Radiation Survey Experiment

Objectives:

- Use calibrated γ and X-ray sources to verify basic radioactivity laws:
  - Inverse-square dependence of count rate (dose) on distance
  - Exponential attenuation of dose on shielding matter
  - Counting statistics (Poisson)
  - Optimum signal vs. background measurements
- Assess counter sensitivity limits:
  - Detection efficiency for γ/X radiation of various energies
  - Fidelity in counting (dead time)
- Discover unexpected radioactive contamination (e.g., of common construction materials and/or of household items).

Reading Assignments :
Knoll Ch. 3.I-3.IV (Counting Statistics & Error Prediction)
Ch. 4 (General Properties of Radiation Detectors)
Reading Bevington Ch. 2 (Probability Distributions), Ch. 3 (Error Analysis),

γ-Induced Processes in Matter

γ-rays (photons): from electromagnetic transitions between different energy states → detect indirectly via effects in detector (**charged particles, e^-, e^+**)

Detection of secondary particles (e^-,e^+) from:

1. Photo-electric absorption
2. Compton scattering
3. Pair production
4. γ-induced nuclear reactions

$$E_{kin} = \hbar\varpi - E_n; \quad E_n = binding\ energy$$

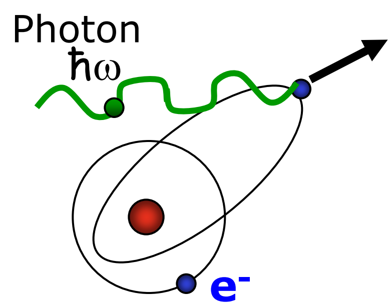
$$E_n = Rhc \cdot \frac{(Z-\sigma)^2}{n^2} \quad Moseley's\ Law$$

$$Rhc = 13.6\,eV \quad Rydberg\ constant$$

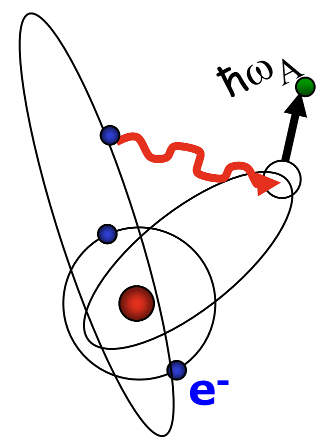
screening constants

$$\sigma_K \approx 3,\ \sigma_L \approx 5,\ different\ subshells$$

1. Photo-electric absorption (Photo-effect)

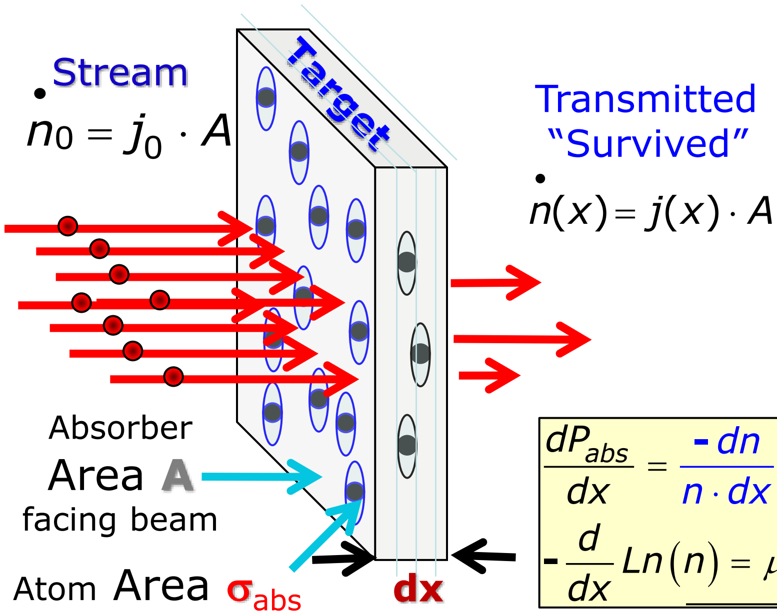


photon is completely absorbed by e^-, which is kicked out of atom



Electronic vacancies are filled by low-energy "Auger" transitions of electrons from higher orbits

Interaction Probability/Atom (=*Cross Section*)



$$\frac{dP_{abs}}{dx} = \frac{-dn}{n \cdot dx} = -\frac{d}{dx} Ln(n) = \mu(\cdots)$$

Assumption: absorption coefficient $\mu(x) \approx$ const.

Transmitted (survived)

$$n(x) = n_0 \cdot e^{-\mu \cdot x} \rightarrow \dot{n}(x) = \dot{n}_0 \cdot e^{-\mu \cdot x}$$

$$P_{abs}(x) = \mu x = \left[\begin{matrix}\#\,atoms/x \\ in\,absorber\end{matrix}\right] \bullet \left(\begin{matrix}P_{absorption}\,/ \\ abs\ atom\end{matrix}\right) \bullet \left(\begin{matrix}Random \\ medium \\ depth\ x\end{matrix}\right)$$

$$\mu x = \left[\left(\frac{L}{M_{abs}}\right) \cdot (\rho_{abs} \cdot A \cdot x)\right] \bullet \left(\frac{\sigma_{abs}}{A}\right)$$ ← Atomic cross section "area"

