A Statistical Model of the Divide-the-Dollar Game

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Abstract

In this paper we derive a statistical estimator for the popular divide-the-dollar bargaining game. Using monte carlo data generated by a strategic bargaining process, we show that the estimator correctly recovers the relationship between dependent variables, such as the proposed division and bargaining failure, relative to substantive variables that comprise players' utilities. We then use the model to analyze bargaining data in a number of contexts. The current example examines the effects of demographics on bargaining behavior in experiments conducted on U.S. and Russian participants. Examples concerning international conflicts and international trade are currently being added.

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Contents

1 Introduction

Over the last twenty years, formal models and quantitative analyses have come a long way toward explaining how strategic actors bargain in a variety of political settings. (Banks 1990, Bennett 1996, Baron 1989, Fearon 1995, Huth and Allee 2002, Laver and Schofield 1990, London 2002, Morrow 1989, Powell 1987, Powell 1996). Game theory, in particular, has proved to be a useful tool for understanding the basic logic of bargaining in the face of conflicting interests. If the frequency with which a single idea or framework is cited or used in the literature is a measure of its importance, then the importance of bargaining models cannot be denied. For example, bargaining models have been applied by political scientists to analyze everything from the effects of open and closed rules on the distributive politics of legislative appropriation to the study of war initiation and termination (Baron and Ferejohn 1989, Fearon 1995, Mansfield, Milner and Rosendorff 2000). In fact, the theoretical and empirical study of bargaining is one of the few places where the different subfields of political science can identify one phenomenon that all can agree is important and worthy of attention.

Results of numerous theoretical studies of the bargaining problem have pointed to the importance of asymmetric information and the "reservation values" of players in distributional politics. Yet, as is usually the case when scholars try to find empirical support for their more abstract theoretical claims, the vagueness of these concepts, which are so important in theory, are difficult to operationalize and even more difficult to test. Additionally, it is often the case that we would like to know the effects of particular substantive variables, like a congressman's district demographics or whether a state possesses nuclear weapons, on the bargaining process. The theoretical models tell us something about the path by which these variables may influence outcomes. However, there has not yet been an appropriate statistical estimator for examining these effects.

As an alternative to the "theory down" approach to understanding bargaining, an increasingly sophisticated body of work has looked directly at the empirical relationship between substantive variables of interest, such as regime type, economic interdependence, institutional rules, legislative composition, and bargaining outcomes (Bennett 1996, McCarty and Poole 1995, Milner 1997, Werner 1999). However, lacking an explicit model of the process that generates the empirical data, and leaving out the choice-based path by which these variables influence decisions, it is often the case that selection and omitted variable bias plague the analysis (King, Keohane and Verba 1994).

In particular, Signorino demonstrates that traditional linear and categorical estimation techniques can lead to faulty inferences when the strategic data generating process is ignored during estimation (Signorino 1999, Signorino 2002). It is unclear how reliable the inferences from these empirical models are given these findings.

What is called for in the bargaining literature is an integration of formal theoretical models and statistical methods. In particular, analysts need a statistical tool that permits them to make theoretically consistent inferences about the relationship between substantive variables, the bargain struck, and the probability of bargaining failure.[1] In other words, we need an estimator that explicitly models the strategic bargaining process.

To move in this direction, we derive a statistical model for divide-the-dollar bargaining games. After verifying that our theoretical model satisfies the minimum criteria for structural estimation, we derive an econometric estimator for the bargaining model. This model explicitly captures the relationship between the variables that affect the players's utilities and the outcomes of the bargaining in a strategic setting. Next, we conduct a Monte Carlo experiment by generating strategic bargaining data and then estimating the relationships between the regressors and the dependent variable(s). We estimate not only the statistical bargaining model, but also traditional OLS and censored variable models. While the statistical bargaining model correctly recovers the parameters on average, the traditional models yield inferences that are not only incorrect, but also appear to be statistically significant.

Finally, we apply our estimator to bargaining data in a number of contexts. Currently, the paper includes an example examining the effects of demographics on bargaining behavior in experiments conducted on U.S. and Russian participants. Results using the statistical bargaining model differ in a number of important ways from those found (using OLS) by the authors of the original study. Further examples, concerning international conflicts and international trade, are currently being added.

[1] For examples of other work that has begun to deal with this problem in alternative settings see Wolpin (1987), Merlo and Wilson (1995), McKelvey and Palfrey (1996), Merlo (1997), and Merlo and Wilson (1998).

2 Theoretical Model

Political decision-making is often fundamentally a bargaining problem. That is, the essence of strategic decision-making between states, parties or leaders is largely about who gets what and when. One of the simplest, and most popular, bargaining models, is the divide-the-dollar game. In this section, we describe this bargaining model and then define a statistical model consistent with that data generating process. The estimator derived in the following section will be a straightforward structural implementation of the statistical model derived here.

2.1 The Divide-The-Dollar Game

Consider the usual bargaining arrangement, depicted in Figure 1, where two players must divide a contested prize, which we denote as $\mathcal{Q}$. Lset the prize $\mathcal{Q} \subset \mathbb{R}_+$ be compact and convex, with lower and upper bounds $\underline{\mathcal{Q}}$ and $\overline{\mathcal{Q}}$, respectively. Without loss of generality, rescale the bounds of the prize $[\underline{\mathcal{Q}}, \overline{\mathcal{Q}}] = [0, \mathcal{Q}^*]$.

