Distributive Politics and the Law of 1/n*

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Distributive politics models often predict that legislators will demand inefficiently large projects, with inefficiency increasing in the number of districts, and that this will translate into larger projects and higher spending. The relationship between efficiency and legislature size is often referred to as the "law of 1/n"(Weingast, Shepsle, and Johnsen 1981). *We demonstrate that the "law of 1/n" result with respect to project sizes and total spending is dependent on several factors, including the type of good being provided, the costs of raising revenue, and whether the local government has to share in the project's cost with the central government. In general, the "law of 1/n" need not hold for total government spending, and in fact a "reverse law of 1/n" often holds. In light of our theoretical findings, we reassess the empirical literature on this topic. The results have implications for a wide variety of applications in American and comparative politics.*

Beginning with Weingast (1979), a number of papers have analyzed various models of distributive politics in which: (1) individual legislators are assumed to care mainly or exclusively about the public projects that flow into their districts; (2) the tax system that finances public projects is fixed and "decoupled" from the projects themselves—e.g., taxes are proportional to income or population; and (3) the legislature is assumed to adopt a norm of universalism, in which all projects proposed are passed (perhaps in one or a few omnibus bills). These assumptions lead to the prediction that legislators will demand inefficiently large projects, with inefficiency increasing in the number of districts.

The relationship between efficiency and legislature size is often referred to as the "law of 1/n" (Weingast, Shepsle, and Johnsen 1981). The intuition for the "law of 1/n" is straightforward: as the number of districts increases, any one district absorbs a smaller share of a project's costs since projects are paid for out of a "common pool." Therefore, each district will want to consume more of that project. The "law of 1/n" is typically interpreted as linking the number of decision makers to the size of projects and to total spending.

The logic is powerful and can be used to understand many political phenomenon beyond the impact of legislature size on total spending, including why multiparty governments might be expected to spend more than single-party governments, and related, why proportional-representation electoral systems might lead to more spending than majoritarian systems. The idea of a "common pool" problem worsening as the number of decision makers increases permeates many questions in political science, so much so that the result is often stated as a given in the literature.[1]

The "law of 1/n" has influenced a variety of empirical and theoretical studies in American and comparative politics and has been applied to state and local politics, Congressional politics, cross-

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[1]The impact has been large by any measure. Weingast's seminal contribution has been cited well over 100 times, according to the Social Sciences Citation Index, mostly in political science journals, and the Weingast, Shepsle, and Johnsen paper has been cited well over 200 times. If other relevant papers are included, the number of citations could easily exceed 1,000.

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national studies, comparisons of electoral systems, and comparisons of legislative organization.[2] Generally, the "law of 1/n" is thought to be reasonably well-supported empirically.[3] However, some scholars have raised questions about its robustness.[4]

This paper shows that questions about the robustness of the "law of 1/n" are justified. Building on existing theory, we show that the link between legislature size and spending decisions is not as clear-cut as the typical characterization would suggest.[5] Instead, we find that the impact of changing the size of the legislature on project sizes and total spending—what researchers typically observe—depends on five factors: the degree of "publicness" and congestion in the goods being distributed, the curvature of the benefit function, the degree of subsidy from the central government, deadweight costs of taxation, and the size of the legislature before the change. In some cases, a "reverse law of 1/n" can hold, in which total spending and/or project sizes are declining in the number of districts.

How can a "reverse law of 1/n" hold? A few examples will provide some intuition. Suppose a state has a small legislature, and one of its metropolitan areas comprises a single district. The representative for this district might push strongly for, say, an expensive area-wide rail system. Now suppose instead that the legislature is several times larger, and the metropolitan area is carved into several districts. Then it can easily be the case that no single legislator from the metro area would be willing to push for the area-wide rail system. Instead, each might advocate subsidies for bus service in his or her district—and the sum of these smaller bus projects might be less than the total costs of the rail project.[6] The same logic applies to the tradeoffs between one large, full-service hospital with the latest medical equipment versus a collection of small health clinics, or a large dam and hydroelectric plant on a river versus a collection of smaller dams for irrigation only. Our model is highly stylized, so it does not exactly capture the richness of these examples. In particular, to simplify the analysis—and to tie it closely to previous work—we assume a continuum of project sizes rather than a discrete set of project "types." But the basic logic is similar.

The Standard Formulation

Previous work typically assumes the following.[7] Let the population of a nation be divided into n equally sized districts. Let X be a publicly provided project of "size" X. Let $C(X)$ be the total cost of the project. Let $B(X)$ be the total benefit received by the citizens in the district where the project is located. Assume $C' > 0$, $C'' > 0$, $B' > 0$ and $B'' \leq 0$. Assume full cost sharing of all projects, and assume taxes are equal for all citizens.

