

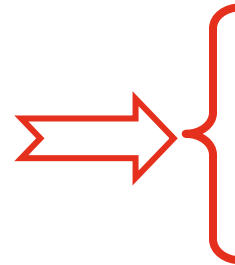
Detection Of Ionizing Radiation Charged Particles

Bethe-Bloch Theory (Metals)



Main Interactions of Charged Particles with Matter

Dominant type of charged-particle interactions: collisions with atomic electrons



Atomic excitation
ionization
fluorescence
phosphorescence

Probability for collisions with nuclei :

$$\frac{\sigma_{nucl}}{\sigma_{atom}} \sim \frac{\pi R_{nucl}^2}{\pi a_Z^2} \sim \frac{Z^2 \cdot 10^{-26} \text{ cm}^2}{10^{-16} \text{ cm}^2} \sim Z^2 \cdot 10^{-10} < 10^{-6}$$

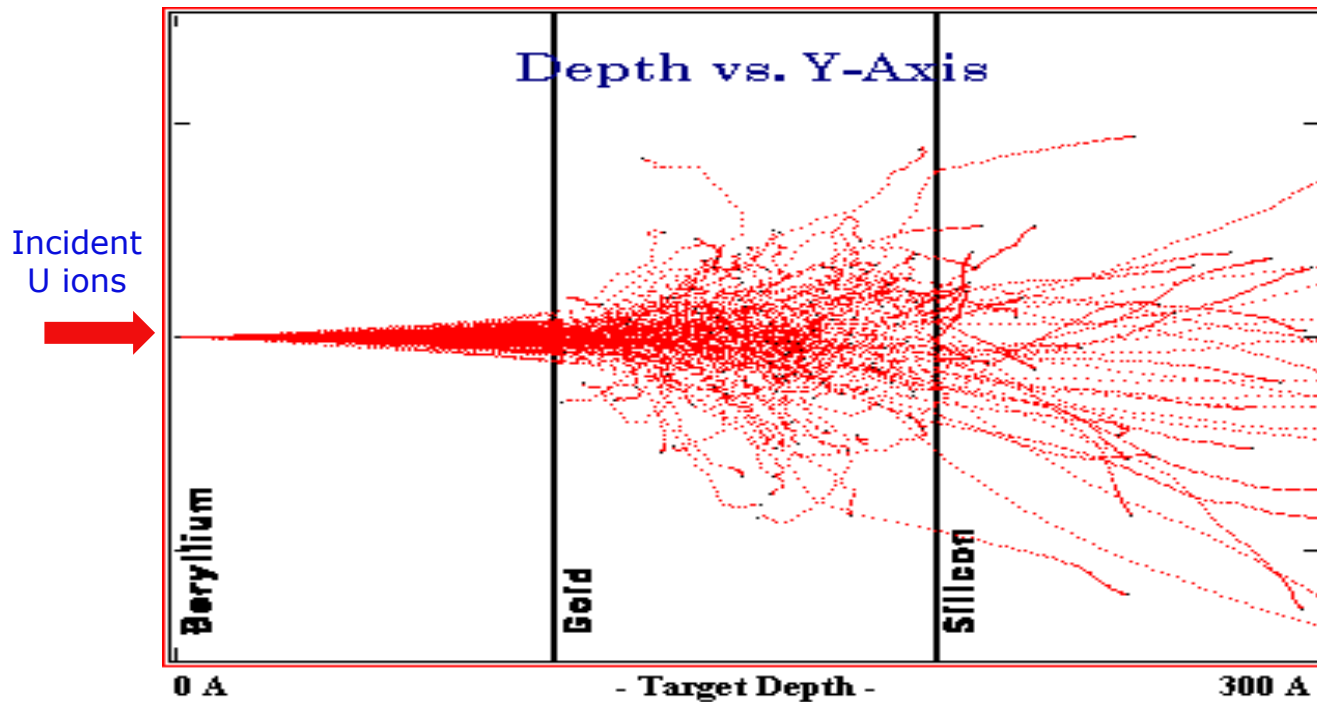
Most interactions of charged particles with material components *are collisions with atomic electrons*.

Nuclear collisions are noticeable only at very low particle kinetic energies.