$$\mu = (L/M_{abs})\rho_{abs} \cdot \sigma_{abs}$$ abs. coeff. =#atoms·σ/Vol

Thin-absorber approximation: $(\mu \cdot x \ll 1)$

$$\dot{n}(x)_{absorbed} = \dot{n}_0 - \dot{n} = \dot{n}_0 \cdot \left(1 - e^{-\mu \cdot x}\right)$$

$$\dot{n}_{absorbed} \approx \dot{n}_0 \cdot (\mu \cdot x) = \frac{\dot{n}_0}{A} \cdot \left(\left(\frac{L\rho_{abs}}{M_{abs}}\right) A \cdot x\right) \sigma_{abs}$$

$$\approx j_0 \cdot N_{abs} \cdot \sigma_{abs}$$ *stream current density j*

Absorption occurs upon photon "hit" atom within cross section area σ_{abs}

j_0, j stream current density (#part/time·area)
A area illuminated by stream
L= 6.022 10^{23}/mol Loschmidt#(Avogadro)
N_{abs} # absorber nuclei in stream
M_{abs} absorber molar weight
ρ_{abs} absorber mass density (g/cm^3)
x absorber linear thickness
[σ]=***1barn = 10^{-24}cm^2***

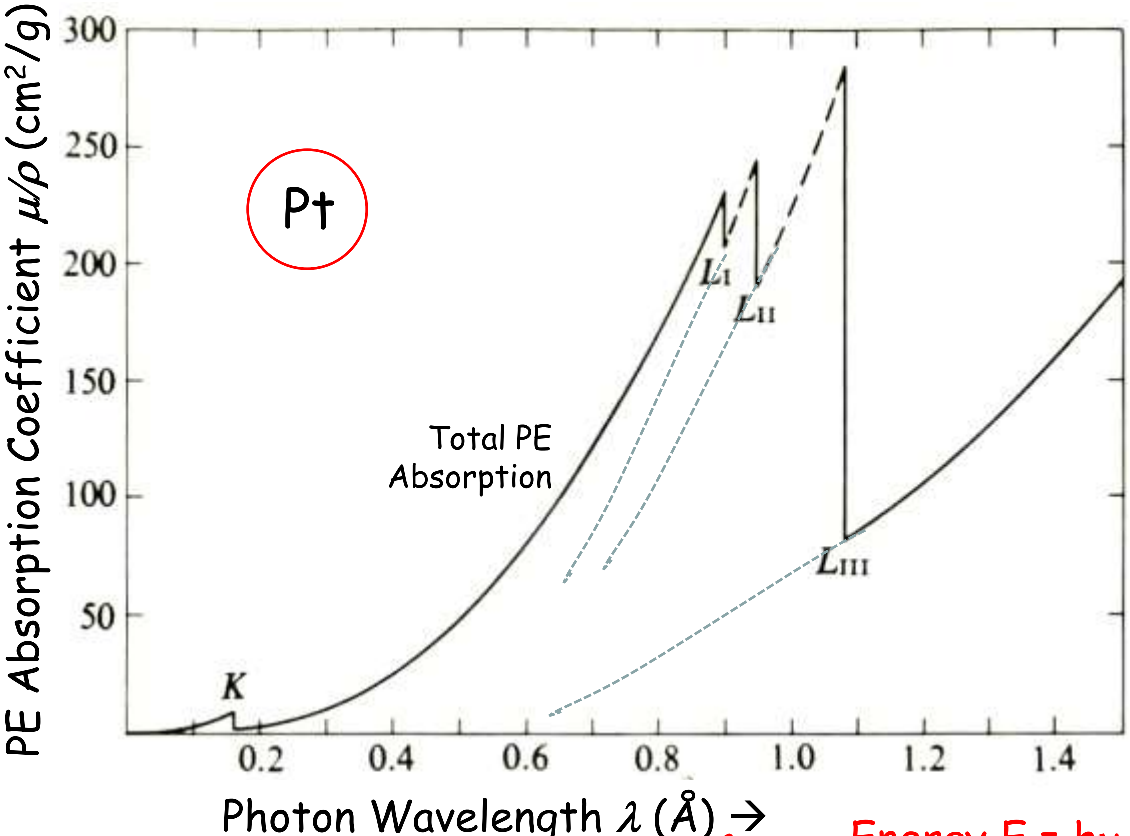
Elementary absorption cross section of one atom

$$\sigma_{abs} = \frac{\dot{n}_{absorbed}}{N_{abs} \bullet j_0}$$

observe sum effect → $\sigma_{abs} = \sum_i \sigma_{abs}(i)$ (*process i*)

1. Photo-Absorption Coefficient

Absorbance $A := Ln(n_0/n) = \mu(E)\cdot x = (\mu(E)/\rho)\cdot(\rho x)$ *Areal mass density* $\rho\cdot x,\ \mu(x) = const.$



Energy $E_\gamma = h\nu$

Absorption coefficient

→ μ (1/cm)

"Mass absorption" is measured per density ρ

→ μ/ρ (cm²/g)

"Cross section" is measured per atom

→ σ (cm²/atom)

Absorption of light is quantal resonance phenomenon: Strongest when photon energy coincides with transition energy (at K, L,… "edges")

Probabilities for independent processes are additive:

$$\mu^{PE}(E_\gamma) = \mu_K^{PE}(E_\gamma) + \mu_L^{PE}(E_\gamma) + \ldots.$$

$$\sigma_{PE}(E_\gamma, Z) \propto Z^5 \cdot E_\gamma^{-7/4} \quad low\ E_\gamma$$

$$\sigma_{PE}(E_\gamma, Z) \propto Z^5 \cdot E_\gamma^{-1/2} \quad high\ E_\gamma$$