Since all citizens in each district are identical, each legislator's payoff should be equal to the payoff of her representative citizen. Alternatively, each legislator's payoff could be set equal to the total payoff of all citizens in her district. Let us take this second approach. Suppose district i receives a project of size X_i, and all other districts receive projects of

[2]For applications to state politics, see Gilligan and Matsusaka (1995), Gilligan and Matsusaka (2001), De Figueiredo (2003), Primo (2006), and Primo and Snyder (2007); for local politics see Bradbury and Crain (2001), Baqir (2002), Bradbury and Stephenson (2003b), and Pettersson-Lidbom (2004); for Congress see Weingast (1979), Inman and Fitts (1990), DelRossi and Inman (1999), and Bradbury and Stephenson (2003a); for studies of political fragmentation, the number of parties, and electoral system variation in a comparative context see Bradbury and Crain (2001), Scartascini and Crain (2001), Franzese (2002), Jordahl and Liang (2006), Lledo (2003), Mukherjee (2003), Pettersson-Lidbom (2004), Ricciuti (2004a), Ricciuti (2004b), and Bawn and Rosenbluth (2006); and for bicameralism see Bradbury and Crain (2001), Ricciuti (2004a), and Chen and Malhotra (2007).

[3]For example, Knight writes, "[E]mpirical evidence has generally supported this prediction of a positive relationship between government spending and legislature size" (2006, 235).

[4]For example, Bradbury and Crain write: "With the exception of a few studies little empirical evidence stands behind this conceptual proposition despite its broad acceptance as a stylized fact" (2001, 310). And Petterson-Lidbom writes: "Previous empirical studies have found a positive relationship between the size of legislature and the size of government. Those studies, however, do not adequately address the concerns of endogeneity" (2004, 1). One can find skepticism in other substantive areas, as well. For instance, Lledo (2003) finds that more fragmented state legislatures in Brazil are associated with smaller state governments. In addition, Jordahl and Liang (2006) find that the merging of municipalities in Sweden, creating several units within one government, does not produce a clear-cut effect on spending, as the "law of 1/n" would suggest. Also, see Baron (1991) for a critique of the theory.

[5]The logic easily extends to party fragmentation and other areas discussed above; we refer to legislatures and legislators in what follows for expositional convenience.

[6]Snyder and Ueda (2007) find empirical evidence consistent with this logic in their study of at-large and single-member districts in U.S. states.

[7]Weingast, Shepsle, and Johnsen (1981) consider a slightly more general version of this model with tax shares not necessarily equal. The case we consider here is the most typical implementation of their model. See the discussion in the section of this article entitled "Implications for Data Analysis" for further details.

size X. Then the payoff for the legislator representing district i is defined as

$$\prod_i (X_i, n) = B(X_i) - C(X_i)/n - C(X)(n-1)/n. \tag{1}$$

Assume each legislator chooses the size of her district's project, X_i, taking all other districts' projects as fixed. Differentiating with respect to X_i yields the first-order condition $B'(X_i^*) = C'(X_i^*)/n$. Differentiating this first-order condition totally with respect to n yields

$$\frac{\partial X_i^*}{\partial n} = \frac{B'(X_i^*)}{C''(X_i^*) - nB''(X_i^*)} > 0. \tag{2}$$

Thus, each district's project size in increasing in the number of districts. The number of projects is also increasing in the number of districts, since each district receives a project. So, total government spending, $nC(X_i^*)$, is clearly increasing in the number of districts.

This formulation leads instantly to any of three versions of the "law of 1/n." Weingast, Shepsle, and Johnsen proved the first version (incorrectly, as we will show), which holds that the "degree of inefficiency in project scale is an increasing function of the number of districts" (1981, 654). They go on to state that this also implies that project sizes are increasing in n, and assuming an increasing cost function, it immediately follows that total spending is increasing in n.

When Projects are Local Public Goods

The intuition above is incomplete, so we build on Weingast, Shepsle, and Johnsen's foundation. Let N be the total number of citizens in a country or state, let n be the number of districts (all equal in population), and let $m = N/n$ be the number of citizens in each district. As before, let X denote a publicly provided project of "size" X. Let $C(X)$ be the total cost of the project, in dollars. Let $b(X, m)$ be the benefit received by each citizen from the project, and let $B(X, m)$ be the aggregate benefits in the district.[8]

[8]Knight (2006, 235) solves a related model, but he also adopts the standard intuition: "As the number of districts increases, the common pool problem becomes more severe, increasing aggregate spending" (2006, 235).