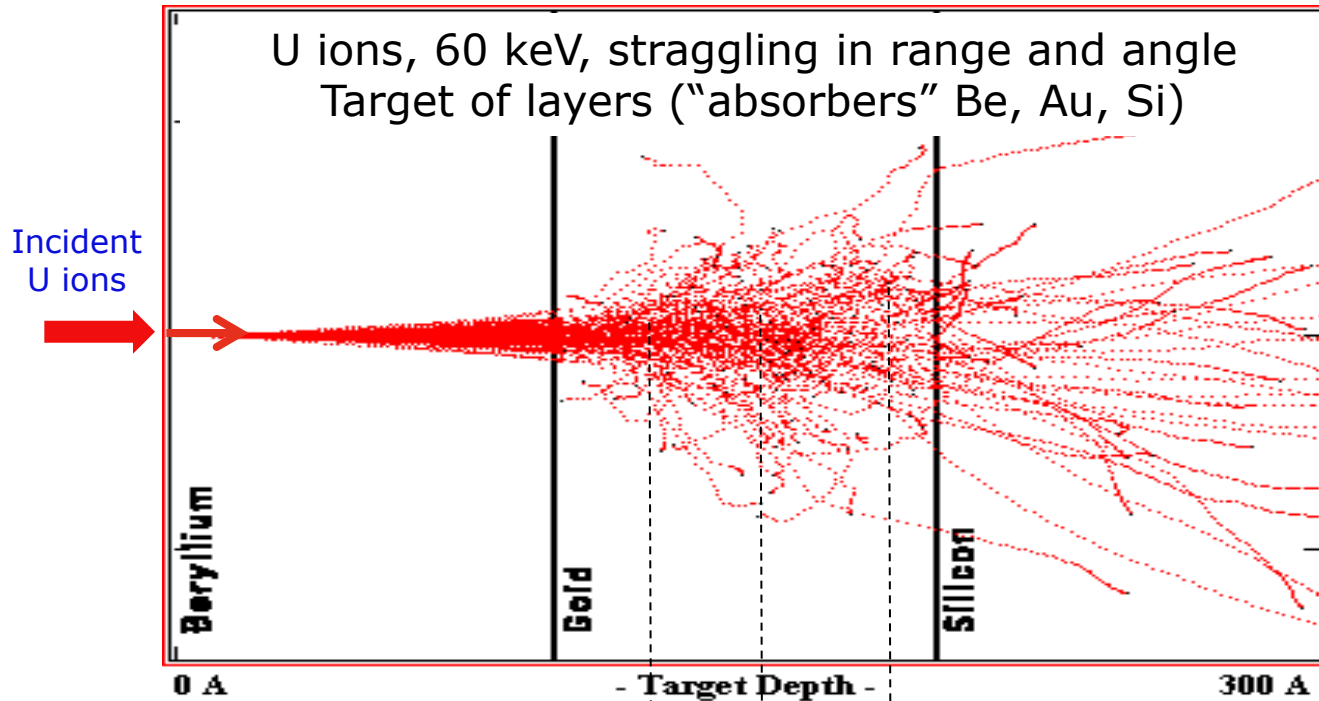
Ionization Mechanisms

- Gamma-rays: Photo-effect, Compton Effect, pair production.
- Charged particles: Coulomb interactions with absorber/target electrons. (G.F. Knoll, Ch.2 I&II)
- Neutrons: Nuclear collisions & reactions

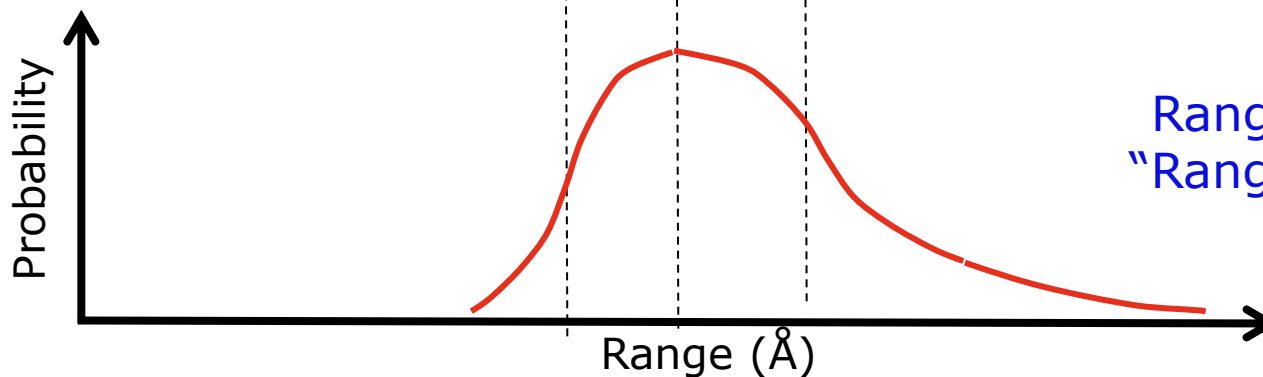
→ Electrons are always the main free charge carriers