Player 1 first offers some division of the prize $(Q^* - y, y)$, where player 1's allocation is $Q^* - y$ and player 2's is y. Player 2 then decides whether to *accept* or *reject* player 1's offer. If player 2 accepts, they divide the prize according to player 1's offer. If player 2 rejects the offer, they receive some reservation amount, which may differ between the players.

Consistent with previous formal models of bargaining, we assume each player's utility for bargaining failure has two components: one that is public knowledge and one that is private. Therefore, we denote player 1's reservation value as $R_1 + \epsilon_1$ and player 2's as $R_2 + \epsilon_2$, where R_i is player i's publicly observable reservation value and ϵ_i is private information.

Let nature draw the type ϵ_i of each player i from a well defined probability distribution. We assume that the players have well defined prior beliefs about the distribution of these types. In particular players assume each type is drawn i.i.d. from the cumulative distribution function $F_i(\cdot)$, with a corresponding density $f_i(\cdot)$, mean $\mu_i = 0$, and variance $\sigma_i^2 \ll \infty$.

Each player's strategy can be characterized by a mapping from types into actions: $\sigma_i : \epsilon_i \rightarrow A^i$, $i = \{1, 2\}$, where A^i defines the action set for player i. Since player 1 is making the ultimatum offer, $A^1 = \{y : y \in [0, \mathcal{Q}^*]\}$. Player 2 is then left to accept or reject the offer, so $A^2 = \{accept, reject\}$. As is standard, we assume that player 2 accepts an offer when she is indifferent between accepting

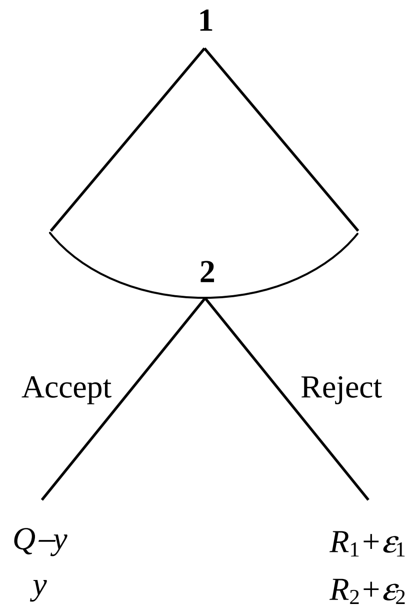

Figure 1: Divide the Dollar Game

and rejecting.

Additionally, for the divide-the-dollar game, we assume that both players utilities are strictly increasing and continuous in their amount of the disputed good, and by the random utility structure, the public and private components of the players' utilities are additively separable.[2] In particular, we assume

$$
\begin{aligned}
u_1(y, accept) &= \mathcal{Q}^* - y \\
u_2(y, accept) &= y \\
u_1(y, reject) &= R_1 + \epsilon_1 \\
u_2(y, reject) &= R_2 + \epsilon_2
\end{aligned}
$$

In equilibrium, the statistical divide-the-dollar game has player 1 making an offer that balances the marginal utility of increasing the probability that an offer is accepted and the marginal utility of a larger amount of y. Player 2, knowing her own type, chooses the alternative that maximizes her utility.

[2]Other ultimatum games may have differently defined preferences.

2.2 Existence and Uniqueness of Equilibrium

As in any game where players play strategies that map a random variable to their action space, such as a traditional Bayesian game or random utility model, the player's actions appear probabilistic rather than deterministic, at least from the perspective of the empirical analyst. Noting that a Nash equilibrium of a statistical divide-the-dollar bargaining game, where each player knows the other has random utilities, is equivalent to a Bayesian Nash equilibrium of an underlying Bayesian game, where the types of the players are private information, we can use well-known game theoretic tools to begin to specify both our theoretical predictions and our empirical estimator.[3] If the Bayesian Nash equilibrium (BNE) of this underlying game can be shown to be unique, then we can solve for the equilibrium strategies and characterize an equilibrium probability distribution over observable outcomes. It is this characteristic of the divide-the-dollar model that will allow for its structural estimation.

Even though the statistical divide-the-dollar model is equivalent to a Bayesian games of two-sided incomplete information, the existence and uniqueness of the equilibrium needs to be demonstrated. Recent research has shown that traditional existence results do not guarantee the existence of an equilibrium in Bayesian games with unbounded type spaces and continuous actions sets (Meirowitz 2002). It is, however, easy to show that there always exists a mutual best response that is sequentially rational, given beliefs. Moreover, this mutual best response is the unique Bayesian Nash equilibrium to the game.[4]

Proposition 1. *Let* $\xi = \frac{F_{\epsilon_2}(y^*-R_2)}{f_{\epsilon_2}(y^*-R_2)}$. *For any prior distribution on* ϵ_1, ϵ_2, *with* $\frac{\partial \xi}{\partial y^*} \geq 0$, *there exists a unique Bayesian Nash equilibrium to the statistical divide-the-dollar game.*

We leave the proof for the appendix, and only sketch each player's optimization problem. Assuming player 1 has made an offer y, player 2 chooses between that offer and her reservation

[3]For examples of work on statistical game theory see McKelvey and Palfrey (1996), Beja (1992), van Damme (1991).