If projects are pure private goods, then we can think of the "size" of a project simply as the number of "units" of project, these units being divided equally among the m citizens. In this case, $b(X, m) = \tilde{b}(X/m)$ and $B(X, m) = \tilde{b}(X/m)m$, with $\tilde{b}' > 0$ and $\tilde{b}'' \leq 0$.

On the other hand, if projects are pure (local) public goods, then each citizen receives the "full" benefits of a project regardless of m. In this case, $b(X, m) = \tilde{b}(X)$ and $B(X, m) = \tilde{b}(X)m$, with $\tilde{b}' > 0$ and $\tilde{b}'' \leq 0$.

Suppose the project size in district i is X_i, and suppose the project size in all other districts is X. Assume full cost sharing, and assume taxes are equal for all citizens. Then the payoff to a representative citizen in the district under consideration is

$$\hat{\pi}_i(X_i, m, n) = b(X_i, m) - C(X_i)/(nm) - C(X)(n-1)/(nm) \tag{3}$$

$$= b(X_i, m) - C(X_i)/N - C(X)(n-1)/N. \tag{4}$$

The denominator of the cost terms is $N = nm$, because the citizens in district i collectively pay $(1/n)$th of the cost of all projects, and this cost is split evenly across the m citizens in the district; so, each citizen pays $(1/N)$th of the total cost of the projects in all districts. Alternatively, the aggregate payoff to the citizens in district i is equal to

$$\widehat{\Pi}_i(X_i, m, n) = B(X_i, m) - C(X_i)/n - C(X)(n-1)/n. \tag{5}$$

In the pure local public goods case, the payoff to the representative citizen in district i and the aggregate payoff to all citizens in district i are, respectively,

$$\hat{\pi}_i(X_i, m, n) = \tilde{b}(X_i) - C(X_i)/N - C(X) \times (n-1)/N, \tag{6}$$

and

$$\widehat{\Pi}_i(X_i, m, n) = \tilde{b}(X_i)m - C(X_i)/n - C(X)(n-1)/n. \tag{7}$$

Either of these is a valid payoff function for the legislator representing district i.

What is the difference between equation (7) and the standard formulation, equation (1)? The cost terms are the same. But the benefit terms are not. *The standard formulation does not account for the fact that if the total population* N *is fixed, then it is impossible to* *change the number of districts,* n, *without also changing the population in each district,* m, *since* N = nm. *And, in the case of public goods, changing m automatically changes the aggregate net benefits to the citizens of district* i—*and, therefore, the payoff of legislator* i.[9]

Alternatively, consider the problem from the point of view of the representative citizen in district i. Dividing equation (1) by m gives the payoff of the representative citizen in the standard formulation,

$$\pi_i(X_i, m, n) = B(X_i)/m - C(X_i)/N - C(X)(n-1)/N. \quad (8)$$

Comparing this with equation (6) we see that the cost terms are again the same. But the benefit terms are not. Rather, equation (8) is much closer to the private goods case than the public goods case. Indeed, if B is linear with slope β then it corresponds exactly to a private goods case, with $b(X, m) = \beta X/m$.

Differentiating equation (6) with respect to X_i yields the first-order condition $\tilde{b}'(X_i^*) = C'(X_i^*)/N$. Since n does not appear explicitly in this equation $\frac{\partial X_i^*}{\partial n} = 0$. The size of the distributive good in each district, therefore, will be invariant with respect to n.

If legislators were required to absorb the full cost of their projects (i.e., there was no cost sharing), each legislator would maximize the net benefit of a representative citizen, or $\hat{\pi}_i(X_i, m, n) = \tilde{b}(X_i) - C(X_i)/m$.Taking first-order conditions and substituting N/n for m gives $\tilde{N}b'(X_i^{**})/n = C'(X_i^{**})$. From this, we see that the efficient level of the public good is declining in n, defining efficiency as the project that maximizes social welfare, given the organization of the legislature.[10]

Weingast, Shepsle, and Johnsen's (1981) "law of 1/n" states that the inefficiency of projects is growing in the number of districts (implicitly holding population constant). Does the "law of 1/n" still hold in the pure public goods case? Yes, since the chosen level of the public good is constant in n while the efficient level is declining in n. However, Weingast, Shepsle, and Johnsen's proof relies on establishing that the size of projects is increasing in n, which is not the case, as the number of districts has no effect on project size. This is not a mere technicality, as scholars routinely extend the "law" to project sizes and total spending.