Stochastic Multiple Scattering → Straggling

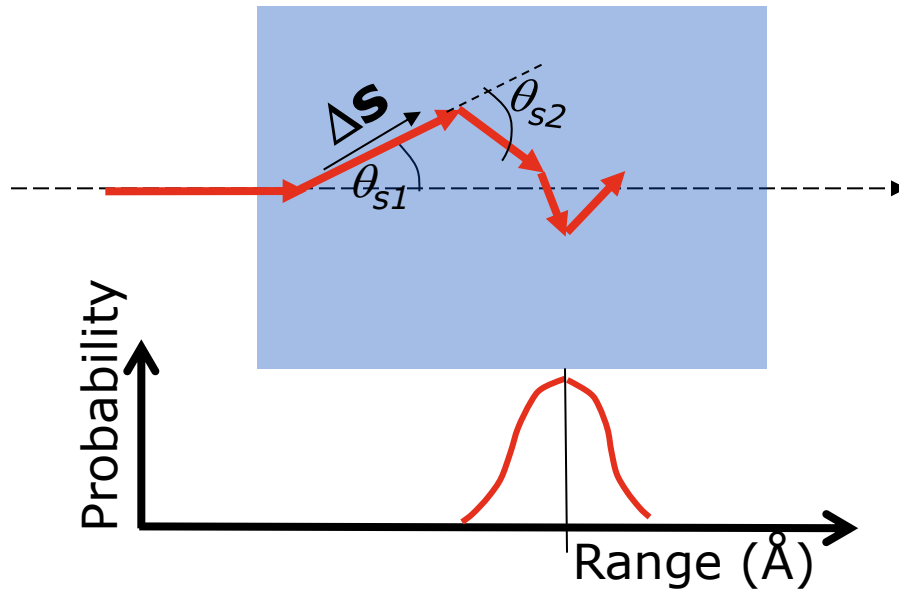


Ziegler
range
program



Range is diffuse:
"Range straggling"

Range and Stopping Power



Scattering angle θ_s path variable s , **mean free path length** $\langle \Delta s \rangle =: \lambda$

Stochastic multiple scattering process produces straggling in range, energy loss, angle \rightarrow

Particles traversing an absorber attain broadened energy spectrum

$$R(E) \approx \int ds \langle \cos \theta_s \rangle = \int_0^E dE' \left[\frac{dE'}{ds} \right]^{-1} \cdot \langle \cos \theta_s \rangle = (\text{mean}) \text{ Range}$$

$$\left[\frac{dE}{ds} \right] =: \text{Stopping Power}$$

$$S(E) = \int ds \geq R(E) \quad \text{Path length of trajectory}$$

Alpha Particle Range: Heuristic Formulas

Range in air ($\rho=1.293 \text{ g/cm}^3$)

$$R_{Air} (cm) = \begin{cases} 0.56 \cdot E_{\alpha} (MeV) & \text{for } E_{\alpha} \leq 4MeV \\ 1.24 \cdot E_{\alpha} (MeV) - 2.62 & 4MeV < E_{\alpha} \leq 8MeV \end{cases}$$

Approximate range in other absorbers (atomic number = A)

$$R_{\alpha} (E_{\alpha}, A) = 0.56 \cdot A^{1/3} \cdot R_{Air} (E_{\alpha}) \text{ mg/cm}^2$$