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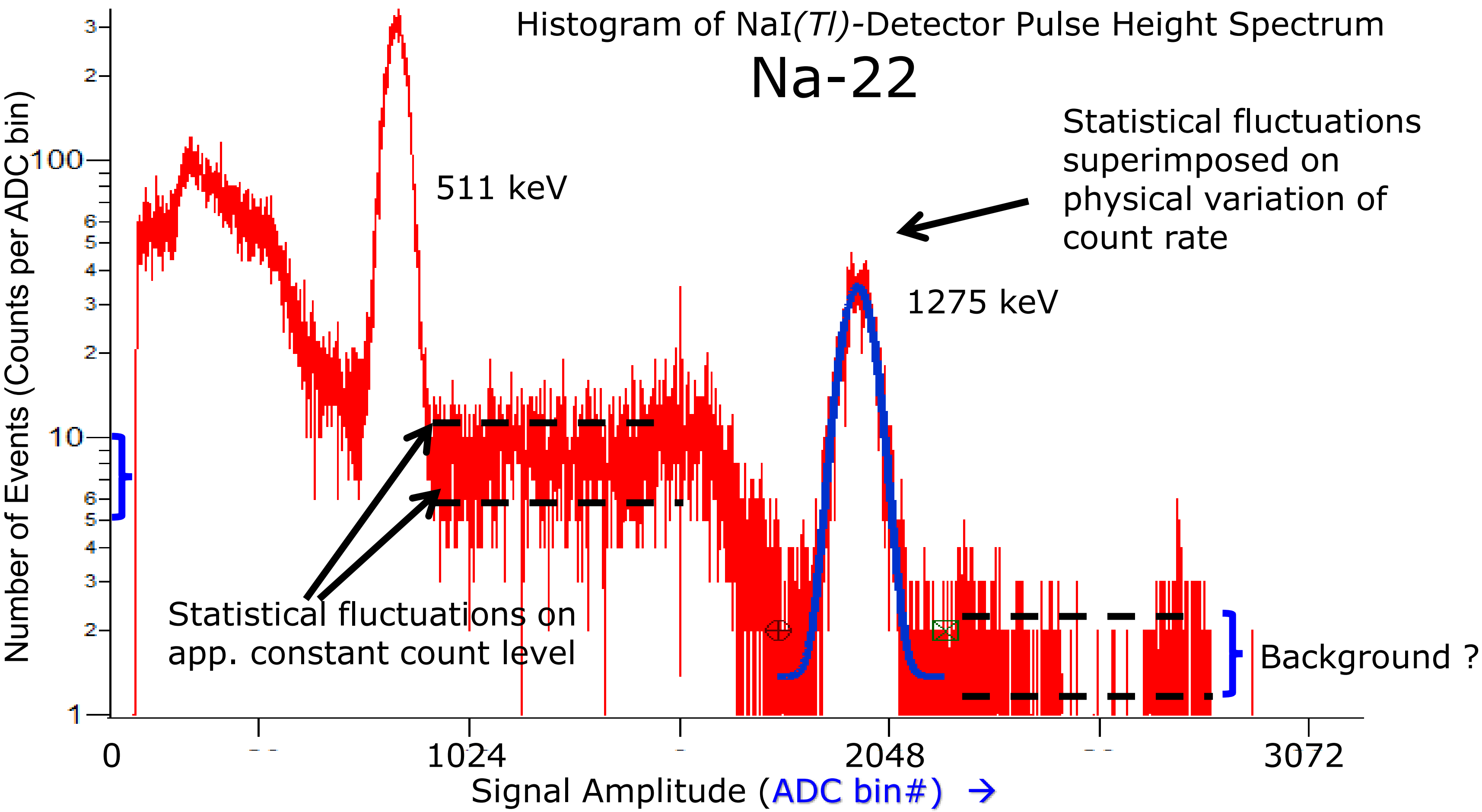
Example: Na-22 Pulse Height Spectrum with NaI Detector (ANSEL Data:)



Signal amplitude = Pulse Height *(PH):* Electrical response by a detector exposed to photon radiation. Pulse height observable is typically given in units of Volts (***V***). → digitized into series of bins. Brief (10 min) measurement of spectrum shows large fluctuations, reduce relatively with duration of measurement.
Statistical fluctuations are superimposed on smooth physical trend of count rate with *PH*.

Binomial Probability (Distribution)

Radioactive decay of a sample of *N* member nuclei is considered to be a stochastic, *quantum-transition process* occurring (or not) with equal probability for every member
>>At any given time ***t*,** every original (parent) sample nucleus has an irreversible *(binary)* option to assume one of two possible states within the following instant Δt: it can "try to decay" into a "daughter" nucleus or remain intact.<<

→ Typical binary yes/no probability theory applies → predict, explain *N(t)*.
→ specific probability to decay $p=-\Delta N/\Delta t$ → *probability* for remaining intact = *(1-p)*

Initiate this process ***N*** times → Q: What is the probability for ***m*** *nuclear decays?*

p^m = **probability** for *m* decays (simultaneous within Δt) among *N(t)* "trials"
$(1-p)^{N-m}$ = **probability** for *N-m* survivals (not detected within Δt)