Why is project size independent of the number of districts? Examining aggregate district payoffs makes this clearer. Differentiate equation (7) to obtain the first-order condition $\tilde{b}'(X_i^*)m = C'(X_i^*)/n$. Substituting for m, this becomes $\tilde{b}'(X_i^*)N/n = C'(X_i^*)/n$. So, increasing n produces two competing forces, which just happen to cancel each other out. On one hand, increasing n exacerbates the "tragedy of the commons" problem inherent under the norm of universalism plus full cost sharing. This is captured by the term on the right-hand side of the equality—it is a force that increases X_i. On the other hand, m decreases as n increases, and since the goods are pure public goods the benefits to increasing X_i fall—there are fewer people in each district to share in the public good. This is captured by the term on the left-hand side of the equality—it is a force that decreases X_i. The more typical interpretation of the "law of 1/n"—that total spending is increasing in the number of districts—holds because the number of projects is increasing in n, not because project sizes are increasing in n. And, as the next section demonstrates, this result holds only under certain conditions.

Congestion, Deadweight Costs, and Partial Cost Sharing

We now consider a formulation that incorporates congestion, deadweight costs of taxation, and partial sharing of project costs.[11] Each of these factors is an important component of any discussion of government budgeting and for understanding the impact of districting. Congestion relates to whether or not the benefits from the goods being provided depend on how many people are consuming them. Two extremes are pure public goods, in which one person's consumption of the good does not affect another person's consumption, and pure private goods, in which only one individual can enjoy a given unit of the good. Streets are an example of a good with partial congestion. If the district population is reduced (i.e., n is increased), then demand for a public good will decline because there are fewer individuals to enjoy the good. This impact will be mitigated when

[9]One way to think about this is in terms of spillovers. If districts are small, then the benefits of a public goods project in district i will be enjoyed by some citizens in neighboring districts, while if districts are larger then more of the benefits will be captured inside the district. Thus, from the point of view of the citizens in district i, aggregate benefits may be well approximated by $B(X, m) = \tilde{b}(X)m$. The examples in the introduction may be viewed this way.

[10]Given production technologies, there will be an optimal districting scheme, but this is not our focus here. For example, at some point the "localness" of the local public goods will imply that further increases in district size do not improve efficiency.

[11]See Tullock (1982) for a discussion of how deadweight taxation might influence the Weingast et al. model.

the good is congested because two forces will be at work: reduced population will lower demand, but reduced congestion will increase each individual's demand. We require a model to determine the precise conditions under which project sizes will be increasing, decreasing, or constant as a result of these countervailing effects.

Deadweight costs of taxation refers to a key principle in microeconomics: it costs more than one dollar for the government to spend one dollar because taxes impose inefficiencies by changing economic behavior. Only "lump-sum" taxes are economically efficient since they do not change behavior, and such taxes rarely if ever exist.

By partial cost sharing, we refer to the phenomenon whereby a central government may provide a set proportion of total spending on a given item. So, for example, the federal government may pay for one-half of the cost of building a bridge. Partial cost sharing reduces tendencies toward inefficiency because it "re-internalizes" the costs of projects.

To simplify the analysis we adopt specific functional forms, but these are not critical for the results. Let total benefits from a project of size X be $B(X, m) = X^{\alpha}m^{\beta}$, where $0 < \alpha < 1$ and $0 < \beta \leq 1$. Per capita benefits then can be written as $b(X, m) = X^{\alpha}m^{\beta-1}$. Congestion occurs when the addition of more individuals reduces the benefits each individual receives from a project. Formally, this occurs when $\beta < 1$. The case of pure local public goods can be represented by setting $\beta = 1$, and the case of pure private goods can be represented by setting $\beta = 1 - \alpha$.

Suppose the cost of a project is linear in its size, with $C(X) = X$. Let s be the fraction of each project's cost that is shared equally by all districts, and let $1 - s$ be the fraction that is paid by the district that enjoys the project's benefits. Let X_i be project size in district i, and assume that project size in all other districts is X. Then each citizen in district i pays taxes of $t_i = (1 - s)X_i/m + s(X_i + (n - 1)X)/N = [(n - ns + s)X_i + (ns - s)X]/N$.

To capture the deadweight costs of taxation, assume that if the amount of taxes paid by a citizen is t, then the total cost borne by the citizen is t^{θ}, where $\theta \geq 1$. If $\theta > 1$, then for all $t > (1/\theta)^{\frac{1}{\theta-1}}$ taxes carry deadweight costs.