Example: Range of ^{210}Po alphas $E_{\alpha} = 5.3 \text{ MeV}$ in Al ($\rho=2.7 \text{ g/cm}^3$)

$$R_{Air} (cm) = 1.24 \cdot 5.3 - 2.62 = 3.95$$

$$R_{\alpha, Al} (5.3, 27) = 0.56 \cdot 27^{1/3} \cdot 3.95 \text{ mg/cm}^2 = 6.64 \text{ mg/cm}^2$$

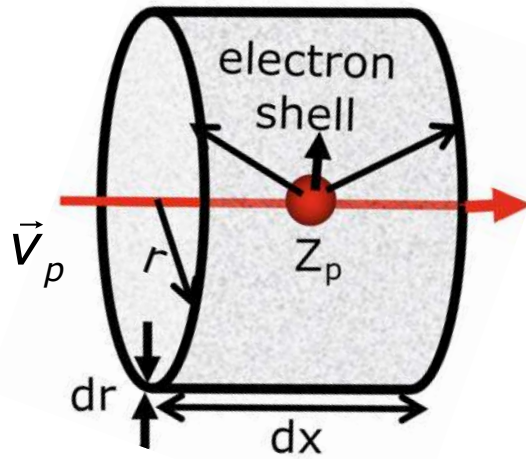
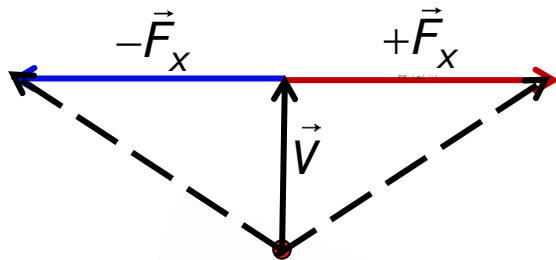
$$R_{\alpha, Al} (5.3, 27) = \frac{6.64 \text{ mg/cm}^2}{\rho_{Al}} = \frac{6.64 \cdot 10^{-3} \text{ g/cm}^2}{2.7 \text{ g/cm}^3} = 246 \mu\text{m}$$

Phenomenological Model of Energy Loss in Matter

Bethe et al. (1930-1953), Lindhardt's electron theory describes energy loss through ionization, incoming ions are fully stripped

Estimate of trends/Order of magnitude E =particle kinetic energy,
 $e^- \approx$ at rest in lab system; but in cms $v_e \approx -v_p$

Lateral force cancellation



$$\Delta p_e = F_{Coul} \cdot \Delta t_{coll} \approx \left(k_0 \cdot \frac{e^2 Z_p}{r^2} \right) \cdot \left(\frac{2r}{v} \right)$$

$$k_0 = 1/4\pi\epsilon_0 = 8.99 \cdot 10^9 \text{ Nm}^2\text{C}^{-2}$$

$$\text{electron density } \rho_e \left[\text{m}^{-3} \right]$$

$$-dE(r, x) \sim [2\pi r dr dx \rho_e] \cdot \left[\frac{(\Delta p_e)^2}{2m_e} \right]$$

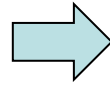
$$\frac{d^2 E(r, x)}{dr dx} \sim \left[\frac{4\pi e^4 Z_p^2}{m_e v^2} \rho_e \right] \cdot \frac{1}{r}$$

Important are only forces \perp to trajectory, others cancel

Phenomenological Model Of Energy Loss in Matter

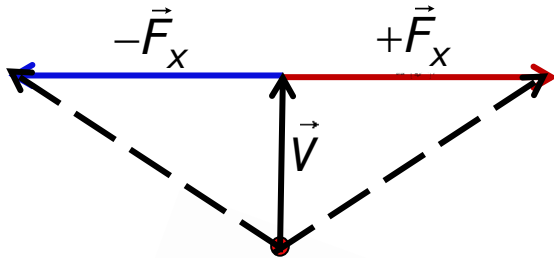
Important forces: \perp to trajectory \rightarrow Integrate only over radial coordinate

$$\frac{dE(x)}{dx} = \int_{r_{min}}^{r_{max}} dr \frac{d^2E(r, x)}{dr \cdot dx}$$



$$\frac{-dE(x)}{dx} \sim \left[k_0^2 \frac{4\pi e^4 Z_p^2}{m_e v_p^2} \rho_e \right] \cdot \text{Ln} \left(\frac{r_{max}}{r_{min}} \right)$$