[4]If we do not assume that indifferent types of player 2 accept, then there exists a generically unique Bayesian equilibrium to the game, i.e. the that conditions necessary for multiple equilibrium are of zero measure, and the statistical model remains unchanged.

value $R_2 + \epsilon_2$. In any equilibrium, player 2 plays the cutpoint strategy:

$$s_2(y, \epsilon_2) = \begin{cases} accept & \text{if } y \geq R_2 + \epsilon_2 \\ reject & \text{if } y < R_2 + \epsilon_2. \end{cases}$$

Player 1 does not observe ϵ_2, but must assess the probability that player 2 will accept or reject his offer:

$$\begin{aligned} \Pr(accept|y) &= \Pr(y \geq R_2 + \epsilon_2) \\ &= \Pr(\epsilon_2 \leq y - R_2) \\ &\equiv F_{\epsilon_2}(y - R_2) \end{aligned} \tag{1}$$

Now consider the optimization problem for player 1, given player 2's strategy. His expected utility is

$$Eu_1(y|\mathcal{Q}^*) = F_{\epsilon_2}(y - R_2) \cdot (\mathcal{Q}^* - y) + (1 - F_{\epsilon_2}(y - R_2)) \cdot (R_1 + \epsilon_1),$$

With no constraints, his optimal offer is the y^* that solves

$$y^* = \mathcal{Q}^* - R_1 - \epsilon_1 - \frac{F_{\epsilon_2}(y^* - R_2)}{f_{\epsilon_2}(y^* - R_2)}, \tag{2}$$

However, $0 \leq y^* \leq \mathcal{Q}^*$. Therefore, in any BNE, player 1 plays:

$$s_1(\epsilon_1|R_1, R_2, \mathcal{Q}^*, F_{\epsilon_2}(\,\cdot\,)\,) = \begin{cases} \mathcal{Q}^*, & \epsilon_1 \leq -\xi - R_1 \\ y^*, & -\xi - R_1 < \epsilon_1 < \mathcal{Q}^* - \xi - R_1 \\ 0, & \epsilon_1 \geq \mathcal{Q}^* - \xi - R_1, \end{cases}$$

3 Empirical Model

Having shown that the statistical divide-the-dollar game has a unique equilibrium, we can now use this theoretical model to construct an econometric model for examining the effects of substantive variables on the bargaining process. To use the statistical bargaining model in empirical analysis, we first need to specify a distribution for the ϵ_i, and then specify the appropriate likelihood given the dependent variable(s).

For the moment, let us assume we have data on both player 1's and player 2's actions — i.e., assume we can measure and code y and $\mathcal{Q}^*$ for each observation, as well as whether player 2 accepted or rejected the offer. Let the public portion of the players' reservation values be $R_1 = X\beta$,and $R_2 = Z\gamma$, where X and Z are sets of substantive regressors. Our interest is in estimating the effects of X and Z on y and player 2's decision.

Because the outcome of the bargaining model consists of two dependent variables — 1's offer and 2's decision — our probability model is a joint density over those random variables. For our estimator, we will assume that the types of players 1 and 2 are drawn i.i.d. from a logistic distribution. The i.i.d. assumption greatly simplifies matters by reducing the joint density to the product of two univariate densities.

Consider player 2's decision. The probability that player 2 accepts the offer y is just the logit probability

$$\Pr(accept|y) = \Lambda(y - Z\gamma)$$

For player 1, the distribution of y^*, is more complicated. A logistic distribution of types implies that

$$y^* = \mathcal{Q}^* - X\beta - \epsilon_1 - \frac{\Lambda(y^* - Z\gamma)}{\lambda(y^* - Z\gamma)}, \tag{3}$$

where $\Lambda(\cdot)$ is the logit c.d.f. and $\lambda(\cdot)$ is the logit p.d.f.

Equation 3 satisfies the condition of Proposition 1 and simplifies to

$$y^* = \mathcal{Q}^* - X\beta - \epsilon_1 - (1 + e^{(y^* - Z\gamma)}) \tag{4}$$

Solving for y^* gives

$$y^* = \mathcal{Q}^* - X\beta - \epsilon_1 - 1 - \mathcal{W}(e^{(\mathcal{Q}^* - X\beta - Z\gamma - \epsilon_1 - 1)}) \tag{5}$$

where $\mathcal{W}$ is Lambert's $\mathcal{W}$, which solves transcendental functions of the form $z = we^w$ for w.[5]

Note that the function $h(\epsilon_1) : \epsilon_1 \to y^*$ is a monotonically decreasing function of ϵ_1. Moreover, $h(\epsilon_1)$ is one-to-one, on-to, and invertible. As a result, the density $f_{y^*}(y^*|X\beta, Z\gamma, \mathcal{Q}^*)$ can be derived

[5] For a discussion of Lambert's W and the algorithm to solve the transcendental equation see Corless, Gonnet, Hare, Jeffrey and Knuth (1996), Fitsch, Shafer and Crowley (1973), Barray and Culligan-Hensley (1995), and Valluri, Jeffrey and Corless (2000).

using the method of transformations,[6] giving

$$f_{y^*}(y^*) = \frac{e^{(\mathcal{Q}^*-1-X\beta-e^{(y^*-Z\gamma)}-y^*)} \cdot (1+e^{(y^*-Z\gamma)})}{(1+e^{(\mathcal{Q}^*-1-X\beta-e^{(y^*-Z\gamma)}-y^*)})^2}, \tag{6}$$

and

$$F_{y^*}(y^*) = \frac{1}{(1+e^{(\mathcal{Q}^*-1-(e^{(y^*-Z\gamma)}+X\beta+y^*)})}. \tag{7}$$

The constraint on the action space of player 1, however, implies that the observed y^* is censored both from above and below. Let y be the observed offer. Define a set of dummy variables δ_k $k \in \{0, y, 1\}$ such that $\delta_0 = 1$ if $y = 0$, $\delta_y = 1$ if $0 < y < \mathcal{Q}^*$, and $\delta_1 = 1$ if $y = \mathcal{Q}^*$. Finally, code player 2's acceptance as $\delta_{accept} = 1$ if she accepted the offer and $\delta_{accept} = 0$ if she rejected the offer.