We can rewrite equation (4) as:

$$\hat{\pi}_i(X_i, m, n) = X_i^{\alpha}m^{\beta-1} - [(n - ns + s)X_i + (ns - s)X]^{\theta}(1/N)^{\theta}. \quad (9)$$

Differentiating with respect to X_i and rearranging yields the first-order condition

$$\left(\frac{\alpha}{\theta}\right)\left(\frac{N^{\beta+\theta-1}\, n^{1-\beta}}{n - ns + s}\right) = [(n - ns + s)X_i + (ns - s)X]^{\theta-1}X_i^{1-\alpha}. \quad (10)$$

This first-order condition implies that Nash equilibrium project sizes will satisfy $X_i = X = X_i^*$. Substituting and rearranging in (10) gives

$$X_i^* = (\alpha/\theta)^{\frac{1}{\theta-\alpha}}(N)^{\frac{\beta+\theta-1}{\theta-\alpha}}\left[\frac{n^{2-\beta-\theta}}{n - ns + s}\right]^{\frac{1}{\theta-\alpha}}. \quad (11)$$

The efficient level of the distributive good, X_i^{**}, will solve (11), letting $s = 0$ (i.e., there is no cost sharing). This gives

$$X_i^{**} = (\alpha/\theta)^{\frac{1}{\theta-\alpha}}(N)^{\frac{\beta+\theta-1}{\theta-\alpha}}\left[n^{1-\beta-\theta}\right]^{\frac{1}{\theta-\alpha}}. \quad (12)$$

From this, it is straightforward to establish the following three propositions, which are discussed in remarks that follow.[12]

Proposition 1. *The degree of project inefficiency, X_i^*/X_i^{**}, is increasing in n.*

Proof. From (11) and (12), $X_i^*/X_i^{**} = \left[\frac{n}{n-ns+s}\right]^{\frac{1}{\theta-\alpha}}$. Differentiating this yields $\left[\frac{s}{(n-ns+s)^2}\right]\left[\frac{n}{n-ns+s}\right]^{\frac{1-\theta+\alpha}{\theta-\alpha}}$, which is clearly positive for $s > 0$. Q.E.D.

Proposition 2. *The size of projects, X_i^*, is increasing [decreasing, constant] in n if and only if $\beta + \theta < [>, =]\frac{n-ns+2s}{n-ns+s}$.*

Proof. In (11), only the term in square brackets depends on n. Differentiating this with respect to n yields $[(n - ns + s)n^{1-\beta-\theta}(2 - \beta - \theta) + (s - 1)n^{2-\beta-\theta}]/[(n - ns + s)^2]$. All terms are always positive except for $(n - ns + s)n^{1-\beta-\theta}(2 - \beta - \theta) + (s - 1)n^{2-\beta-\theta}$. This term can be rewritten as $n - ns + 2s - (\beta + \theta)(n - ns + s)$. This implies that $\frac{\partial X_i^*}{\partial n} > 0$ if and only if $\beta + \theta < \frac{n-ns+2s}{n-ns+s}$. Reversing the inequality or making it an equality establishes the related results. Q.E.D.

Proposition 3. *The amount of total spending, nX_i^*, is increasing [decreasing, constant] in n if and only if $\beta + \alpha < [>, =]\frac{n-ns+2s}{n-ns+s}$.*

Proof. Total spending across all districts is $nX_i^* = (\alpha/\theta)^{\frac{1}{\theta-\alpha}}(N)^{\frac{\beta+\theta-1}{\theta-\alpha}}\left[\frac{n^{2-\beta-\alpha}}{n-ns+s}\right]^{\frac{1}{\theta-\alpha}}$. Only the term in square brackets depends on n. Differentiating this with respect to n yields $[(n - ns + s)n^{1-\beta-\alpha}(2 - \beta - \alpha)$

[12] A corollary of Proposition 1, which follows immediately, is that the degree of inefficiency of total project spending nX_i^*/nX_i^{**} is increasing in n.

TABLE 1 Maximum value of $\beta + \theta$ [$\alpha + \beta$] such that $\partial X_i^*/\partial n > 0[\partial nX_i^*/\partial n > 0]$

n	s	value
20	.95	1.49
20	.90	1.31
50	.95	1.28
50	.90	1.15
100	.95	1.16
100	.90	1.08

$+ (s - 1)n^{2-\beta-\alpha}]/[(n - ns + s)^2]$. All terms are always positive except for $(n - ns + s)n^{1-\beta-\alpha}(2 - \beta - \alpha) + (s - 1)n^{2-\beta-\alpha}$. This term can be rewritten as $n - ns + 2s - (\beta + \alpha)(n - ns + s)$. This implies that $\frac{\partial X_i^*}{\partial n} > 0$ if and only if $\beta + \alpha < \frac{n-ns+2s}{n-ns+s}$. Reversing the inequality or making it an equality establishes the related results. Q.E.D.