Lateral force cancellation



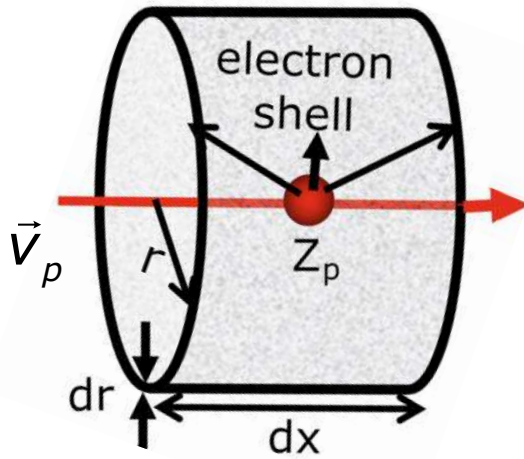
Estimate of radial limits: $\rho_e = e^-$ density
 $\lambda_e =$ electronic wavelength, cms $|v_e| \approx |v_p|$

$$r_{min} \gtrsim 2 \cdot \lambda_e; \lambda_e = h / p_e = h \sqrt{1 - \beta_p^2} / m_e v_p$$

non-adiabatic motion (fast particle, low Δt_{coll})

$$\Delta t_{coll} \sim \frac{r}{\langle v_e \rangle} \rightarrow \frac{r_{max}}{v_p} = \frac{T_e}{\sqrt{1 - \beta_p^2}} = \frac{h}{[IE \cdot \sqrt{1 - \beta_p^2}]}$$

$T_e =$ average e^- orbital period $\rightarrow IE / h =$ frequency
 $IE =$ ionization energy, $\beta_p := v_p / c$



Rest mass energy of electron

$$m_e c^2 = 0.511 \text{ MeV}$$

Electronic Stopping Power: Bethe-Bloch Formula

$$\left(\frac{-dE}{dx}\right)_p \approx \left(\frac{k_0^2 4\pi e^4}{m_e c^2}\right) \left(\frac{Z_p}{\beta_p}\right)^2 \cdot \rho_{Abs}^{el} \cdot \left(\text{Ln} \frac{2m_e c^2 \beta_p^2}{I_E (1 - \beta_p^2)} - \beta_p^2 + \dots \right)$$

$k_0^2 \frac{4\pi e^4}{m_e c^2} = \frac{5.08 \cdot 10^{-25}}{\text{MeV} \cdot \text{cm}^2}$
 $m_e c^2 = 0.511 \text{ MeV}$

Specify absorber material : density, volume, atomic number Z_{abs} ; mass number A_{abs}
 = # of nucleons (n, p) per particle, Number of electrons per particle $n_{el} = Z_{abs}$
 → Density of electrons in absorber = $\rho_{Abs}^{el} \uparrow$

"Molecular weight" $W_{mol} = A_{abs} \cdot 1g$ contains $L = 6.02 \cdot 10^{23}$ particles
 Volume = $V_{abs} [m^3]$, Density = $\rho_{abs} [g/m^3]$
 → $\rho_{Abs}^{el} = L \cdot \frac{W_{abs}}{W_{mol}} \cdot \frac{n_{el}}{V_{abs}} = L \cdot \frac{\rho_{abs}}{W_{mol}} \cdot n_{el} \rightarrow \rho_{Abs}^{el} \sim \frac{Z_{abs}}{A_{abs}}$ ← Contains absorber isotopic information

Absorber Ionization energy

$$IE = h \langle v_e \rangle \approx \begin{cases} (12 \cdot Z_T + 7) \text{ eV} & \text{for } Z_T < 13 \\ (9.76 \cdot Z_T + 58.8 \cdot Z_T^{-0.19}) \text{ eV} & \text{for } Z_T \geq 13 \end{cases}$$

For a compound $\left\{ \text{rel. density } \rho_i = \frac{n_{el,i}}{n_{el}}, Z_i \right\} \rightarrow \text{Ln} I_E = \sum_i \rho_i \cdot Z_i \cdot \text{Ln} I_{E,i}$
 Water : $I_{E,H} = 19 \text{ eV}, I_{E,O} = 105 \text{ eV} \rightarrow I_{E,H_2O} = 75 \text{ eV}$

