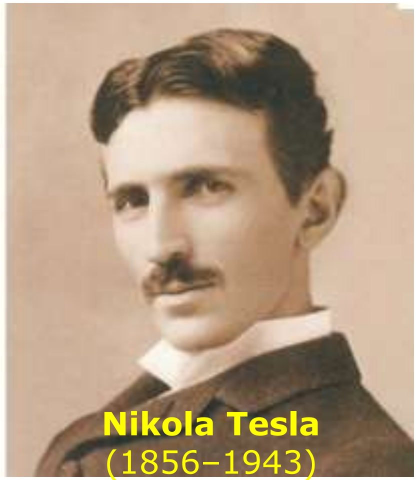
Mechanical rotation of wire loop in field of permanent magnets generates voltage at commutator/collector → **Dynamo/Alternator**
Direction of current depends on orientation of loop in magnetic field → AC or DC currents





Advent of Hydroelectric Power

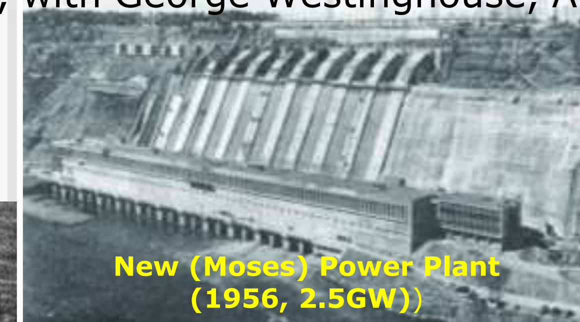


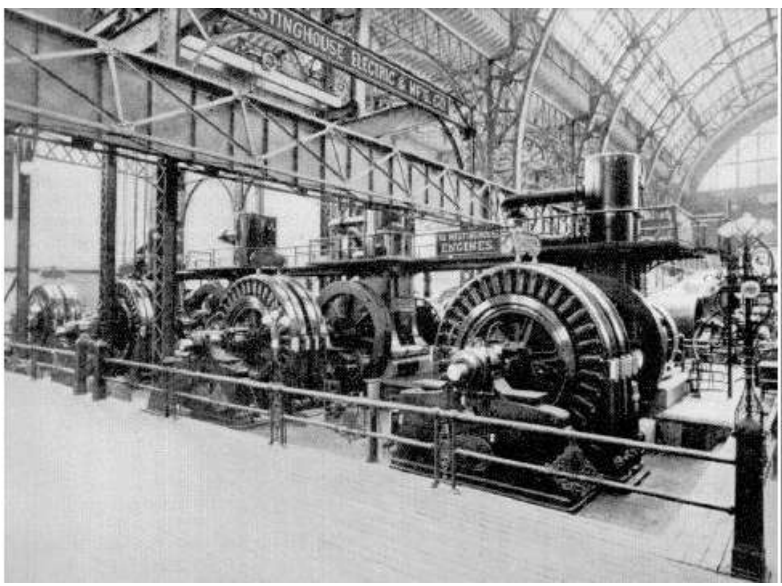


Influential inventors of DC (Edison) and AC (Tesla) electrical power transmission over large distances.

Electrical lighting, wireless radio,….
Power wars (→ J.P. Morgan).

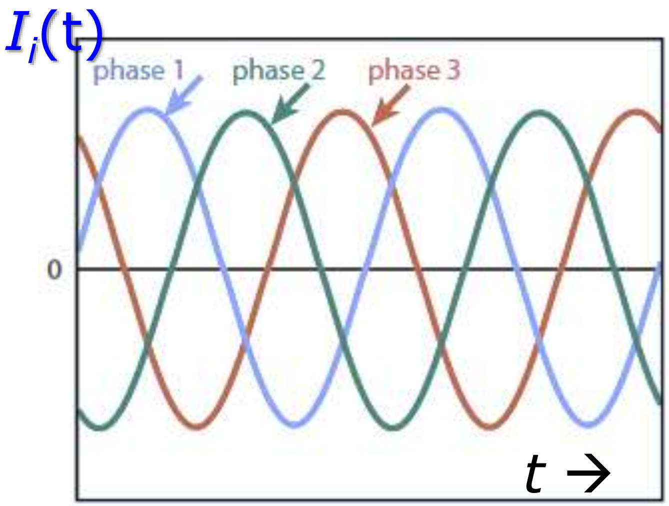
1895: Built 1. hydro electric power station (Niagara, with George Westinghouse, AC)