Assuming we have data on both player 1's and player 2's actions (i.e., y and δ_{accept}), then the likelihood would be

$$\begin{aligned} L &= \prod_{i=1}^{n} \Pr(y^* < 0)^{\delta_0} \cdot \Pr(y^* = y)^{\delta_y} \cdot (1 - \Pr(y^* < \mathcal{Q}^*))^{\delta_1} \times \\ &\quad \Pr(accept)^{\delta_{accept}} \cdot \Pr(reject)^{1-\delta_{accept}} \end{aligned} \tag{8}$$

where the observation index has been omitted. Deriving the log-likelihood from this is straightforward. We then have a log-likelihood function for our data in terms of distributions already derived, which are functions of our regressors, and which explicitly models the divide-the-dollar game. Estimates of β and γ may be obtained using maximum likelihood estimation.

Finally, suppose we had data only on player 2's or player 1's actions, but not both. For a given dependent variable (i.e., player 1 or player 2's action), we would then simply use the appropriate density (already derived) as the basis of our log-likelihood equation.

4 Monte Carlo Analysis

The maximum likelihood estimator for the logit bargaining model should be consistent and asymptotically efficient. In this section, we demonstrate that (on average) the bargaining estimator correctly recovers the parameters of the bargaining model as a data generating process. Because

[6] See (Wackerly, Mendenhal and Scheaffer 1996, p. 265).

the norm in analyzing bargaining data has been to use traditional techniques, such as logit, OLS, and censored (Tobit) regression, we also examine the inferences that produced using those models.

For the Monte Carlo analysis the divide-the-dollar game in Figure 1 was used as the data generating process. For a given observation, the value of the disputed good, $\mathcal{Q}^*$, was drawn from a uniform distribution on $[0, 10]$. The public reservation values consisted of a single variable for each player, X for player 1 and Z for player 2, with β and γ set to one. X and Z were drawn from i.i.d. uniform distributions on the interval $[0, \text{Max R}]$. Lastly, the private information for players 1 and 2, ϵ_1 and ϵ_2, respectively, were drawn from i.i.d. logistic distributions. Based on this, player 1 determines his optimal offer y using Equation 5 and the constraint $0 \leq y \leq \mathcal{Q}^*$. Given that, player 2 makes a decision by comparing y to her reservation $Z\gamma + \epsilon_2$. The data for a given observation then consists of $\mathcal{Q}^*$, y, δ_{accept}, X, and Z.

For a given monte carlo iteration, N=1000 observations were generated. The logit bargaining model was estimated, along with a Normal model and a censored-Normal model, where the censored model was essentially a Tobit model censored below at 0 and above at $\mathcal{Q}^*$. A traditional logit model was also estimated for player 2's decision by regressing δ_{accept} on y and Z. After estimating each model, the parameter estimates were saved. This was repeated for 2000 iterations of the Monte Carlo to form a density of the estimates. Finally, we noticed that the larger Max R, the greater the number of the observations that were censored by the bargaining constraint. Therefore, to assess the effect of strategic censoring on the inferences from the traditional models, separate Monte Carlo experiments were conducted for each value of Max R $\in \{2, 4, 6, 8, 10\}$.

Tables 1 displays the results of the logit bargaining model ("Ultimatum") estimated using the full set of data, as well as the results of a simple logit model estimated using only player 2's data. The table is divided into five sections, one for each value of Max R. For each value of Max R, the number of censored observations is displayed (out of 1000 observations for each sample). Each cell in the table reports the mean (top) and the standard deviation (bottom) of the distribution of the estimates for each group of 2000 iterations.

As is clear in Table 1, $\hat{\beta}_u$ and $\hat{\gamma}_u$ are recovered by the logit bargaining model on average, and the standard deviations of the estimates are small. Moreover, the estimates continue to be unbiased and have low variance, even as the censoring exceeds half of the observations (i.e., as Max R increases to 8 or 10).

Max R	Number Censored	Ultimatum $\hat{\beta}_u$	$\hat{\gamma}_u$	Logit $\hat{\omega}_l$	$\hat{\gamma}_l$
2	253	.99	.99	1.00	−1.00
		.06	.02	.07	.10
4	331	1.00	1.00	1.00	−1.00
		.03	.02	.07	.07
6	421	1.00	1.00	1.00	−1.01
		.02	.02	.08	.07
8	511	.99	.99	1.00	−1.01
		.02	.02	.08	.07
10	593	1.00	1.00	1.01	−1.01
		.02	.02	.09	.08

For each Monte Carlo, N=1000 observations were generated. For the parameter estimates, the top value displays the mean and the bottom displays the standard deviation.

Table 1: Monte Carlo Results for Bargaining Estimators

What is also clear is that the effects of substantive variables on the success or failure of bargaining, i.e. the probability player 2 accepts or rejects an offer, can be estimated with a properly specified logit. In fact, the logit's estimates are consistently close to the true values. The negative sign on $\hat{\gamma}_l$ is correct and simply a result of specifying the regression equation as $y\omega_l + Z\gamma_l$. As before, increased censoring due to Max R does not affect the consistency of the estimates. Not surprisingly, the effect of Z is estimated more precisely when all of the data is used with the "Ultimatum" model, rather than when limited only to data on player 2's decision.