The Bethe-Bloch Formula

$$\left(\frac{-dE}{dx} \right)_p \approx 5.08 \cdot 10^{-25} \text{ MeVcm}^2 \cdot \rho_{Abs}^{el} \cdot \frac{Z_p^2}{\beta_p^2} \cdot \left(\text{Ln} \frac{2m_e c^2 \beta_p^2}{I_E (1 - \beta_p^2)} - \beta_p^2 \right)$$

IE= Ionization energy

"Molecular weight" $W_{mol} = A_{abs} \cdot 1g$ contains $L = 6.02 \cdot 10^{23}$ particles

Volume = $V_{abs} [m^3]$, Density = $\rho_{abs} [g/m^3]$

$$\rightarrow \rho_{el} = L \cdot \frac{W_{abs}}{W_{mol}} \cdot \frac{n_{el}}{V_{abs}} = L \cdot \frac{\rho_{abs}}{W_{mol}} \cdot n_{el} \rightarrow \rho_{el} \sim \frac{Z_{abs}}{A_{abs}}$$

Example : Proton kinetic energy $E_p = 1\text{MeV}$, in $H_2O \rightarrow \beta^2 = 0.0021$, $IE = 75\text{eV}$

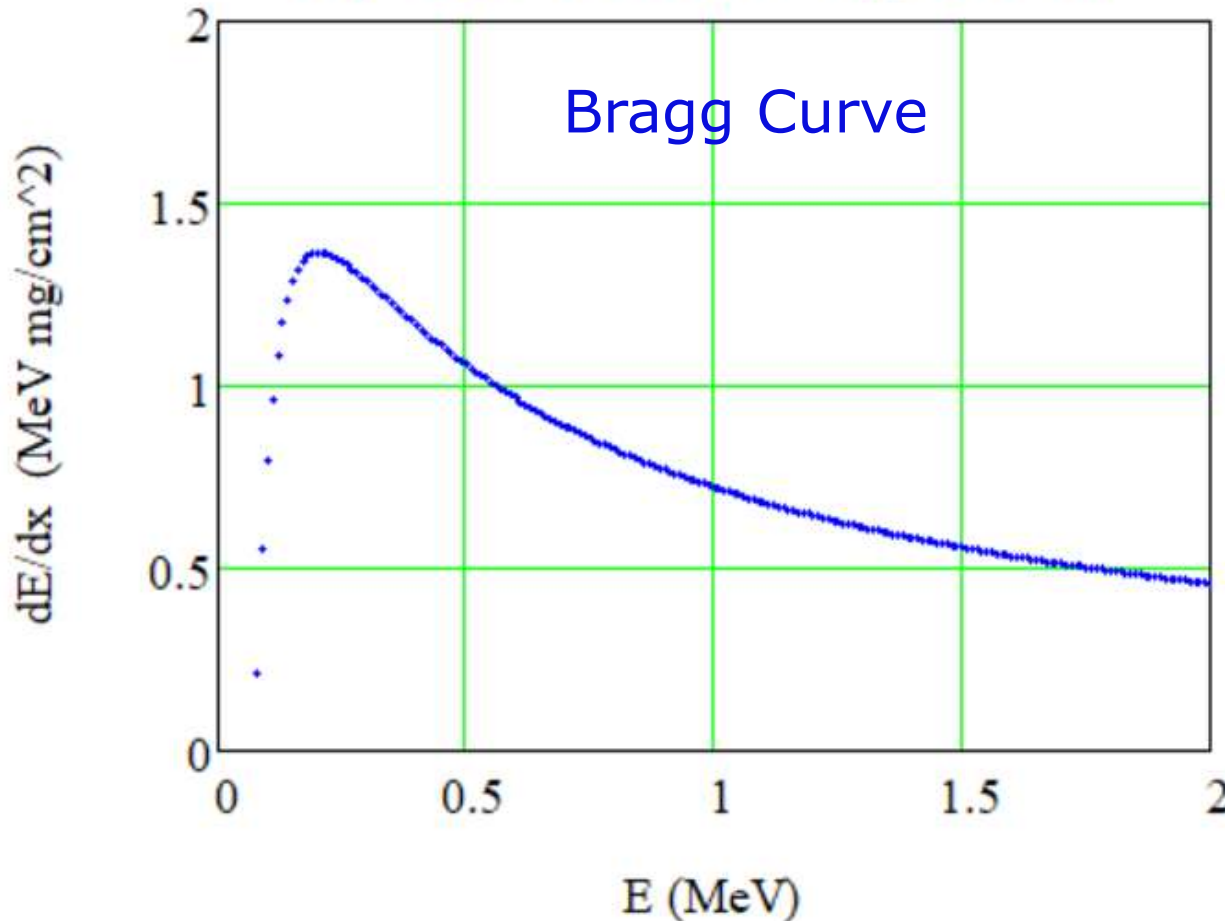
$$\frac{-dE}{dx} \approx \frac{0.1697}{\beta^2} \cdot \left(\text{Ln} \frac{0.0021 \cdot 1.022 \cdot 10^6 \text{eV}}{75\text{eV} \cdot 0.9979} - 0.0021 \right) \frac{\text{MeV}}{\text{cm}} = 270 \frac{\text{MeV}}{\text{cm}} = 2.70 \frac{\text{keV}}{\mu\text{m}}$$