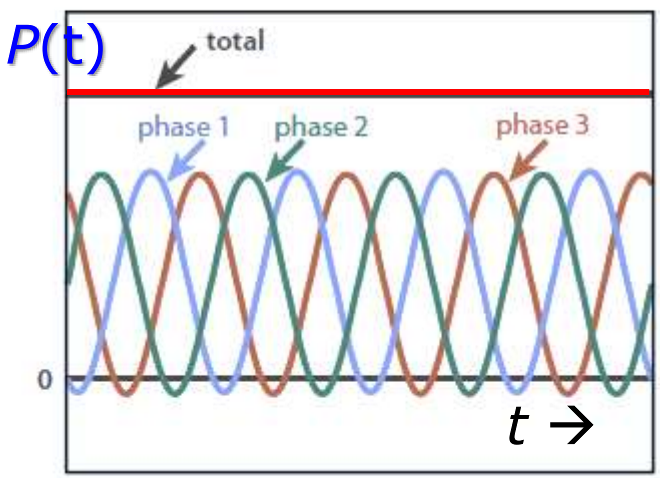


Start of new chapter in hydropower
→ many hydro-electric power plants

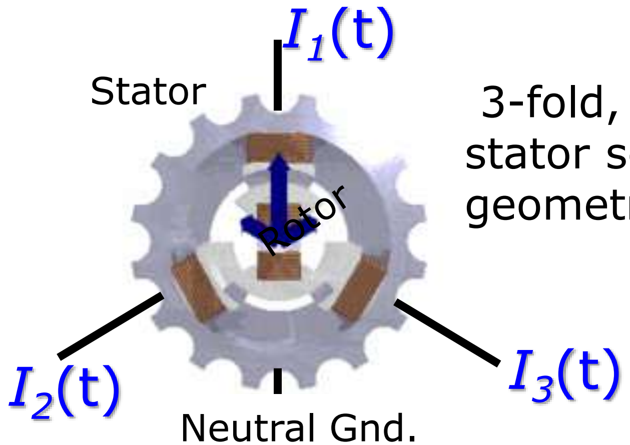
3-Phase Current



(a) Voltage

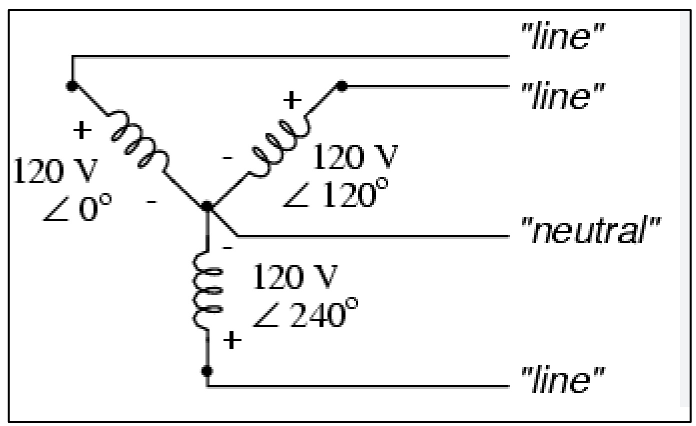


(b) Instantaneous Power



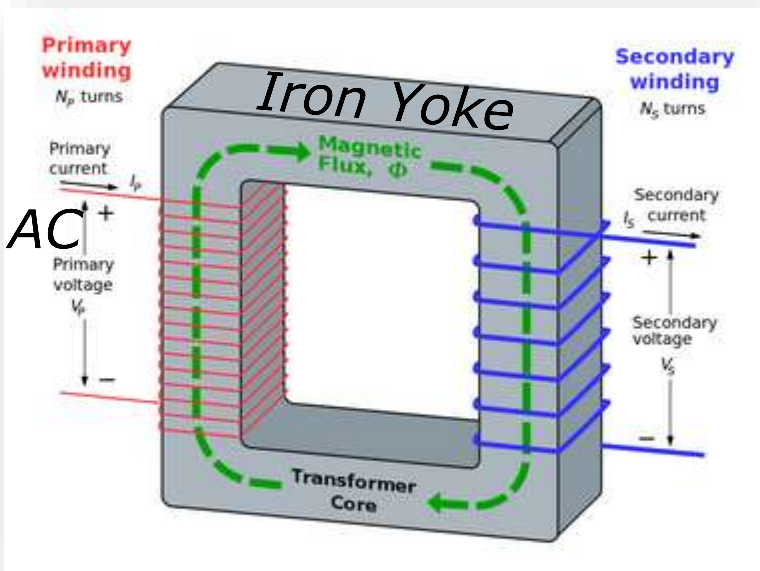
3-fold, 120^0 stator segment geometry

3-phase AC current invented by Nikola Tesla.
Stator has 3 segments, $\Delta\Theta = 120^0$ rotation.
→ Combined (sum @R) power always > 0.



P(t) = const.

AC Advantage:Voltage Level Transformers

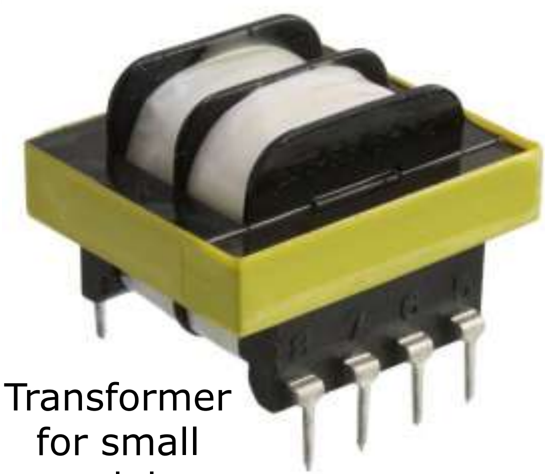


$$V_{primary} = -N_{primary}\, d\Phi/dt$$
$$V_{secondary} = -N_{secondary}\, d\Phi/dt$$
$$V_{secondary}/V_{primary} = N_{secondary}/N_{primary}$$
$$Changes\ load\ impedance\ Z_{primary}$$

Laminated or toroidal transformer cores.

Iron/steel laminations prevent eddy currents. Insulated with a nonconducting material, such as varnish or epoxy.

Toroidal: coils wrapped around cylindrical core.