Probability for exactly *m decays (*successes) observed out of a total of *N* trials

$$P(m) = \binom{N}{m} \cdot p^m \cdot (1-p)^{N-m} \to \sum_m P(m) = 1$$

$$\binom{N}{m} = \frac{N!}{m!(N-m)!}$$

How many ways can *m* success events be 'chosen' out of N ? → Binomial coefficient

→ **ν's moment** $\langle m^\nu \rangle = \sum_{m=0}^{N} m^\nu \cdot P(m) = \sum_{m=0}^{N} m^\nu \cdot \binom{N}{m} p^m (1-p)^{N-m}$

$$\text{Mean value } \mu = \langle m \rangle = N \cdot p; \qquad \text{Variance } \sigma_m^2 = N \cdot p \cdot (1-p)$$

Radioactive Decay as a Poisson Process

Only a (low) mean rate $-dN/dt = p \cdot N(t) \rightarrow -d(Log\ N)/dt = p$ is known for a process (radiative decay, background emission, or rare reaction events).

Example: ^{137}Cs = unstable isotope, decays with mean lifetime τ **= 43a**, or half-life $t_{1/2} = 30a$ (years). ***N***= number of decays, Activity ***A = N/τ*** *(decays per s)*

$$N(t)/N(0) = A(t)/A(0) = e^{-(t/\tau)} = (1/2)^{+(t/t_{1/2})}$$

$\rightarrow p = A/N = Ln2/30a = 0.023/a = 7.32 \cdot 10^{-10} s^{-1} \rightarrow$ **small (→Poisson)**

Sample of ***1*** μg: $N = 10^{15}$ nuclei (=trial candidates for decay)

How many will decay (on average, for many measurements)?

$\mu = N \cdot p = 7.32 \cdot 10^{+5}\ s^{-1}$ $= dN/dt$ Count rate estimate

$(7.32 \cdot 10^{+5} \pm 854)\ s^{-1}$

Probability for observing *m* decays within (per) Δt =1s:

$$Lim_{p \to 0, N \to \infty} P_{binomial}(N,m) = P_{Poisson}(\mu, m) = \frac{\mu^m \cdot e^{-\mu}}{m!} = \frac{(7.32 \cdot 10^5)^m \cdot e^{-7.32 \cdot 10^5}}{m!}$$

Poisson Probability Distribution

Limit of binomial (yes/no) distribution for very unlikely events → $p \approx 0$ but $N \to \infty$

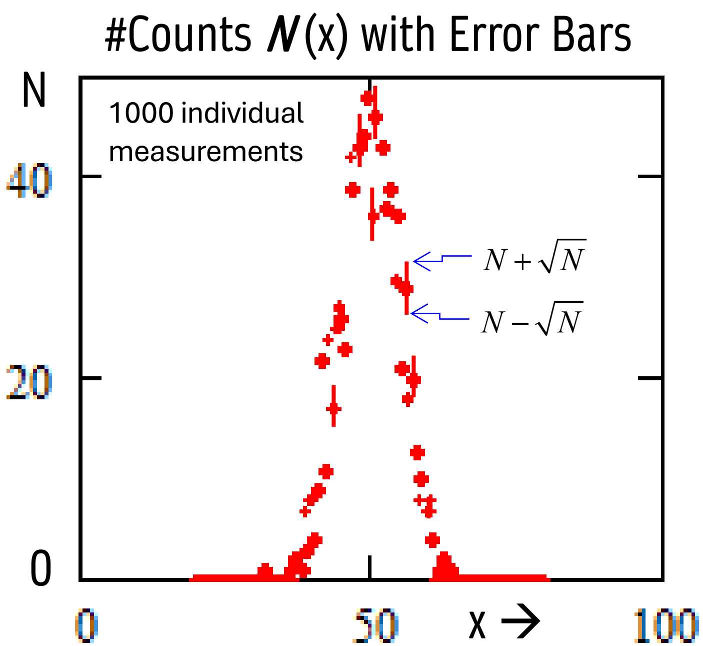
$$Lim_{p\to 0, N\to\infty} P_{binomial}(N,m) = P_{Poisson}(\mu,m)$$

Probability for observing m events when N=?? but mean=average is known as <m> = μ

$$P_{Poisson}(\mu,m) = \frac{\mu^m \cdot e^{-\mu}}{m!}$$

μ = <m> = $N \cdot p$ for $N \to \infty$

and variance $\sigma^2 = \mu$



For radioactive decays per time $[\Delta t^{-1}]$

$$\rightarrow p = \frac{Activity \cdot \Delta t\ (\#decays)}{Number\ of\ candidates}$$

$$p \ll 1 \rightarrow \sigma_m^2 \approx \langle m \rangle\ \#counts$$

Observe large #N of counts (e.g., decay events) for a given rxn variable x
→ statistical uncertainty is $\pm\sigma_N = \pm\sqrt{N}$