In popular absorber units (g/cm^2)

$$\left(\frac{-dE}{d(\rho_{Abs} \cdot x)} \right)_p \approx 0.3071 \left(\frac{Z}{\beta} \right)_p^2 \cdot \left(\frac{Z}{A} \right)_{Abs} \cdot \left(\text{Ln} \frac{2m_e c^2 \beta^2}{I_E (1 - \beta^2)} - \beta^2 \right) \frac{\text{MeV}}{g/cm^2}$$

Theoretical E-Loss in Thin Absorbers

Alpha Bethe-Bloch Energy-Loss Al



^{210}Po alphas
 $E_\alpha = 5.3$ MeV

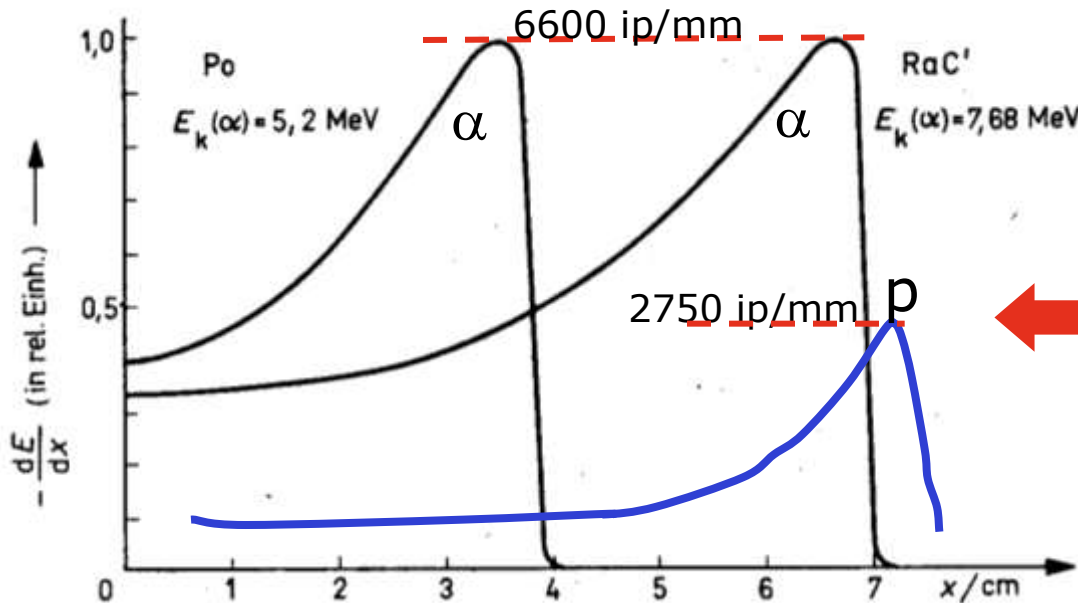
Loss:
MeV per mg/cm^2

Bragg maximum

Semi-quantitative only: Expt values smaller both at small and large energies
→ recharging effects for projectile