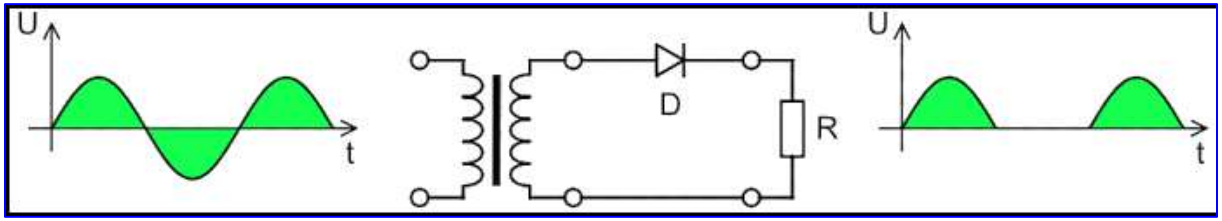


Transformer for small modular electronics

Cooled power transformer on national e-grid

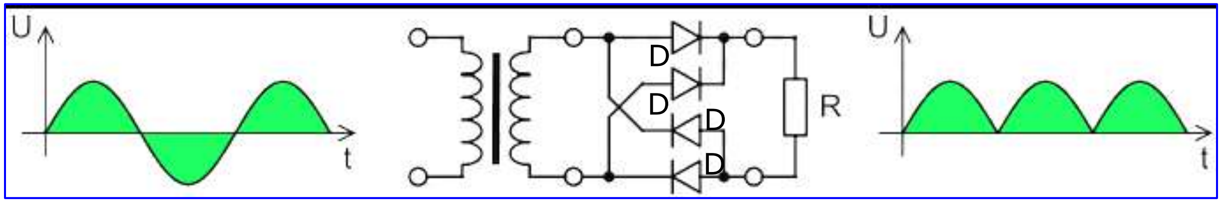
AC→DC Rectification with Diodes

Half-wave rectifier, unidirectional current flow enforced by diode D



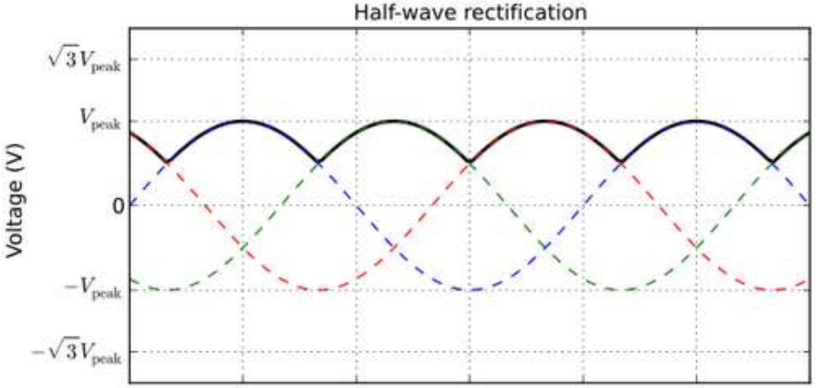
$$V_{rms} = (1/2)\,V_{peak}$$
$$V_{dc} = (1/\pi)\,V_{peak}$$

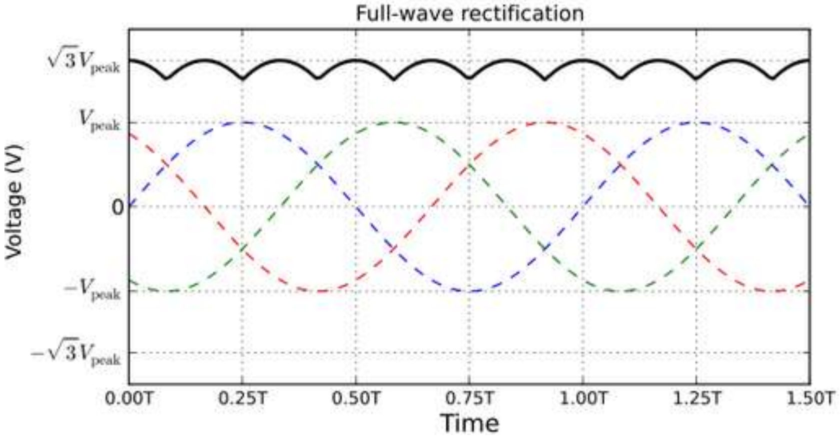
Full-wave rectifier, current flow patterns enforced by 4 diodes D



$$V_{rms} = (1/\sqrt{2})\,V_{peak}$$
$$V_{dc} = (2/\pi)\,V_{peak}$$

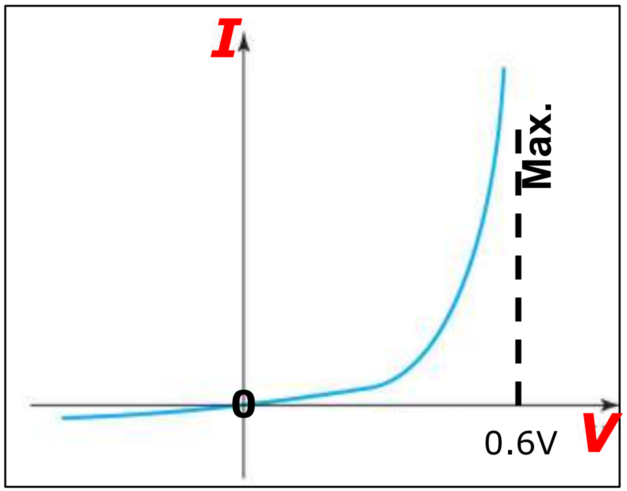
3-Phase current, half-wave and full-wave rectified.

