PSCI 407 Mathematical Modeling

Fall, 2023
MW 10:00–11:50
Harkness 112

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Hours: by appointment

This course is the first half of a two-course sequence consisting of PSCI 407 and PSCI 408. The goal of the sequence is to give students a rigorous introduction to the main concepts and models used in positive political theory and game theory. At the same time, we will teach you the mathematical tools necessary to understand and analyze these models, to incorporate them in your research, and (if it suits you) to push the frontier of mathematical modeling in political science. The sequence emphasizes rigorous logical and deductive reasoning—this skill will prove valuable, even to the student primarily interested in empirical analysis, rather than modeling.

The sequence is designed to serve both as a rigorous foundation for students planning to take further courses in the positive political theory field, and as a self-contained overview of the field for students who do not intend to do additional coursework in it. PSCI 407 is mainly concerned with rational decision making and preference aggregation—analyzing the choice problem of a rational agent and considering the challenges of reconciling conflict of preferences across agents—whereas PSCI 408 will focus on strategic interaction and game-theoretic analysis.

Students should have, at a minimum, a sound familiarity with basic algebra (solving equations, graphing functions, etc.) and a knowledge of basic calculus. Consistent with department policy, students are required (unless explicitly exempted) to attend the "math camp" offered in the weeks before the fall semester.

Homeworks, a midterm, and a final will be assigned to help develop and test your mathematical modelling skills. Students are allowed to collaborate on homework, but after discussion with others, each student is expected to write up his or her answers independently. The date and time of the final are set by the University Registrar: it will take place on **Tuesday, December 19, at 12:30,** and you will have three hours to complete it. This date is firm, so keep it in mind when making your travel plans for winter break.

An outline of the topics to be covered is as follows.

1. *Logic, proofs, and set theory*—rules of proof, quantifiers, sets, induction
2. *Application: individual and social choice*—preference and utility, voting rules, majority core, Arrow's theorem
3. *Multidimensional mathematics*—sequences, open/closed/compact sets, continuity, linearity, differentiability, implicit function theorem, comparative statics
4. *Application: spatial model*—median voter theorem, Pareto optimality, Plott's theorem

5. *Application: expected utility*—von Neumann-Morgenstern axioms, simple lotteries, quadratic utility
6. *Optimization theory*—existence, unconstrained optimization, concave problems, single equality/inequality constraint
7. *Application: rational choice*—social welfare maximization, contest/public good model, proposer model

There are multiple resources available in the course Dropbox folder. The main sources for class lectures will be selections from three volumes,

- *Formal Methods in Political Theory, I: Mathematical Foundations*
- *Formal Methods in Political Theory, II: Axiomatic Rational Choice Theory*
- *Formal Methods in Political Theory, III: Mathematics of Multiple Dimensions,*

which I have compiled over many years of teaching the formal theory sequence at Rochester. In addition, I recommend that all students obtain:

- Simon and Blume, *Mathematics for Economists.*

Simon and Blume is a valuable compendium of the mathematics used in the social sciences, but the applications are oriented toward economics. Nevertheless, it will be a valuable complement to the main volumes. For students who want more advanced references in positive political theory or selected topics in mathematics, I am glad to provide recommendations.

The teaching assistant for the course is **Shusuke Ioku,** who will hold a weekly recitation and office hour. Keep in mind that the TA's primary responsibility during recitation is to answer your questions, so come prepared.

*All assignments and activities associated with this course must be performed in accordance with the University of Rochester's Academic Honesty Policy. Visit the academic honesty webpage for more information.

** The University of Rochester respects and welcomes students of all backgrounds and abilities. In the event you encounter any barrier(s) to full participation in this course due to the impact of disability, please contact the Office of Disability Resources. Visit the office website for more information.

*** This course follows the College credit hour policy for four-credit courses. This course meets twice per week for a total of four hours per week; in addition, the course includes one hour of recitation per week.