Remark 1 *Under full cost sharing ($s = 1$), inefficiency and total spending are always increasing in n, while project sizes are increasing [decreasing, constant] in n when $(\beta + \theta) < [> , =]2$.*

In the case of full cost sharing (s = 1), the condition of Proposition 3 simplifies to $\beta + \alpha < [>, =]2$. Since $\alpha < 1$ and $\beta \leq 1$, this implies that total spending is always increasing in n. The conditions for project sizes are a bit more involved. For the simple case of a pure public good and no deadweight costs of taxation, $\beta = 1$ and $\theta = 1$. Then $\beta + \theta = 2$, demonstrating that while inefficiency and total spending are increasing in n, the size of projects is not. Some congestion is a necessary condition for the "law of 1/n" to hold for project sizes; the result obtains whenever there is sufficient congestion (i.e., β is not too close to 1) and deadweight costs are not too high (i.e., θ is close to 1), because in those cases increasing the number of districts will mitigate problems related to congestion. On the other hand, with a sufficiently public good (i.e., β near 1) and sufficiently large deadweight costs of taxation (i.e., $\theta > 1$), we will have $\beta + \theta > 2$ and therefore a law that is the *opposite* of the "law of 1/n." Deadweight costs cause this because for any fixed project size, the marginal cost of taxation increases with n, which tends to reduce X_i^*.

Remark 2 *Under partial cost sharing ($0 < s < 1$), inefficiency is always increasing in n, but total spending and project sizes may be increasing, decreasing, or constant in n, depending on congestion, the curvature of the benefit function (for spending only), deadweight costs of taxation (for project sizes only), and the initial value of n.*

With partial cost sharing the "law of 1/n" is even more likely to fail for the size of projects, and for total spending, as well. Interestingly, if we begin with a fairly large number of districts, then even with "nearly full" cost sharing (i.e., s close to 1) we often find that project size *decreases* as the number of districts increases further. The number of districts, n, looms large in the calculations.

TABLE 2 Summary of Results Three Versions of the "Law of 1/n"

	Inefficiency	Project Size	Total Spending
Full Cost-Sharing	Increasing in n	Varies	Increasing in n
Partial Cost-Sharing	Increasing in n	Varies	Varies

Table 1 gives several examples. The table shows that even when there are no deadweight costs (i.e., $\theta = 1$), project sizes can be declining in n even when goods are relatively uncongested (i.e., with values of β close to 1). For example, in a small legislature of 20, if $\beta = 1$ and $\theta = 1$, then project sizes are declining in n. Cost sharing, by forcing legislators to absorb a greater share of own-project costs, greatly diminishes any pro-spending bias caused by the addition of districts. Moreover, for any $s < 1$, as n gets large, the maximum value in column 3 of the table tends to 1. In other words, as the number of districts increases, it is harder for the "law of $1/n$" result to hold because the effect of adding in just a little cost sharing is magnified dramatically. In a legislature of 20, the effect of moving from $s = 1$ (full cost sharing) to $s = .9$ is to move a legislator's share of own-project costs from .05 to (.1+.05×.9), or a .145 share of costs, a three-fold increase. When a legislature's size is 100, the effect is to move costs from .01 to (.1+.01×.9), or .109, a ten-fold increase.

In addition, under partial cost sharing, the spending version of the "law of 1/n" may fail. This is clear from the table above, since the values in the last column give the maximum values for $\alpha + \beta$ such that $\partial(nX_i^*)/\partial n > 0$. Thus, if we begin with a fairly large number of districts, then total spending may be *decreasing* in the number of districts even with "nearly full" cost sharing. For example, in all of the cases shown in the table, all we need is $\alpha > .75$ and $\beta > .75$, which corresponds to a relatively public good. In addition, the impact of legislature size on spending, just as with efficiency, will depend on the magnitude of n one starts with, all else equal. For instance, when $s = .9$ and $\alpha = \beta = .6$, the impact of increasing legislature size on spending will be positive when $n = 20$ but negative when $n = 50$, for the same reasons as discussed above. Table 2 summarizes the theoretical findings.