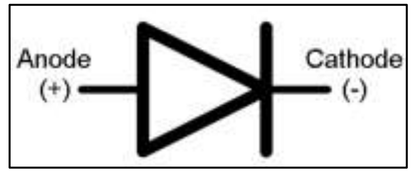




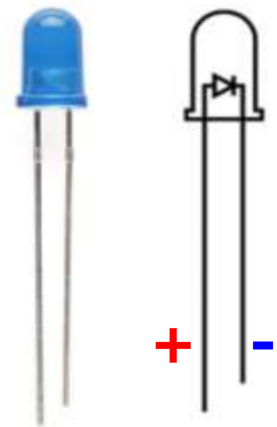
Semiconductor Diodes

Diode I-V Characteristic





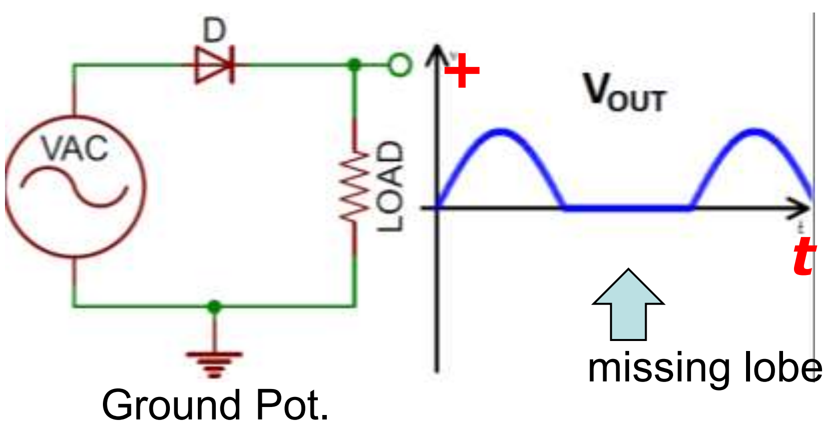
Forward bias



Forward bias

Ideal Diode Characteristics		
Operation Mode	On (Forward biased)	Off (Reverse biased)
Current Through	I>0	I=0
Voltage Across	V≈0.6V	V<0
Diode looks like	Short circuit	Open circuit

Half-wave rectifier



Missing lobe is recovered in more complicated circuit with several diodes. → Full-wave rectifier

Complex Notation

Phase differences are conveniently handled in complex notation *V(t), I(t)*

$$V(t) = |V(t)| \cdot e^{i\cdot(\varpi \cdot t + \phi_V)}; \quad I(t) = |I(t)| \cdot e^{i\cdot(\varpi \cdot t + \phi_I)}$$

Amplitudes and phases are determined from initial conditions, *V(t=0), I(t=0).*

Example: $V(t=0) = V_0 \rightarrow V(t) = V_0 \cdot \cos(\varpi \cdot t)$

Ohm's Law $V(t) = Z \cdot I(t)$ *in circuits at all times*

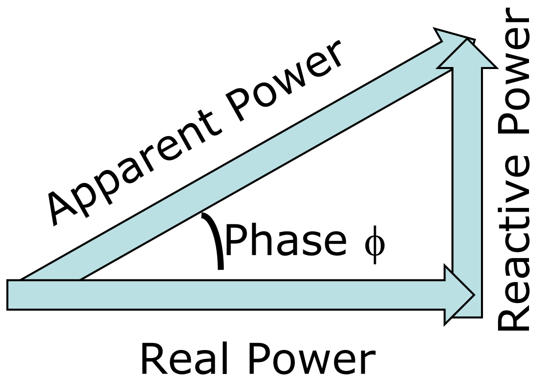
Impedance $Z(\omega) = R + i \cdot [X_C(\omega) + X_L(\omega)]$ ($i := \sqrt{-1}$)

1. Ohm resistance *R*
1. Capacitive reactance $X_C = -1/\omega C$ ($\Delta\phi_I = +90^0$)
2. Inductive reactance $X_L = \omega L$ ($\Delta\phi_I = -90^0$)

$$Euler's\ Formula: i = e^{i\cdot\pi/2} = \cos(\pi/2) + i \cdot \sin(\pi/2)$$

Real and Reactive Power

$Power\ P(t) = V(t)\cdot I(t)\ or,\ in\ complex\ notation\ P = V\cdot I^*\ (* = complex\ conjugate)$



$Purely\ resistive\ loads: P_R = V_R \cdot I = V^2/R = I^2\cdot R$

$Real = apparent\ power\ P_R(t) = V_R(t)\cdot I(t)$

$General:\ Apparent\ power: P_A(t) = \sqrt{P_R^2(t) + P_{LC}^2(t)}$

$Real\ power\ P_R(t) = P_A(t)\cdot \cos\phi$

$Power\ Factor = P_R/P_A$

Real and reactive power are "out of phase"

