PSC 407 Mathematical Modeling

Fall, 2010
MW 10:00-11:50
Harkness 329

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Hours: by appointment

This course is the first half of a two-course sequence consisting of PSC 407 and PSC 408. The goal of the sequence is to give a rigorous introduction to the main concepts and results in positive political theory. At the same time, we will teach you the mathematical tools necessary to understand these results, to use them, and (if it suits you) to surpass them in your own research in political science. The sequence emphasizes rigorous logical and deductive reasoning — this skill will prove valuable, even to the student primarily interested in empirical analysis rather than modelling.

The sequence is designed to be both a rigorous foundation for students planning on taking further courses in the positive political theory field and serve as a self-contained overview of the field for students who do not intend to do additional coursework in the field. PSC 407 is mainly concerned with social choice applications and the mathematics behind them, drawing mainly on logic, real analysis, and calculus, whereas PSC 408 will focus on strategic interaction.

Students should have, at a minimum, a sound familiarity with basic algebra (solving equations, graphing functions, etc.) and a knowledge of basic calculus. Consistent with department policy, students are required (unless explicitly exempted) to attend the "math camp" offered in the weeks before the fall semester.

Homeworks, a midterm, and a final will be assigned to help develop and test your mathematical modelling skills. Students are allowed to collaborate on homework, but after discussion with others, each student is expected to write up his or her answers independently. The date and time of the final are set by the University Registrar: it will take place on **Saturday, December 18, at 12:30,** and you will have three hours to complete it. This date is firm, so keep it in mind when making your travel plans for winter break.

The primary source for material covered in the lectures will be lecture notes distributed throughout the semester. In addition, I will assume students own or have access two textbooks for the course.

- Simon and Blume (SB), *Mathematics for Economists*
- Ordeshook (O), *Game Theory and Political Theory*

Simon and Blume is a valuable compendium of the mathematics used in the social sciences, but the applications are oriented toward economics. Nevertheless, it will be a valuable complement to the lecture notes. Ordeshook is not a math book, but it draws together many of the social-choice theoretic applications we cover in class.

The teaching assistant for the course is Brenton Kenkel, who will hold a weekly recitation and have office hours. Tentatively, recitation will be held in [ROOM], [DAY & TIME]. Keep in mind that Brenton's primary responsibility during recitation is to answer your questions, so come prepared.

An outline of the topics to be covered is as follows. Next to each, I list readings from the texts. It will be clear from the selections of readings that the authors, especially Simon and Blume, organized their books differently than I'm organizing the course. Later chapters, say Simon and Blume's Chapter 22 on "Economic Applications," won't be easily accessible the first time through.

1. Logic and Set Theory [my notes]
   - logical connectives, necessary and sufficient conditions, direct proof, proof by contradiction, quantifiers, set operations, natural numbers, real numbers, maximum and supremum
2. Relations, Functions, and the Rational Choice Model [O 1.1–1.2; notes]
   - relations, transitivity, weak orders, maximal elements, strict and weak preference, mappings, utility functions
3. The Social Choice Model [O 2.1–2.2; notes]
   - majority preference, core/Condorcet winner, Pareto dominance, Pareto optimality, aggregation rules, voting paradoxes, sincere agendas, May's theorem, Arrow's Theorem
4. Analysis of the Real Line and the Unidimensional Spatial Model [SB 2–3, 5, A2; O 4.6; notes]
   - monotonicity, concavity and quasi-concavity, continuous/differentiable functions, maximization, single-peakedness, median voter theorem
5. Mathematics of Multiple Dimensions [SB 10, 13–15, 21; notes]

- vector addition, scalar multiplication, lines and hyperplanes, convex sets, simplexes, multivariate functions and level sets, concavity and quasi-concavity, dot products and orthogonality, Euclidean norm, open and closed sets, gradients, maximization

6. The Multidimensional Spatial Model [O 1.3–1.4, 2.3, 4.7; notes]

   - convex and continuous preferences, indifference curves, utility representations, Plott's theorem, McKelvey's theorem and the top cycle set, uncovered set

7. Optimization and Pareto Optimality [SB 3, 17, 21; notes]

   - first and second order necessary conditions and sufficient conditions for local maximizers, global maximizers, cost-benefit analysis, profit maximizing firms, the envelope theorem, comparative statics