This model is a logical starting point for many extensions and applications to American and comparative politics. Extensions include incorporating district heterogeneity into the model, either by allowing taxes and/or preferences for publicly provided goods to vary by district.[13] It is straightforward to incorporate varying tax shares. For example, we solved the model for the case when tax shares vary and costs are fully paid through a common tax pool ($s = 1$) and the results regarding "the law of 1/n" are identical to those in Remark 1 above. Another extension would be to incorporate multiple chambers with overlapping jurisdictions in the model.[14] It is also possible to use the functional forms in this analysis in a noncooperative bargaining environment. And, by allowing costs to be shared between central governments and local governments, the model speaks to the federalism literature, which addresses the balance of power between central and state governments, as well as the literature on "flypaper effects," which studies why a dollar of grant funding for a local government tends to be treated differently than an extra dollar of state personal income.[15] Overall, the findings and setup of our model offer a foundation for many future applications, and as the next section demonstrates, have many important empirical implications.

Implications for Data Analysis

These results can be used to inform data analysis. First, our results suggest the importance of making a distinction between efficiency, project sizes, and total spending. An increase in inefficiency due to a change in legislature size need not be associated with an increase in total spending. Second, partial cost sharing is a necessary condition for a "reverse law of 1/n" to hold with respect to *spending*. This means that localities must pay part of their own costs if they want to dip into the common pool, which is a very weak assumption. Once this condition is met, the impact of legislature size on total spending is dependent on congestion, the curvature of the benefit function, and the starting value of n. Deadweight costs of taxation are not relevant for total costs in this setup.

Based on this, it is easy to see why the empirical evidence on the "law of 1/n" is mixed. As note 2 indicates, there are about 25 studies that deal with this question in various domains. To keep the discussion focused, we focus primarily on studies of state governments and studies of local governments. The details of these studies vary depending on the context, but they all have a similar general structure. Each conducts regression analyses in which measures of government spending are regressed on measures of legislative size together with various controls. Government spending is measured in per-capita terms or as a fraction of income. The controls typically include measures of income, population, partisan composition of government, and demographics (e.g., percent elderly and percent school-aged). What differs across the studies is (a) the years under study and (b) the governments under study. (Pettersson-Lidbom (2004) also uses instrumental variables.) The relatively similar methodologies enable us to focus on the differences that arise out of the different governments studied, thereby giving us leverage on our question.[16]

The results from these studies can be summarized succinctly as follows. First, at least four papers study spending in U.S. states. Two of these find a generally significant and positive relationship between spending and the size of the upper chamber, but an insignificant and negative relationship between spending and the size of the lower chamber.[17] Primo (2006) find similar

[13]See Crain (1999) and Knight (2006) for a discussion of heterogeneous preferences.

[14]One puzzle is why the effect of the lower chamber on total spending tends to be negative (but statistically insignificant) in studies of the U.S. states and positive (and statistically significant) in the case of upper state chambers. Averaging over the postwar period, the median state upper house has 38 members and the median state lower house has 100 members. Given the results presented later in the paper that the "law of 1/n" is more likely to hold in small chambers, holding the lower house size fixed and increasing the size of the upper house is more likely to put upward pressure on spending than holding upper house size fixed and increasing the size of the lower house. Chen and Malhotra (2007) provide a start toward developing this intuition more fully; they incorporate bicameralism into a distributive politics model and find that the "law of 1/n" holds for upper chambers, while a "reverse law of 1/n" holds for the *ratio* of lower chamber seats to upper chamber seats when considering the distribution of private goods.

[15]See Rodden (2006) for an overview of fiscal federalism issues; see Hines and Thaler (1995) for a discussion of flypaper effects.

[16]The same type of analysis can be conducted on the cross-national studies, with similar results.

[17]Gilligan and Matsusaka (2001) study the period 1902–42 and find that the effect of legislature size on state and local government spending is dependent on specification. The size of the upper chamber has a positive and statistically significant coefficient in 15 of 19 regressions. By contrast, the size of the lower chamber a negative but statistically insignificant coefficient in 15 cases. Gilligan and Matsusaka (1995) study the period 1960–90 and also find a consistently statistically significant positive effect of upper chamber size, and a positive but a statistically insignificant effect for the lower chamber. Gilligan and Matsusaka also consider spending on specific budget categories but we do not discuss these here.

patterns, except both chambers are statistically significant. The primary difference across these papers is in the years under study, which likely accounts for differences in statistical significance. De Figueiredo (2003) studies a long time period (1865–1994) and finds a negative effect for both upper and lower chambers (only the latter is statistically significant). In sum, there is nearly as much evidence for a negative effect of lower chamber size in the United States as there is for a positive effect of upper chamber size.

