Derivation of p_4 in Appendix A (p.259) of

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The probability p_4 that state 1 will choose to fight at node 4 is

$$\begin{aligned} p_4 &= \Pr[U_1^*(W2) > U_1^*(C1)] && (1)\\ &= \Pr[U_1(W2) + \epsilon_{w2} > U_1(C1) + \epsilon_{c1}] && (2)\\ &= \Pr[\epsilon_{c1} < U_1(W2) - U_1(C1) + \epsilon_{w2}] && (3) \end{aligned}$$

Let $f(\epsilon_{w2}, \epsilon_{c1})$ and $F(\epsilon_{w2}, \epsilon_{c1})$ be the joint pdf and cdf of $(\epsilon_{w2}, \epsilon_{c1})$ and let $F_{w2}(\epsilon_{w2}, \epsilon_{c1}) = dF(\epsilon_{w2}, \epsilon_{c1})/d\epsilon_{w2}$. Then

$$\begin{aligned} p_4 &= \int_{-\infty}^{\infty}\int_{-\infty}^{U_1(W2)-U_1(C1)+\epsilon_{w2}} f(\epsilon_{w2}, \epsilon_{c1}) d\epsilon_{c1} d\epsilon_{w2} && (4)\\ &= \int_{-\infty}^{\infty} F_{w2}[\epsilon_{w2}, U_1(W2) - U_1(C1) + \epsilon_{w2}] d\epsilon_{w2} && (5) \end{aligned}$$

This holds for any joint density $f(\epsilon_{w2}, \epsilon_{c1})$. Here, we now assume ϵ is distributed iid according to a type I extreme value density: $f(\epsilon) = \lambda \exp\left(-\lambda\epsilon - e^{-\lambda\epsilon}\right)$ and $F(\epsilon) = \exp\left(-e^{-\lambda\epsilon}\right)$, with $E[\epsilon] = \gamma/\lambda$ and $V[\epsilon] = \pi^2/(6\lambda^2)$, where γ is Euler's constant, approximately equal to 0.577.

With that assumption, Equation 5 becomes

$$\begin{aligned} p_4 &= \int_{-\infty}^{\infty} f(\epsilon_{w2}) F[U_1(W2) - U_1(C1) + \epsilon_{w2}] d\epsilon_{w2} && (6)\\ &= \int_{-\infty}^{\infty} \lambda \exp\left\{-\lambda\epsilon_{w2} - e^{-\lambda\epsilon_{w2}}\right\} \exp\left\{-e^{-\lambda[U_1(W2)-U_1(C1)+\epsilon_{w2}]}\right\} d\epsilon_{w2} && (7)\\ &= \int_{-\infty}^{\infty} \lambda \exp\left\{-\lambda\epsilon_{w2} - e^{-\lambda\epsilon_{w2}}\left[1 + e^{-\lambda[U_1(W2)-U_1(C1)]}\right]\right\} d\epsilon_{w2} && (8)\\ &= \int_{-\infty}^{\infty} \lambda \exp\left\{-\lambda\epsilon_{w2} - e^{-\lambda\epsilon_{w2}}\left[\frac{e^{\lambda U_1(W2)} + e^{\lambda U_1(C1)}}{e^{\lambda U_1(W2)}}\right]\right\} d\epsilon_{w2} && (9) \end{aligned}$$

Now, let $z = \frac{1}{\lambda} \ln\left[\frac{e^{\lambda U_1(W2)} + e^{\lambda U_1(C1)}}{e^{\lambda U_1(W2)}}\right]$. Then

$$p_4 = \int_{-\infty}^{\infty} \lambda \exp\left\{-\lambda\epsilon_{w2} - e^{-\lambda\epsilon_{w2}} e^{\lambda z}\right\} d\epsilon_{w2} \qquad (10)$$

$$= \int_{-\infty}^{\infty} \lambda \exp\left\{-\lambda\epsilon_{w2} - e^{-\lambda(\epsilon_{w2}-z)}\right\} d\epsilon_{w2} \quad (11)$$

$$= \int_{-\infty}^{\infty} \left(e^{-\lambda z} e^{\lambda z}\right) \lambda \exp\left\{-\lambda\epsilon_{w2} - e^{-\lambda(\epsilon_{w2}-z)}\right\} d\epsilon_{w2} \quad (12)$$

$$= e^{-\lambda z} \int_{-\infty}^{\infty} \lambda \exp\left\{-\lambda(\epsilon_{w2} - z) - e^{-\lambda(\epsilon_{w2}-z)}\right\} d\epsilon_{w2} \quad (13)$$

Letting $\epsilon^* = \epsilon_{w2} - z$,

$$p_4 = e^{-\lambda z} \int_{-\infty}^{\infty} \lambda \exp\left\{-\lambda\epsilon^* - e^{-\lambda\epsilon^*}\right\} d\epsilon^* \quad (14)$$

$$= e^{-\lambda z} \int_{-\infty}^{\infty} f(\epsilon^*) d\epsilon^* \quad (15)$$

$$= e^{-\lambda z} \quad (16)$$

$$= \frac{e^{\lambda U_1(W2)}}{e^{\lambda U_1(W2)} + e^{\lambda U_1(C1)}} \quad (17)$$