Table 2 displays the results of using the Tobit model and the uncensored Normal model to analyze the relationship between the regressors and the size of the offer y. When it comes to estimating how substantive variables affect strategic offers in bargaining, these traditional techniques produce estimates far from the true values. First consider the uncensored Normal model. A number of scholars, looking at the continuous nature of the offer variable, have chosen to use OLS and FGLS to analyze such data.[7] For the uncensored Normal model, a constant was estimated, along with β_n and γ_n (the effects of X and Z, respectively). The n subscript simply denotes that these estimates are from the Normal model. Since this model does not allow for censoring, it seemed reasonable to include the size of the prize $\mathcal{Q}^*$ as a regressor. The parameter associated with it is θ_n. Also shown is the estimate of the variance σ_n^2.

The uncensored Normal results highlight two potential problems with using OLS on bargaining data. First, the estimates of the parameters change significantly as the number of censored observations increases. In particular, $\hat{\gamma}_n$ ranges from .09 to .47. Second, every estimate looks as though it is a very good estimate, i.e., the standard deviation of the density is small.

It is already well known, however, that censoring can significantly affect estimates in a traditional OLS model. Moreover, a sophisticated empiricist would likely conclude that the strategic process would, in some way, censor the observed data. Therefore, she may account for this in her estimation, and analyze the data using a Tobit model, which is censored above and below.

The Tobit results are also found in Table 2. Again, the estimates from the censored regression model are far from the set values of β and γ. And, again, the standard deviations of the estimates are small, leading one to believe the estimates are, in some sense, good. Perhaps most interesting,

[7] See Bothelho, Harrison, Hirsch and Ruström (2002) and Henrich, Boyd, Bowles, Camerer, Fehr, Gintis and McElreath (2001).

Max	**N**umber	**T**obit				**N**ormal				
R	**C**ensored	$Const_t$	$\hat{\beta}_t$	$\hat{\gamma}_t$	$\hat{\sigma}_t^2$	$Const_n$	$\hat{\beta}_n$	$\hat{\gamma}_n$	$\hat{\theta}_n$	$\hat{\sigma}_n^2$
2	253	.92	−.35	.56	1.69	−.42	−.25	.47	.29	.29
		.11	.07	.07	.08	.05	.03	.03	.01	.02
4	331	1.13	−.48	.43	3.23	−.49	−.31	.35	.36	.58
		.15	.05	.05	.17	.07	.02	.02	.01	.03
6	421	1.52	−.59	.31	4.93	−.22	−.34	.23	.38	.91
		.18	.05	.04	.27	.09	.02	.02	.01	.04
8	511	1.92	−.69	.22	6.32	.19	−.34	.15	.35	1.17
		0.21	.04	.04	.39	.09	.02	.02	.01	.06
10	593	2.29	−0.78	.16	7.34	.53	−.30	.09	.31	1.33
		0.23	.04	.04	.51	.09	.01	.01	.01	.07

Table 2: Tobit and Uncensored Normal Regression Results for Offers

even when we account for the censoring in the model, strategic censoring still affects the estimates as the number of censored cases increase. The estimates for $\hat{\beta}_t$ change from $-.35$ to $-.78$, an increase in magnitude of 122 percent. Similarly, the estimates of $\hat{\gamma}_t$ change from .56 to .16, a decrease in magnitude of 71 percent. Moreover, as the proportion censored observations increase in the sample, the Tobit model looks as though it is making better and better estimates.

From these simulations, we see that the appropriate statistical method can depend on the question one wishes to answer. For example using a logit model on bargaining data is acceptable when one wishes to investigate how different variables affect bargaining failure, given data on the offers. However, if we are interested in saying something about how substantive variables affect the kind of bargain that is struck, then traditional techniques are inappropriate. The Monte Carlo experiments not only show that these techniques produce incorrect estimates of β and γ, but also that the estimates change as a function of the distribution of the reservation values of the players. Since we cannot know, *ex ante*, the empirical distribution of the reservation payoffs, independent of the estimation of $\hat{\beta}$ and $\hat{\gamma}$, OLS and Tobit results for real world data should be viewed with extreme skepticism. On the other hand, the logit bargaining model is shown to be unbiased even with largely censored samples.

5 Bargaining Behavior and Demographics

One thing that makes the study of bargaining special is the interdisciplinary interest in the process. Everything from bargaining between multinational cooperations and states over terms of foreign investment, to the resolution of territorial disputes, to anthropologist's interests in how social and personal characteristics affect the "rational" behavior of individuals, has been studied in the context of bargaining models.

At this point, however, we analyze a data set on experimental ultimatum games conducted by Bothelho et al. (2002). Their work and the experimental data they present provides an effect foil and allows us to point to the substantive differences between using our logit bargaining estimator versus OLS.