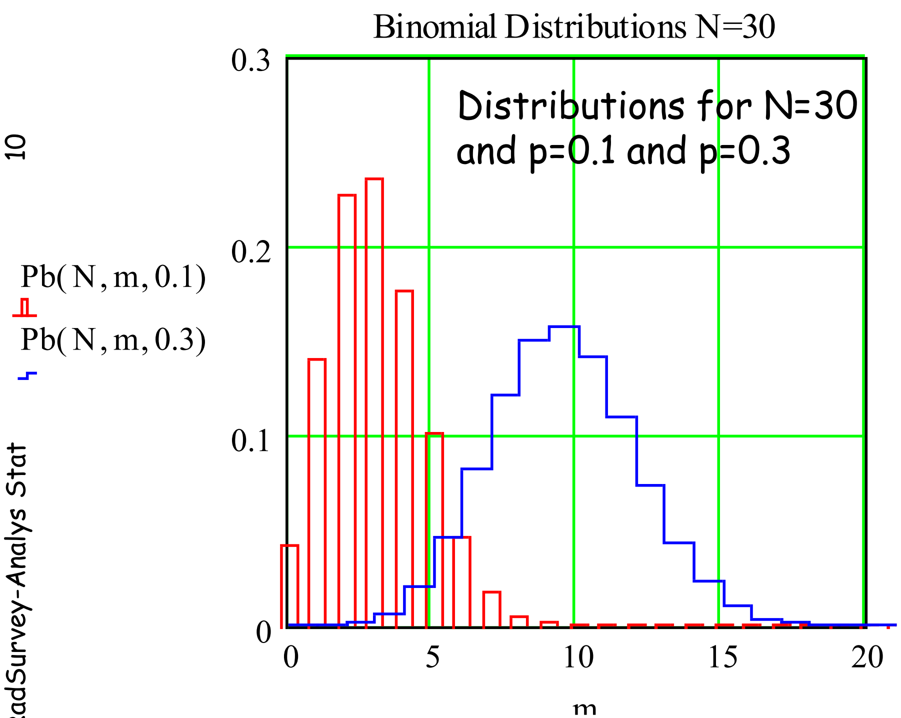
Distribution Moments and Limits

$$P_{binomial}(N,m,p) = \binom{N}{m} p^m (1-p)^{N-m}$$

Probability for m decays out of N trials, individual probability p

Normalization

$$1 = \sum_{m=0}^{N} P_{bin}(m,p) = \sum_{m=0}^{N} \binom{N}{m} p^m (1-p)^{N-m}$$



Mean and variance ('uncertainty')

$$\bar{m} = N \cdot p \approx N_{obs} \quad and \quad \sigma_m^2 = N \cdot p \cdot (1-p) \approx N_{obs}$$

$$N_{obs} = \#\ of\ \text{"counts" observed}, \quad p \ll 1.0$$

$$\text{Statistical "error" of } N_{obs} : \sigma_m \approx \sqrt{N_{obs}}$$

$$\frac{\sigma_m}{\bar{m}} = \frac{\sqrt{N \cdot p \cdot (1-p)}}{N \cdot p} \approx \frac{1}{\sqrt{N_{obs}}} \rightarrow \text{more counts = smaller error}$$

Distributions P(m) approximates Gaussian very fast, already good for p=0.2-0.3

Observe Change Poisson → Gaussian

$$\lim_{\substack{p \to 1 \\ N \to \infty}} P_{bin}(N,m,p) = \frac{1}{\sqrt{2\pi\sigma_m^2}} \cdot \exp\left\{ -\frac{\left(m - \langle m \rangle\right)^2}{2\sigma_m^2} \right\}$$

Central-Limit Theorem

The means (averages) of different samples from a given population tend to cluster together closely. → general property of samples of stochastic variables:

The distribution of **sample means** of a population approaches a Gaussian ***Normal Distribution***, as the size ***n*** of the sample increases, *regardless of the form of the original (population) distribution*.

The mean (average) of a distribution of stochastic data does not contain information on the actual shape of the distribution.

The average of any truly random sample of a population is already close to the true population average. Considering many samples, or large samples, narrows the choices. The Gaussian width becomes narrower for larger samples. → The standard error of the mean decreases as the sample size increases.

Experimental Mean Count Rate and Variance

What can be measured: ensemble (sampling) averages (expectation values) and uncertainties

Task: ^{236}U (0.25mg) source, count # α particles emitted during M = 10 time-intervals Δt (samples @ Δt ≈ 1 min). λ = ??

n	n-<n>	(n-<n>)^2
36076	129.6	16796.16
35753	-193.4	37403.56
35907	-39.4	1552.36
36116	169.6	28764.16
35884	-62.4	3893.76
36136	189.6	35948.16
35741	-205.4	42189.16
35640	-306.4	93880.96
36124	177.6	31541.76
36087	140.6	19768.36
35946	-1.5E-12	**3463.76**
<n>	**<n-<n>>**	**σ^2_n**

Set $\{m = 1, ..., M\}$ of *M independent, identical* samplings of large population. $N_m = \#measurements\ in\ sampling\ m$

Sample $m : \{n_i;\ i = 1...N\}_m$ counts → average #counts in sample m:

$$\bar{n}_m = \frac{1}{N_m}\sum_{i=1}^{N_m} n_i \rightarrow Sample\ average\ \langle n \rangle = \frac{1}{M}\cdot\sum_{m=1}^{M}\bar{n}_m,\quad \sum_{i=1}^{M} N_i = N = M\langle n \rangle$$

Variance of counts n_i in sample m: $s_m^2 = \frac{1}{N_m - 1}\cdot\sum_{i=1}^{N_m}(n_i - \bar{n}_m)^2$

2 observables fluctuate statistically: M averages $\bar{n}_m$ AND M variances s_m^2

$$Sample\ variance\ \sigma^2 = \frac{1}{N}\left\{\sum_{m=1}^{M}(N_m - 1)s_m^2 + \sum_{m=1}^{M} N_m\left(\bar{n}_m - \langle n \rangle\right)^2\right\}$$