Range and Specific Ionization

E-loss in Air: 1atm, 15°C



Stopping power dE/dx (specific energy loss) depends on energy E and therefore on x

Bragg Curve

Highest E loss close to end of path \rightarrow Bragg maximum

$$\frac{dE}{dx} \triangleq \frac{\#(e^- - \text{ion pairs})}{\text{unit length}}$$

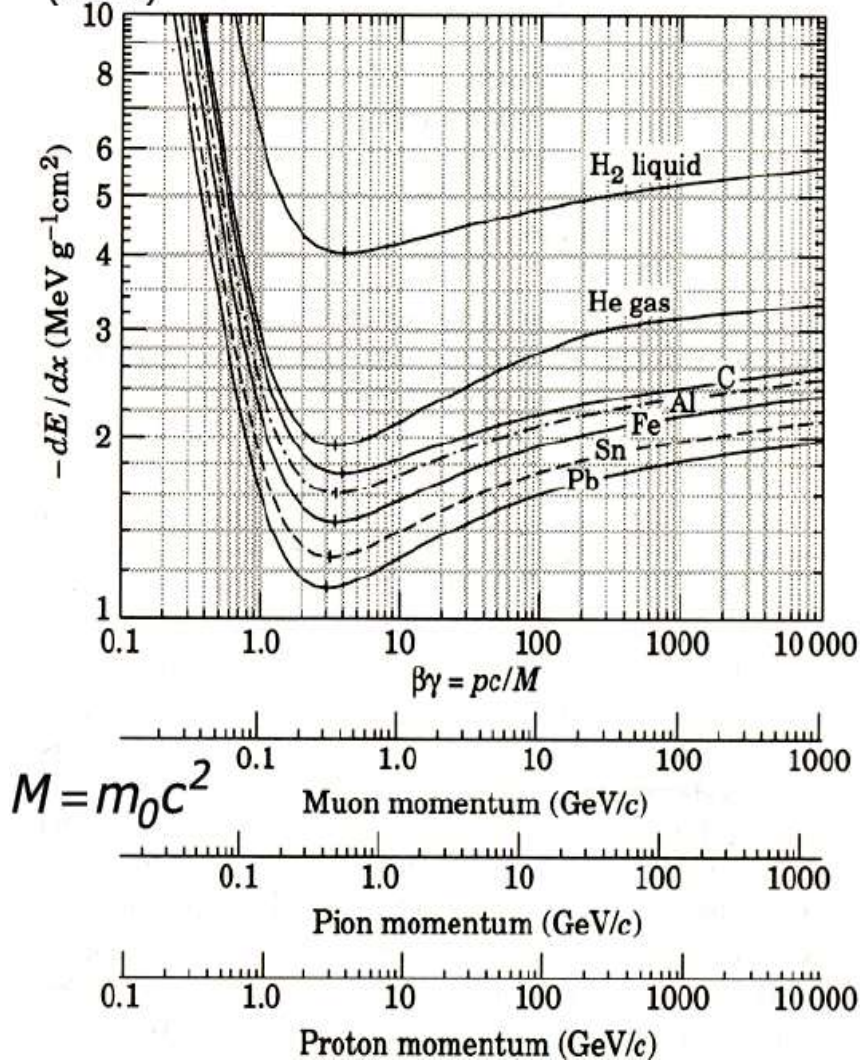
α particles :

$$\frac{dE}{dx} \leq \frac{7 \cdot 10^3 \text{ pairs}}{\text{mm}}$$

Main E-loss mechanism: ionization, production of " δ electrons," electron-ion pairs

ρ, π, μ Stopping Power Curves

Review of Particle Properties, Phys. Rev. D50, 1173 (1994)



$$E^2 = (pc)^2 + (m_0 c^2)^2$$

$m_0 =$ particle rest mass

$v =$ particle velocity

$$c = 2.9979 \cdot 10^8 \text{ ms}^{-1}$$

$$\beta = v/c$$

$$\gamma = \left(\sqrt{1 - \beta^2} \right)^{-1}$$

$$pc = (\gamma m_0) v c = \gamma \beta m_0 c^2$$

"mips"
= minimum ionizing
particles

End Charged Particles