In the experiments that generated this data, the researchers sought to isolate the effects of

OfferP	β	S.E.	t
Russia	-.08914	1.6632	**-0.75**
SlavS	**3.13**	.7623	4.107
ParInc 50-100kS	**-1.1783**	.6187	**-1.905**
Years Worked	**-.0157**	.0740	**-0.212**
fbusS	3.2226	.5821	**5.536**
HHInc 50-100k	5.0542	.6355	**7.953**
HHInc 100k+	5.9341	.6737	**8.809**

Table 3: Bargaining Model

OfferP	β	S.E.	t
Russia	-7.5605	3.4552	**-2.01**
SlavS	**5.5880**	3.1346	1.78
ParInc 50-100kS	**-2.8885**	2.4920	**-1.16**
Years Worked	**-1.397**	.504275	**-2.77**
fbusS	-2.6599	2.390862	**-1.11**
HHInc 50-100k	-2.2123	1.8711	**-1.18**
HHInc 100k+	-2.4205	1.7601	**-1.38**

Table 4: OLS Model

demographic variables on the bargaining process. They start from the widely known experimental observation that, when people "play" the ultimatum game, they do not play the (complete information) game's Nash equilibrium. In particular, divide-the-dollar experiments consistently show that proposers giving receivers larger shares of the pie than is predicted by strict income maximizing behavior. In a number of previous studies, most noticeably Roth, Prasnikar, Okuno-Fujiwar and Zamir (1991) and Henrich et al. (2001), economists and anthropologists have joined together to conduct experiments in various countries. They found that, not only do people not play the Nash equilibrium, but there also appears to be systematic differences in the way people play the game across countries and cultures.

Bothelho et al. (2002) enter the debate by claiming that variance across countries does not necessarily imply a "cultural" or "national" effect. They correctly point out that these studies fail to control for the potential effects of demographic variables. In their paper, they report on two sets of experiments, one in the US (at the University of Southern California) and one in Russia (at the Moscow Institute of Electronic Technology). In these experiments, the authors collect information on the demographic characteristics of the players, and attempt to assess their influence on the bargaining process.

For our purposes, this data provides a perfect opportunity to use the logit bargaining model, to compare its results with other relevant research, and to compare some of the substantive implications of our approach with the results obtained by using traditional statistical analysis.

For illustrative purposes, we replicated the Bothelho et al. (2002) results for their logit regression on acceptance propensities, with no interaction terms, and their least squares regression on offers, also with no interaction terms. Noting that the comparative static on $X\beta$ in the logit bargaining model is linear, we might suspect that using least squares regression would produce the same results as the estimation of the bargaining model. It turns out, however that is not the case.

Consider the variables presented in Tables 3 and 4.[8] These are variables selected from the total set of demographic variables (24 plus a constant) used for each player in the analysis. Cells in bold highlight differences between the two models. What becomes clear is that there are stark substantive differences between the effects inferred from the least squares regression and those of the bargaining model.

First, in the least squares regression we see that the so called "country" or cultural effect is both substantively and statistically significant. The result is interpreted as demonstrating that Russian participants tend to make smaller offers than Americans. The estimate from the bargaining model finds there is no such effect. Similarly, we find no statistically significant effect years worked. It is also the case that the bargaining model finds demographic variables that are not significant in the least squares regression, significant. In fact, the variables: parents's income in the \$ 50-100k (ParInc 50-100kS), father's occupation is business (fbusS), household income \$ 50-100k (HHInc 50-100k), and household income over \$ 100k (HHInc 100k+) are all significant in the bargaining model, but not significant in Table 4. Finally, the effect of being of Slavic culture, while marginally significant and positive in the Bothelho et al. (2002) study, in the bargaining model estimation it has a strongly significant effect, *in the other direction.* That is, people of Slavic decent make smaller offers, not larger ones.

While not a full fledged analysis of this bargaining data, the results form this comparison point to some potentially interesting conclusions. We see that in the experimental data, the structural bargaining model finds substantively different effects of demographic characteristics than one finds using a least squares approach. Where intuitively we may believe that parents's income in the \$ 50-100k range (ParInc 50-100kS), father's occupation is business (fbusS), household incomes of \$ 50-100k (HHInc 50-100k), and household income over \$ 100k (HHInc 100k+) would influence player's

[8]When comparing model results, recall that in a least squares regression the $X\beta$s are additively related to the offer. In the logit bargaining model, the $X\beta$s are the reservation utilities, and therefore negatively related to the size of the offer. So, in table (3) negative β decrease the reservation value and increase the size of the offer. In the OLS, negative values represent a decrease in offer size.

reservation values,i.e., their marginal utility of the experiment, if we believe the OLS estimation, they do not. The logit bargaining model, however, finds that these variables have the same intuitive directional effect as OLS, but also finds them to be statistically significant.

We also see that it may not be true that if the comparative statics of a model are linear, that an OLS model will accurately infer the effects of substantive variables of interest. As an exercise, suppose that the bargaining model were simplified and assumed a uniform distribution of types. Then, suppose we find the equilibrium offer. It is easy to see that such an offer would be linear in $X\beta$. Given the linearity of the relationship, we may think it appropriate to use a classical linear regression model to analyze the effect of variables in X on the offers. We see here, however, that it is not the case.

6 Conclusion

In the final analysis, the investigation of the logit bargaining model points to a number of conclusions. First, traditional logit models will be appropriate to answer some questions, particularly about bargaining failure, if properly specified. Second, the results show that the approach of using logit or OLS to analyze monotonic comparative statics may be more problematic than previously thought. The logit bargaining model also shows that the structural estimation approach is not limited to games where players have finite action paces. In particular, there are number of games — such as the Romer-Rosenthal setter model and the Rubenstein bargaining model — that may be estimable in similar ways.