$Std.deviation: \sigma_n = \sqrt{\sigma_n^2} = 59 \rightarrow$

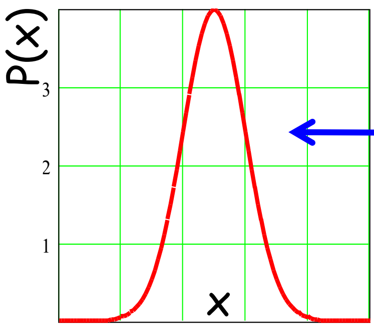
$Std.\ error\ of\ mean\ \delta = \sqrt{\sigma_n^2 / N} = 19$

$Result: \langle n \rangle \approx \langle n \rangle_{pop} = (35946 \pm 19)\,\mathrm{min}^{-1}$

Sample average $\langle n \rangle \neq \langle n \rangle_{population}$ has "standard error" δ depending on the measured sampling variance σ_n^2 and the total number N of measurements: $\delta \approx \sqrt{\sigma_n^2 / N}$

"Error" δ<n>of <n> much smaller than σ. Reduced by 1/10 for 100 times larger sample

Sample Statistics (Simulation)

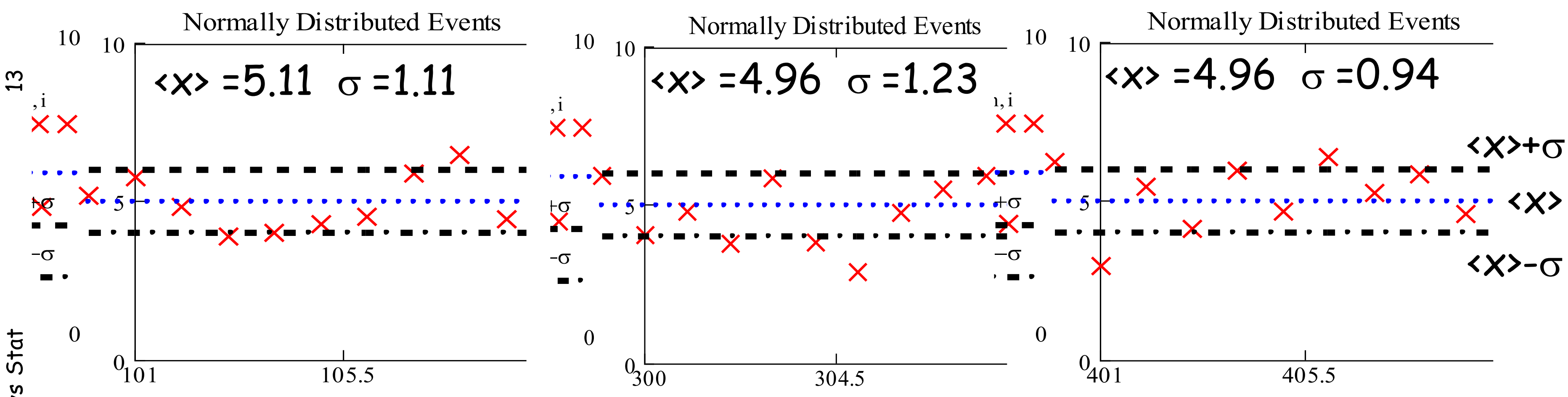


Assume true population distribution for variable x

$$P(x)=\frac{1}{\sqrt{2\pi\sigma_X^2}}\cdot\exp\left\{-\frac{\left(x-\langle x\rangle_{pop}\right)^2}{2\sigma_X^2}\right\}$$

with true (N=30 "population") mean $\langle x\rangle_{pop}$ = 5.0, ν_x =1.0

Sample of M=three 10-count "measurements" of some variable x:



Equally weighted sample average $\langle x\rangle_s$ = (5.11+4.96+4.96)/3 = 5.01

Sample variance in means $s^2=\sigma^2$ = $10\cdot[(5.11-5.01)^2+\mathbf{2}(4.96-5.01)^2]/29$ = 0.01 **s = 0.0034**

$\sigma_x^2 = 9\cdot(1.11^2+2\cdot1.23^2)/29 = 1.18$ $\sigma_x = 1.09$ → Error of mean: $\delta^2 \approx \sigma_x^2/30 = 3.6\cdot10^{-2}$,

Combined result of 3 independent small samples $\langle x\rangle_s = 5.01+0.19$

Functions of Stochastic Variables

Random independent variables $N_1, N_2, \ldots, N_n$
corresponding variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$
Function $\boldsymbol{f}(N_1, N_2, \ldots, N_n)$ of random variables: Uncertainty $\Delta N_i \rightarrow \Delta f(\{N_i\})$

Gauss' law of error propagation:

$$\sigma_f \approx \left\{ \underbrace{\left(\frac{\partial f}{\partial N_1}\right)^2 \sigma_1^2}_{\left(\Delta f\,|_{N_2,N_3,..}\right)^2} + \underbrace{\left(\frac{\partial f}{\partial N_2}\right)^2 \sigma_2^2}_{\left(\Delta f\,|_{N_1,N_3,..}\right)^2} + \ldots + \underbrace{\left(\frac{\partial f}{\partial N_n}\right)^2 \sigma_n^2}_{\left(\Delta f\,|_{N_1,N_2,..,N_{n-1}}\right)^2} \right\}^{1/2}$$

Further terms if N_i are not independent ($\rightarrow$ correlations, co-variance tensor)
Otherwise, individual independent component variances $(\Delta f)^2$ add.