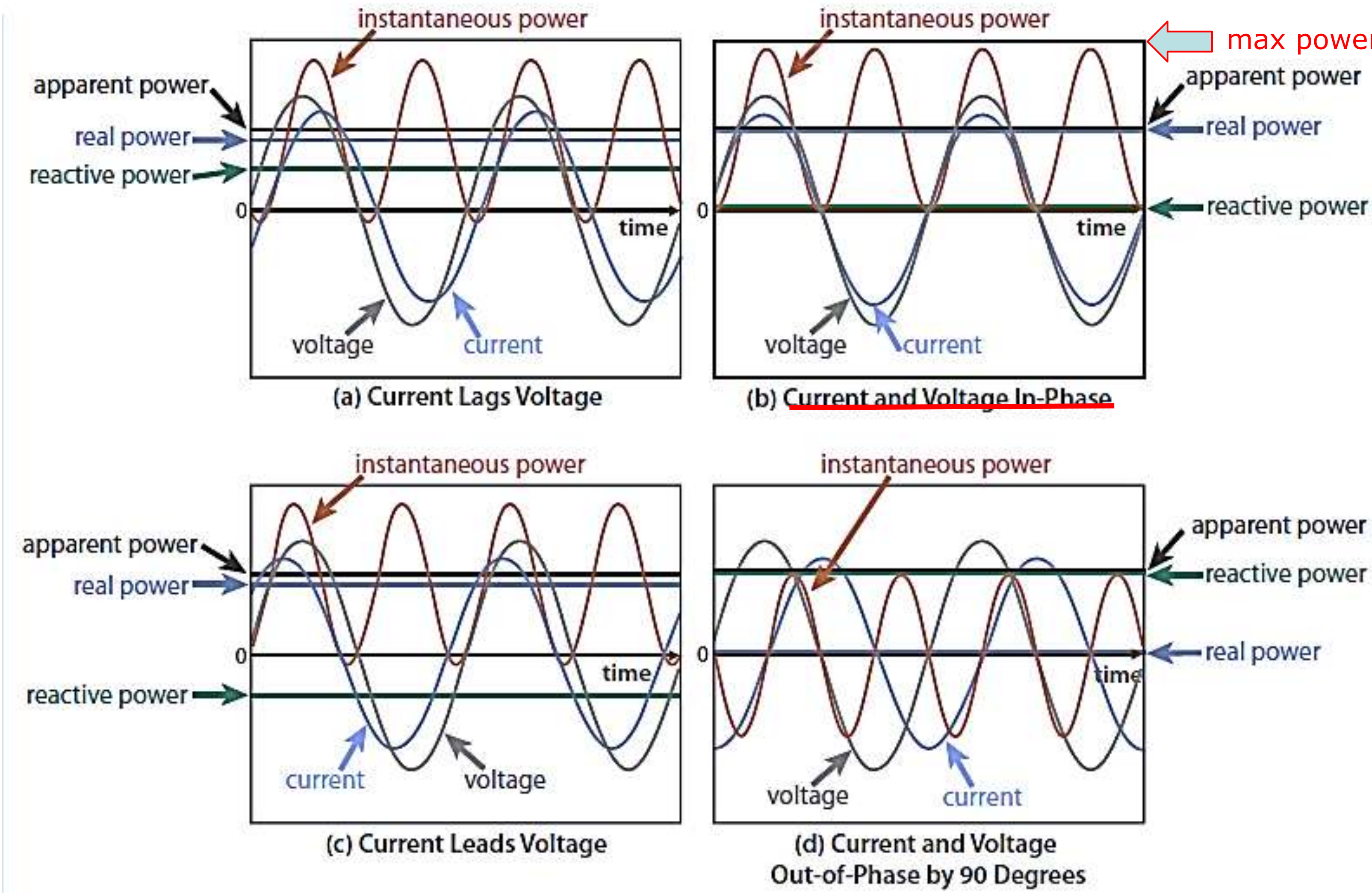
$Apparent\ power: P_A^2(t) = P_R^2(t) + \left[P_C^2(t) + P_L^2(t)\right]$

$Oscillating\ reactive\ power\ P_C(t) \rightleftarrows P_L(t)$

$Reactive\ power\ P_{LC}(t) = P_A(t)\cdot \sin\phi$ "var" *voltage – ampere – ractive*

Actual loads on the power supply (e-grid) like an e-motor are always complex (Ohm + capacitive + inductive) → have feedback effect on supply → Affect power factor (available power) and frequency.
→ General effect on stability of grid.

Effect of Reactance in AC System

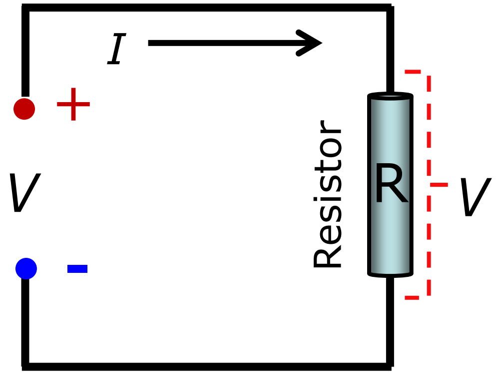


Components of Electrical DC Circuits: Resistors

Electric current I unit=[1 A(mpere)= 1C/s]:
e^- stream [dq/dt=#e^-/sec] transfers power through metallic wires, dissipates e^- energy→ heat. All metallic wires have intrinsic (distributed) resistivity ≠0.

→ Unidirectional (DC) electrical (I_{tech}= -I_{e-}) current sustained by applied electric potential = Voltage differential V ≈ constant in time, depending on potential energy PE content/supply source (=Q for battery).

Always finite electric conductivity σ → 1/Ohm resistance $R \neq 0$!!



Ohm's Law, R real, no ω dependence

$$I = \frac{V}{R} \rightarrow V = I \cdot R$$

$$\left[R\right] = 1\Omega \; (Ohm) = \frac{1V}{1A}$$

Power dissipated in resistor $(T \nearrow)$

$$p = V \cdot I = I^2 \cdot R$$

$$\left[p\right] = W \; (Watt) = J/s$$

Commercial resistor elements are made of materials with rel. low electric conductivity.

Components of Electrical AC Networks: Resistors

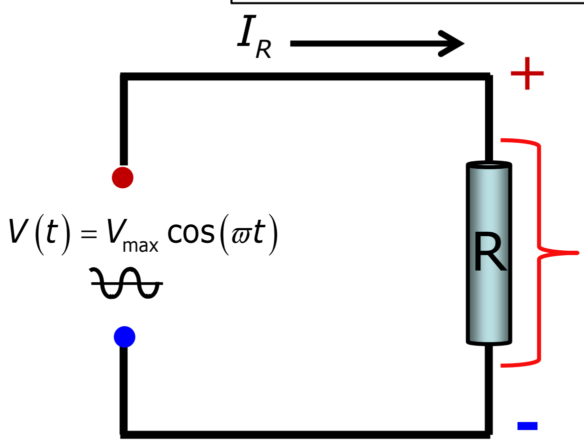
Transfer of electrical power through metallic wires → Electric conductivity σ → Electrons dissipate kinetic energy through scattering
→ Ohm resistance $R \sim 1/\sigma \quad [R] = \Omega\ (Ohm)$ Extension: R=generic workload