Finally, the broad interest in bargaining games leaves open a number of substantive areas where our estimator can be applied. As such, we are currently processing a number of data sets for analysis with the logit bargaining model. First, we are collecting data on the resolution of territorial disputes and intend to re-analyzes the Huth and Allee (2002) model. Given the Monte Carlo results, we expect to find significant differences between our results and theirs when it comes to the relationship between bargaining, democracy, and territorial disputes. Second, we are in the process of obtaining multinational corporation bargaining over ownership rights in developing countries. This data is part of a much larger project managed at Harvard Business School. This data is exciting because it has been used in many forums, both in political science and economics, and it has been the empirical standard for testing claims of multinational corporate bargaining models. Finally, in the last few years anthropologists and economists have come together to begin to sort out the effect of demographic and "cultural" variables on the bargaining behavior of individuals and what these differences may imply for rational models of human behavior. We look to analyze this data, collected from bargaining experiments conducted across the world, and to begin to test some anthropological claims about non-material influences on bargaining behavior. In fact, out model is particularly well suited to test these alternative theories of quasi-rational bargaining. Moreover, this particular project presents an opportunity for serious interdisciplinary work between experimental economists, anthropologists, and political methodologists.

APPENDIX

Proposition 1. *Let* $\xi = \frac{F_{\epsilon_2}(y^*-R_2)}{f_{\epsilon_2}(y^*-R_2)}$. *For any prior distribution on* ϵ_1, ϵ_2, *with* $\frac{\partial \xi}{\partial y^*} \geq 0$, *there exists a unique Bayesian Nash equilibrium to the statistical divide-the-dollar game.*

Proof. Define a best response such that $B_i(s_{-i}) = \{s_i \in S^i : u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \forall S' \in S^i | F_{-i}\}$, and note that by definition of a Bayesian Nash equilibrium, a BNE $\Leftrightarrow$ mutual best response, given beliefs. Assume each player has a well defined prior belief about the distribution of types. It is trivially true that player 2 always has a best response, given her finite strategy space. To prove the existence of a mutual best response, show that $B_1(s_2|F_{\epsilon_2}(\cdot))$ is not empty. This requires a demonstration the the Bayesian decision problem of player 1 has a solution. Note that $\mathcal{Q}^*$ is a compact, convex set and that player 1's best response is always to chose a y that maximizes his expected utility. Since $Eu_1(y, \mathcal{Q}^*)$ is continuous in y, there exists a maximum on $[0, \mathcal{Q}^*]$ and $B_1(s_2|F_2(\cdot))$ is not empty (Sundaram 1999). This proves existence.

To show uniqueness, note that player 2 has a strict best response to any offer y. In any equilibrium player 2 plays a cutpoint strategy:

$$s_2(y, \epsilon_2) = \begin{cases} \textit{accept} & \text{if } y \geq R_2 + \epsilon_2 \\ \textit{reject} & \text{if } y < R_2 + \epsilon_2. \end{cases}$$

Note also that the $\Pr(\text{accept}|y) = \Pr(y > R_2 + \epsilon_2) = \Pr(\epsilon_2 < y - R_2)$, and $\Pr(\epsilon_2 < y - R_2) \equiv F_{\epsilon_2}(y - R_2)$.

Now consider the optimization problem for player 1, given player 2's strategy. His expected utility is:

$$Eu_1(y, \mathcal{Q}^*) = F_{\epsilon_2}(y - R_2) \cdot (\mathcal{Q}^* - y) + (1 - F_{\epsilon_2}(y - R_2)) \cdot (R_1 + \epsilon_1),$$

and his optimal offer is the y^* that solves,

$$y^* = \mathcal{Q}^* - R_1 - \epsilon_1 - \frac{F_{\epsilon_2}(y^* - R_2)}{f_{\epsilon_2}(y^* - R_2)}, \tag{9}$$

Let $\xi = \frac{F_{\epsilon_2}(y^*-R_2)}{f_{\epsilon_2}(y^*-R_2)}$ and assume that $\frac{\partial \xi}{\partial y^*} \geq 0$,[9] then ξ is increasing in y^* and equation (1) is a special case of a class of functions such that

$$y^* = C - \frac{F_{\epsilon_2}(y^*)}{f_{\epsilon_2}(y^*)},$$

[9]This condition is more strict than necessary, but is sufficient to show uniqueness for the most common statistical distributions, such as the normal, logistic, and uniform.

which is equivalent to

$$C = y^* + \frac{F_{\epsilon_2}(y^*)}{f_{\epsilon_2}(y^*)}. \tag{10}$$

Note that for any C, the function has at most one solution such that $y^* + G(y^*) = C$ and, therefore, so does $y^* = C - G(y^*)$.

So, on the real line, the optimization problem for player 1 has a unique solution. If that solution is an element of the constraint set, it is the unique optimal offer. If the solution is outside the constraint set, it is easy to see that the closest endpoint (0 or 1) is player 1's unique optimal offer. Therefore, given the constraints, in any BNE player 1 plays:

$$s_1(\epsilon_1 | R_1, R_2, \mathcal{Q}^*, F_{\epsilon_2}(\,\cdot\,)\,) = \begin{cases} \mathcal{Q}^*, & \epsilon_1 \leq -\xi - R_1 \\ y^*, & -\xi - R_1 < \epsilon_1 < \mathcal{Q}^* - \xi - R_1 \\ 0, & \epsilon_1 \geq \mathcal{Q}^* - \xi - R_1, \end{cases}$$

where y^* is the optimal offer that satisfies equation (2).

Uniqueness of s_1 follows from the fact that there is no other offer strategy that does better, given F_{ϵ_2}, for player 1. For player 2, given any offer y^*, she also has a unique best response, so for each possible pairing of types (s_1^*, s_2^*) is unique. □

References

Banks, Jeffrey S. 1990. "Equilibrium Behavior in Crisis Bargaining Games." *American Journal of Political Science* 34(3):579–614.