W. Udo Schröder, 2026

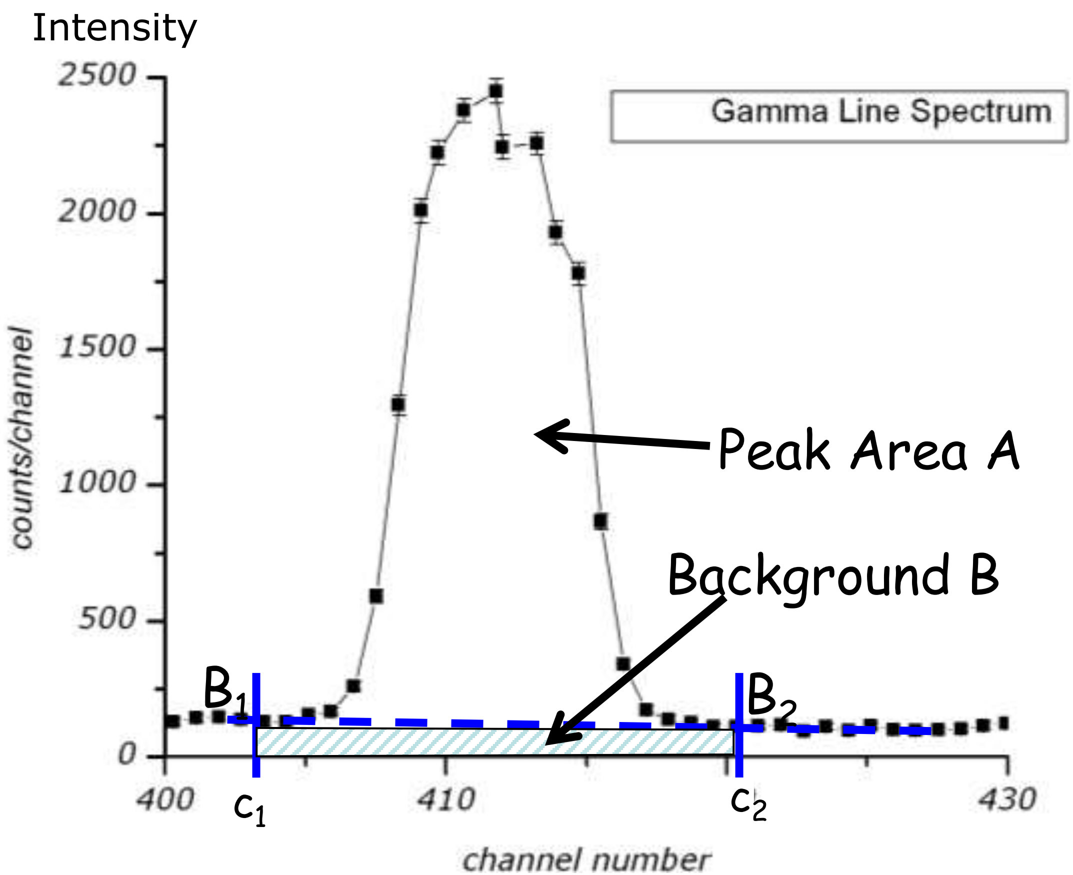
Example: Spectral Analysis (Local Background Subtraction)

Add or subtract 2 Poisson-distributed numbers N_1 and N_2:
Variances σ^2 always add

Std. dev $\sigma_{1\pm2}$

$$N := \left[N_1 \pm \sqrt{N_1}\right] \pm \left[N_2 \pm \sqrt{N_2}\right] \triangleq (N_1 \pm N_2) \pm \sqrt{N_1 + N_2}$$

Std. dev σ_1 Std. dev σ_2



Analyze peak in range channels $c_1 - c_2$: beginning of background left and right of peak

$$n = c_1 - c_2 + 1.$$

Total area c_1 - c_2 → N_{12}

$N(c_1)=B_1$, $N(c_2)=B_2$,

Linear (≈constant) background
$B = n(B_1+B_2)/2$

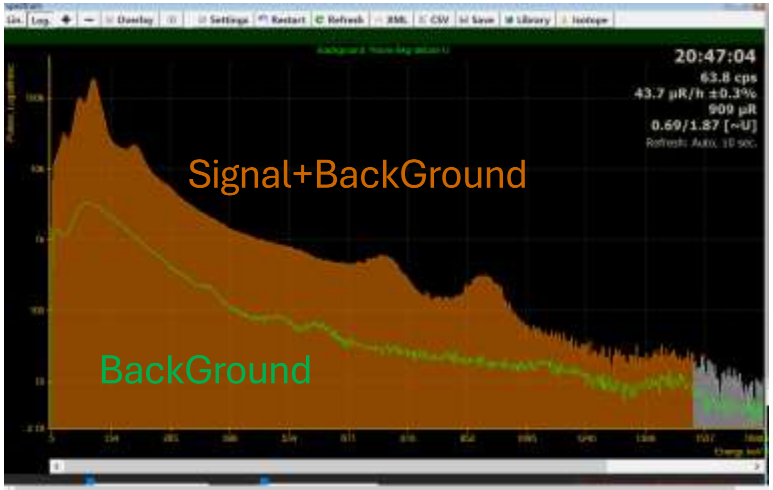
$$Peak\ area\ A = \sum counts_i / ch$$

$$A = N_{12} - n \cdot (B_1 + B_2)/2$$

Stat uncertainty std.dev.

$$\sigma_A = \sqrt{N_{12} + n \cdot (B_1 + B_2)/2}$$

Economic Division of Allocated Experimental Time *T*



Measured spectra of intensity N_{S+B} (counts) vs. energy (E) of detected radiation

$$\frac{dN_{S+B}(E)}{dE} = \frac{dN_S(E)}{dE} + \frac{dN_B(E)}{dE}$$

"Signal" ***S*** = proper object of exptl. study of a certain radiation source.