$$Applied\ AC\ voltage\ [V] = V\ (Volt)$$

$$V(t) = V_{\max}\cos(\varpi t) \rightarrow effective\ V = \langle V(t)\rangle = V_{\max}/\sqrt{2}$$

$$effective\ I = \langle I(t)\rangle = I_{\max}/\sqrt{2}$$

Averaged over 1 period



$$I_R(t) = I_{\max}\cos(\varpi t); \quad I_{\max} = \frac{V_{\max}}{R}$$ Ohm's Law

$$Current\ I_R(t)\ \{[I] = A\ (Ampere)\}\ in\ phase\ with\ V(t)$$

Power dissipated in resistor $[p] = W\ (Watt)$

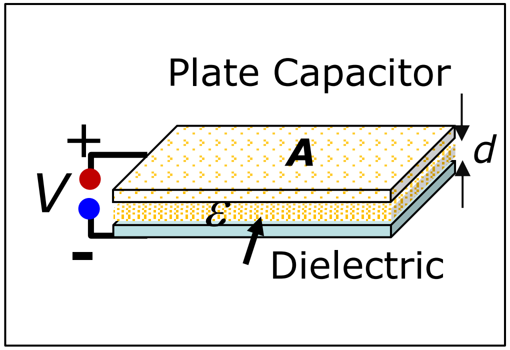
$$p_R(t) = V(t)\cdot I_R(t) = V_{\max}I_{\max}\cos^2(\varpi t)$$

$$= \frac{1}{2}V_{\max}I_{\max}\{1+\cos(2\varpi t)\} = V\cdot I_R\{1+\cos(2\varpi t)\}$$

effective

Effective dissipated power $\langle p_R(t)\rangle = V\cdot I_R = \dfrac{V^2}{R} = I_R^2\cdot R$ This is real power loss.

Components of Electrical DC Circuits: Capacitors

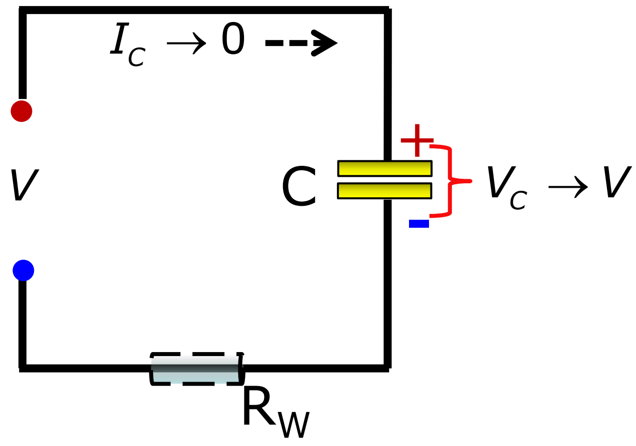


Metal plates (area *A*) separated by dielectric medium (ε) of thickness *d* form a capacitor w. capacitance

$$C = \frac{\varepsilon \cdot A}{d} \qquad [C] = F\,(Farad)$$

→Carries static charge $Q = C \cdot V$

As "load" element :



$$Switch-on\ voltage\ V \to brief\ \ current\ I_C(t)$$

$$V_C(t) = \frac{1}{C}Q(t) = \frac{1}{C}\int^{t} I_C(t')dt' \to V$$

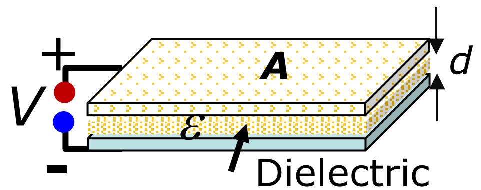
Energy content W_C of capacitor C

$$W_C = \frac{1}{2}Q \cdot V = \frac{C}{2}V^2 = \frac{Q^2}{2C}$$

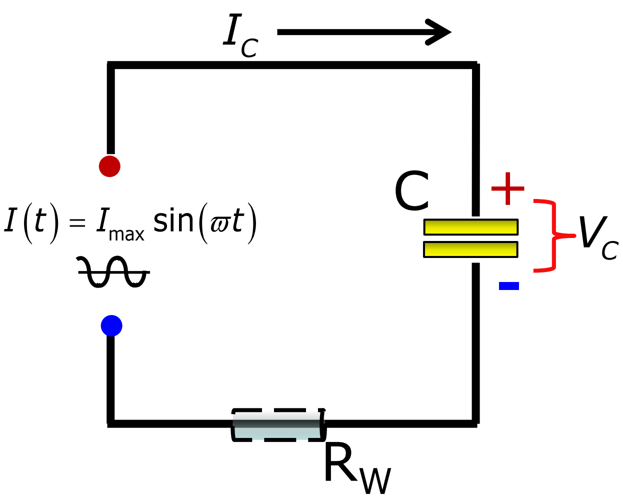
A purely capacitive load in an electrical circuit does not dissipate (lose) power.
But circuit wire conductors have always an additional $R_W \neq 0$.
→ Slow dissipation of energy content of source, *t-const.* $\tau = R_W \cdot C$

Components of Electrical AC Networks: Capacitors

Parallel Plate Capacitor



$$C = \frac{\varepsilon \cdot A}{d}; \left[C\right] = F\left(Farad\right)$$



Metal plates (area *A*) separated by dielectric medium (*ε*) of thickness *d* form a capacitor w. capacitance

As "load" element in AC (frequency ω) circuit:

→Carries electric charge $Q = C \cdot V$

$$V_C\left(t\right) = \frac{1}{C}Q\left(t\right) = \frac{1}{C}\int^{t} I_C\left(t'\right)dt' = \frac{I_{max}}{\varpi C}\cos\left(\varpi t\right)$$

$$= V_{max}\cos\left(\varpi t\right) \rightarrow V_{max} = I_{max} \cdot X_C \quad \text{Ohm's Law}$$

$$\text{Reactance } \left|X_C\right| = \frac{1}{\varpi C}$$

$$\textit{Current leads voltage}: \textit{phase difference} + \frac{\pi}{2}$$

Power in capacitor

$$p_C\left(t\right) = V\left(t\right) \cdot I_R\left(t\right) = V_{max}I_{max}\sin\left(\varpi t\right) \cdot \cos\left(\varpi t\right)$$

$$= \frac{1}{2}V_{max}I_{max}\left\{\sin\left(2\varpi t\right)\right\} \rightarrow \left\langle p_C\left(t\right)\right\rangle = 0$$

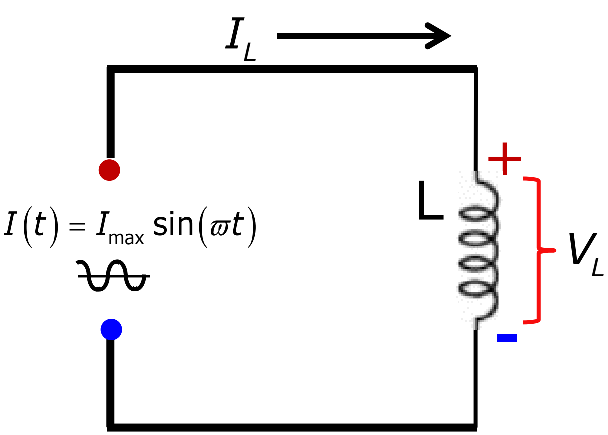
A pure capacitive load in an AC circuit does not dissipate (lose) power, it moves energy between electron currents and electric field and changes relative phase of current vs. voltage → reactive power

Components of Electrical AC Networks: Inductors

Helical coil insolated Cu wire wound around plastic ("solenoid") or ferrite/carbon-iron core. Connected to electric battery it produces static axial magnetic field ("electro-magnet")

Inductance L, $L_{solenoid} = \mu_0 K \left(N^2 A/\ell\right)_{coil}$ $[L]$=H=Vs/A (Henry)

As "load" element in AC (frequency ω) circuit:



$$V_L(t) = -L\frac{dI}{dt} = -\varpi L \cdot I_{\max}\cos(\varpi t)$$

$$= V_{\max}\cos(\varpi t) \rightarrow \boxed{V_{\max} = I_{\max} \cdot X_L}$$ Ohm's Law

Reactance $|X_L| = \varpi L$

$$Current\ lags\ voltage: phase\ difference -\frac{\pi}{2}$$

Power in inductor

$$p_L(t) = V_L(t) \cdot I(t) = -V_{\max} I_{\max} \sin(\varpi t) \cdot \cos(\varpi t)$$

$$= -\frac{1}{2} V_{\max} I_{\max} \{\sin(2\varpi t)\} \rightarrow \boxed{\langle p_L(t) \rangle = 0}$$

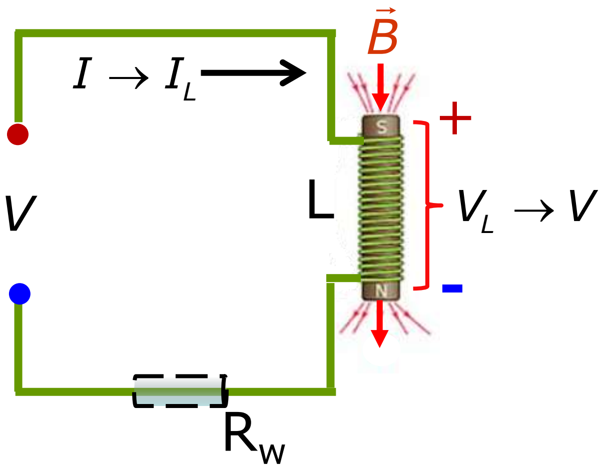
A pure inductive load in an AC circuit does not dissipate (lose) electric power, it moves energy between electron currents and magnetic field and changes relative phase of current vs. voltage → reactive power

Components of Electrical DC/AC Circuits: Inductors

Helical coil of insolated Cu wire wound around plastic ("solenoid") or ferrite/carbon-iron core. Connected to electric battery it produces static axial magnetic field ("electro-magnet")→ large resistance to high-frequency current spikes.

$$\text{Inductance}\quad \boxed{L = \mu_0 K \left(N^2 A/\ell\right)_{coil}} \qquad [L] = H = Vs\,/\,A \;\; (Henry)$$

As "load" element in circuit:



$$\boxed{V_L(t) = -L\frac{dI}{dt}} \rightarrow Work\;\; \frac{dW_L}{dt} = -L \cdot I \cdot \frac{dI}{dt}$$

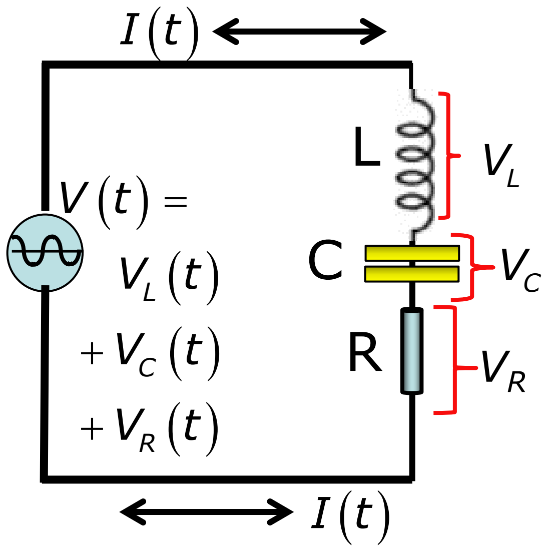
Energy content W_L stored in inductor L

$$\boxed{W_L = \frac{1}{2} L \cdot I_L^2}$$

A purely inductive load in an electric circuit does not dissipate electric energy.
But circuit wire conductors have always an additional $R_W \neq 0$.
→ Slow dissipation of energy content of source, *t-const.* $\tau = L/R_W$