Baron, David P. and John Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 33(4).

Baron, David P. 1989. "A Noncooperative Theory of Legislative Coalitions." *American Political Science Review* 33(4).

Barray, D. A. and P. J. Culligan-Hensley. 1995. "Real Values of the W-Function." *ACM Transactions on Mathematical Software* 21(2):161–171.

Beja, Avraham. 1992. "Imperfect Equilibrium." *Games and Economic Behavior* 4.

Bennett, D. Scott. 1996. "Security, Bargaining, and the End of Interstate Rivalry." *International Studies Quarterly* 40(2).

Bothelho, Anabela, Glenn W. Harrison, Marc A. Hirsch and Elisabeth E. Ruström. 2002. "Bargaining Behavior, Demographics and Nationality: What Can Experimental Evidence Show?" mimeo.

Corless, R. M., G. T. Gonnet, D.E. G. Hare, D.j. Jeffrey and D. E. Knuth. 1996. "On the Lambert W Function." *Advances in Computational Mathematics* 5:329–359.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379–414.

Fitsch, F. N., R.E. Shafer and W. P. Crowley. 1973. "Algorithm 443: Solution of the Transcendental Equation $we^w = x$ [C5]." *Communications of the AMC* 16(2):123–124.

Henrich, Joseph, Robert Boyd, Samuel Bowles, Clin Camerer, Ernst Fehr, Herbert Gintis and Richard McElreath. 2001. "In Search of Homo Economicus: Behavioral Experiments in 15 Small-Scale Societies." *American Economic Review* ?(?).

Huth, Paul K. and Todd L. Allee. 2002. *The Democratic Peace and Territorial Conflict in the Twentieth Century.* New York, NY: Cambridge University Press.

King, Gary, Robert Keohane and Sidney Verba. 1994. *Designing Social Inquiry: Scientific Inference in Qualitative Research.* Princeton, NJ: Princeton University Press.

Laver, Michael and Norman Schofield. 1990. *Multiparty Government.* Ann Arbor, MI: University of Michigan Press.

London, Tamar R. 2002. Leaders, Legislatures, and International Negotiations: A Two-Level Game with Different Domestic Conditions. Phd thesis University of Rochester.

Mansfield, Edward D., Helen V. Milner and B. Peter Rosendorff. 2000. "Tree to Trade: Democracies, Autocracies, and International Trade." *American Political Science Review* 94:305–321.

McCarty, Nolan and Keith Poole. 1995. "Veto Power and Legislation: An Empirical Analysis of Executive and Legislative Bargaining from 1961 to 1986." *Journal of Law, Economics and Organization* 11:282–312.

McKelvey, Richard D. and Thomas R. Palfrey. 1996. "A Statistical Theory of Equilibrium in Games." *The Japanese Economic Review* 47(2):186–209.

Meirowitz, Adam. 2002. "On the existence of equilibria to Bayesian games with non-finite type and action spaces." working paper.

Merlo, Antonio and Charles Wilson. 1995. "A Stochastic Model of Sequential Bargaining with Complete Information." *Econometrica* 63(2):371–399.

Merlo, Antonio and Charles Wilson. 1998. "Efficient delays in a stochastic model of bargaining." *Economic Theory* 11:39–55.

Merlo, Antonio. 1997. "Bargaining over Governments in a Stochastic Enviroenment." *Journal of Political Economy* 105(1):101–131.

Milner, Helen V. 1997. *Interest, Institutions, and Information: Domestic Politics and International Relations.* Princeton, NJ: Princeton University Press.

Morrow, James D. 1989. "Capabilities, Uncertainty, and Resolve: A Limited Information Model of Crisis Bargaining." *American Journal of Political Science* 33(4):941–972.

Powell, Robert. 1987. "Crisis Bargaining, Escalation, and MAD." *American Political Science Review* 81(3):717–736.

Powell, Robert. 1996. "Bargaining in the Shadow of Power." *Games and Economic Behavior* 15:255–289.

Roth, Alvin E., Venesa Prasnikar, Msahiro Okuno-Fujiwar and Shmuel Zamir. 1991. "Bargaining and Market Behavior in Jerusalem, Ljubljana, Pittsburgh, and Tokyo: An Experimental Study." *American Economic Review* 81(5).

Signorino, Curits. 1999. "Strategic Interaction and the Statistical Analysis of International Conflict." *American Political Science Review* 93(2):279–297.

Signorino, Curtis. 2002. "Strategy and Selection in International Relations." *International Interactions* ?(1):?

Sundaram, Rangarajan K. 1999. *A First Course in Optimization.* New York, NY: Cambridge University Press.

Valluri, S. R., D. J. Jeffrey and R. M. Corless. 2000. "Some Applications of the Lambert W Function to Physics." *Canadian Journal of Physics* 78:823–831.

van Damme, Eric. 1991. *Stability and Perfection of Nash Equilibrium.* New York, NY: Springer-Verlag.

Wackerly, Dennis D., William Mendenhal and Richard L. Scheaffer. 1996. *Mathematical Statistics with Applications.* New York, NY: Duxbury.

Werner, Suzane. 1999. "Choosing Demands Strategically: The Distribution of Power, the Distribution of Benefits, and the Risk of Conflict." *Journal of Conflict Resolution* 43(6):705–726.

Wolpin, Kenneth I. 1987. "Estimating A Structural Search Model: The Transition from School to Work." *Econometrica* 55(4):801–817.