Superimposed on the *signal component* is an undesirable, always present "Room Background" and/or noisy random "Background" ***B***.

Room Background *B* can be measured separately, with the source switched off, to be later appropriately scaled and subtracted from the Signal+Background data.

Desired: optimal measurement $\frac{dN_S(E)}{dE} = \frac{dN_{S+B}(E)}{dE} - \frac{dN_B(E)}{dE}$ @ highest precision during ***T***

$$\text{Uncertainties add quadratically}: \sigma_S^2 = \sigma_{S+B}^2 + \sigma_B^2, \text{ rel. importance depends on rates}$$

$$\text{Total available time } T = T_{S+B} + T_B \rightarrow \text{Minimize } \sigma_S^2(T) = \frac{N_{S+B}}{T_{S+B}} + \frac{N_B}{T_B} \rightarrow \boxed{\frac{T_{S+B}}{T_B} = \sqrt{\frac{N_{S+B}}{N_B}} = \boxed{s_{sb}}}$$

$$\boxed{\text{Optimum}: T_{S+B} \approx \left(\frac{s_{sb}}{1+s_{sb}}\right)T, \quad T_B \approx \left(\frac{1}{1+s_{sb}}\right)T}$$

Example: Available T=30min @ S+B=100cps, B=20cps → S_{sb}=2.2; T_{S+B}=21min, T_B=9min

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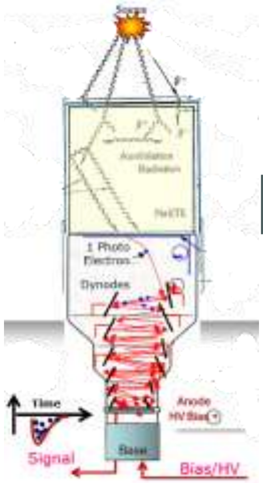
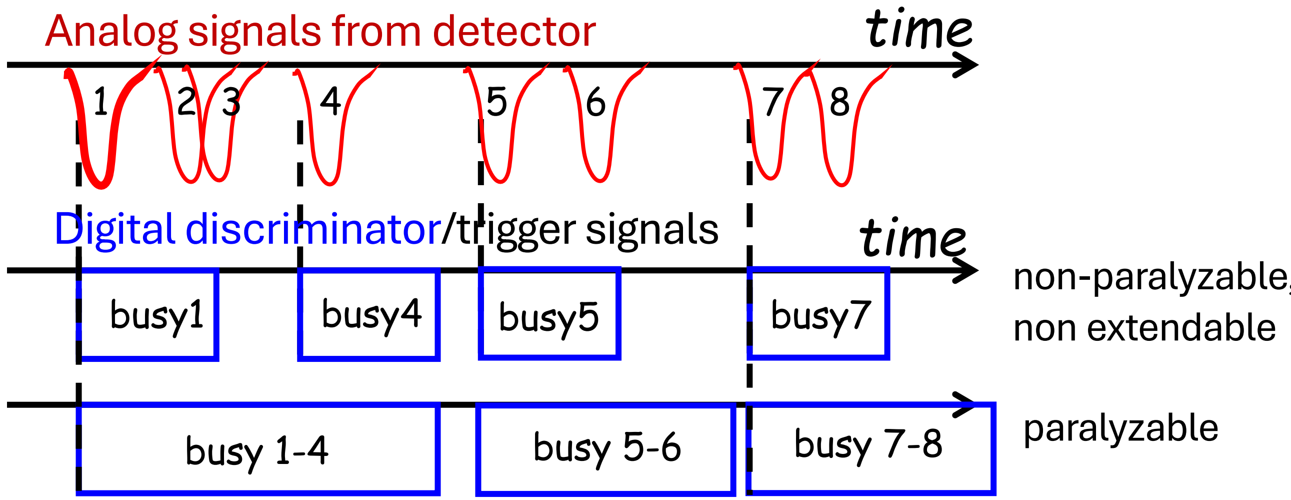
Electronic Busy/Dead-Times





Discriminator modules with non-paralyzable, non extendable busy time τ_d. accept analog signals #1,4,5,7 and miss all others. Paralyzable deadtimes lead to more losses at high rates. Typically: $\tau_d \sim 50ns$ (fast electronics) - $\tau_d \sim 10\mu s$ (DAQ electronics)

$$\textit{Apparent, measured \# counts } N_m \le \textit{True \# counts } N_{tr}$$

$$N_m = \frac{N_{tr}}{1 + N_{tr} \cdot \tau_d} \; (non-paralyzable); \qquad N_m = N_{tr} \cdot e^{-N_{tr} \cdot \tau_d} \; (paralyzable)$$

Non – monotonic with $N_{tr} \to \exists \max(N_m)$

$$\textit{Safe count rates } \dot{N}_m \ll 1/\tau_d$$

Signal Pulses: Output from PM Base