Basic Electrical Circuit Laws

Ohm's Law $V(t) = Z \cdot I(t)$ $(\Delta\phi_I = \text{phase difference } I \text{ rel } V)$

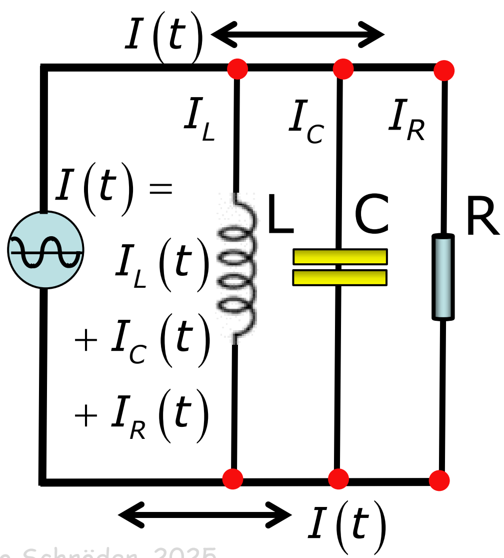


1. Ohm resistance *R*
1. Capacitive reactance $X_C = -1/\omega C$ $(\Delta\phi_I = +90^0)$
2. Inductive reactance $X_L = \omega L$ $(\Delta\phi_I = -90^0)$

Kirchhoff's Loop (or mesh) Rule

The directed sum of the potential differences (voltages) around any closed loop is zero.

$$V(t) = V_L(t) + V_C(t) + V_R(t)$$

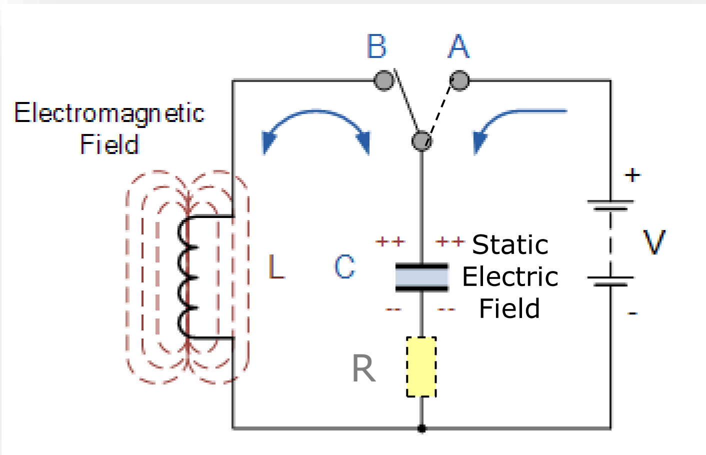


Kirchhoff's Junction Rule (Parallel Loops)

The algebraic sum of currents in a network of conductors meeting at a point is zero.

$$I(t) = I_L(t) + I_C(t) + I_R(t)$$

Electronic LC Oscillator



Kirchoff's Laws:

Closed circuit :

$$I_C = I_L = I$$

$$V_C + V_L = 0,\ V_C = C \cdot \frac{dI_C}{dt},\ V_L = L \cdot \frac{dI_L}{dt}$$

Charge $Q(t)$, $I(t) = dQ/dt$

$$I_C = \frac{dQ_C}{dt} = C \cdot \frac{dV_C}{dt} = I \quad (Neglect\ R)$$

$$\rightarrow I = C \cdot \frac{dV_C}{dt} = -C \cdot \frac{dV_L}{dt} = -L \cdot C \cdot \frac{d^2 I}{dt^2}$$

Define $\omega_0 := \frac{1}{\sqrt{L \cdot C}}$

Differential Equation for Oscillator

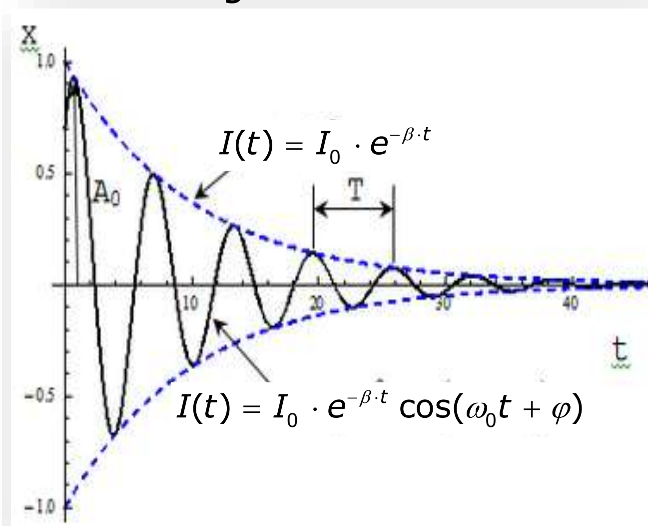
$$\rightarrow \frac{d^2 I(t)}{dt^2} + \omega_0^2 I(t) = 0 \quad \rightarrow I(t) = a \cdot e^{i\omega_0 \cdot t} + b \cdot e^{-i\omega_0 \cdot t}$$

Constants a, b *defined by initial conditions* $(t = 0)$

$$a := \frac{I_0}{2} e^{i\varphi}, \quad b = a^* = \frac{I_0}{2} e^{-i\varphi} \rightarrow I(t) = I_0 \cdot \cos(\omega_0 \cdot t + \varphi)$$

Including Ohm resistance → damped oscillations

Including Ohm Resistance R

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