Next, there are at least three studies of local governments. Bradbury and Stephenson (2003b) find consistently positive effects of the number of Georgia county commissioners on spending. Baqir (2002) finds similarly consistent positive effects in a study of city governments. However, Pettersson-Lidbom (2004) finds that spending in local Swedish and Finnish governments is declining in the number of council members.[18]

Our theory offers an explanation for these seemingly divergent findings, since it accounts for the varying size of decision-making bodies under study. As Table 1 shows and our earlier discussion explains, the "law of 1/n" is much more likely to hold in smaller legislatures than in larger ones. The local councils studied by Bradbury and Stephenson and Baqir are much smaller than those studied by Pettersson-Lidbom. In Bradbury and Stephenson's case, the mean number of county commissioners is 4.80. In Baqir's case, the mean council size is 6.86. In Pettersson-Lidbom's case, the mean council size is 28.66 for Finland and 47.35 for Sweden. Thus, for example, if $s = .9$, then the "law of 1/n" holds for Bradbury and Stephenson's case whenever $\alpha + \beta < 1.65$ and for Baqir's case whenever $\alpha + \beta < 1.57$. For Pettersson-Lidbom's values, the same condition is $\alpha + \beta < 1.24$ for Finland and 1.16 for Sweden.

Finally, *none* of the studies above demonstrates that the "law of 1/n" holds for project sizes, since they all focus on total spending. In fact, a simple analysis of the U.S. states suggests that while the "law of 1/n" might hold for total spending (at least with respect to upper chambers), it does not hold for project sizes. To examine this, we conducted an analysis of 46 states over a 40-year period, from 1957 to 2000.[19] We used a proxy for project size as the dependent variable instead of total per capita spending. The proxy for project size is the log of spending divided by the number of seats in the upper chamber. (We focus on the upper chamber since only it has typically been shown to be associated with higher spending in state-level analyses.)

The following controls are included in the analysis: the log of per capita income, the log of per capita federal aid, the log of population, percent elderly, percent school-aged, the share of state legislative seats held by Democrats, the proportion of state and local spending done at the state level, the average Democratic vote share in the state over the past 10 years, the presence of divided government, and a dummy for southern states (when state fixed effects are not used). To control for the fact that state spending may be higher, all else equal, the more that the state shoulders the overall state and local fiscal burden, we also include a variable for the proportion of state and local spending done at the state level. Standard errors are clustered by state, and year fixed effects are included in the analysis. Both with and without state fixed effects, we find that the upper chamber has a statistically significant and *negative* effect on project sizes. The existing "law of 1/n" framework cannot explain this result. Our framework can, since we can easily find reasonable values of the model's parameters that generates this result (for example, full cost-sharing [$s = 1$], a nearly public good [$\beta = .9$], and a moderate amount of deadweight costs of taxation [$\theta = 1.2$]).

In short, then, the conventional wisdom—that the "law of 1/n" is a robust empirical finding—is not borne out by examining an existing set of studies. Moreover, our theory, by allowing legislature size, congestion, and deadweight costs of taxation to vary in meaningful ways, offers an explanation for why some existing findings are consistent with the "law of 1/n" and others are consistent with a "reverse law of 1/n." We have shown this both across studies (comparing existing work) and with a new analysis showing that divergent results are achieved when the dependent variable is changed from spending to project size.

Conclusion

This paper has important implications for formal and empirical research. On the formal side, it suggests that the effects of districting depend crucially on the types of goods being provided by government. On the empirical side, it suggests that more careful attention

[18]Pettersson-Lidbom focuses exclusively on at-large council seats. If we assume that each legislator has a sub-constituency within the district, our model extends easily to his analysis.

[19]Alaska and Hawaii were not yet states for some of the years in the dataset, Nebraska has a nonpartisan unicameral legislature, and Minnesota had a nonpartisan legislature for some of the time period under study.

should be paid to how one uses models of legislative organization to motivate data analysis. More specifically, several important results emerge from the setup above:

1. *Efficiency*: The "law of 1/n," as originally presented by Weingast, Shepsle, and Johnsen (1981), states that the inefficiency of projects is increasing in the number of districts. We show that this result holds generally, but we correct the Weingast, Shepsle, and Johnsen proof.
2. *Project Sizes*: We show that the "law of 1/n" does not necessarily hold for project sizes. By incorporating congestion and deadweight costs of taxation, we demonstrate that the "law of 1/n" result for project sizes requires some congestion and sufficiently small deadweight costs of taxation. A "reverse law of 1/n" for project sizes can hold for sufficiently public goods and large deadweight costs of taxation. In these cases, increasing the number of districts reduces the number of individuals in each district (holding population fixed), which reduces the benefit of the government-provided good and also increases the cost of generating revenue to fund it. Partial cost sharing causes the deadweight costs of taxation to loom larger, making the "law of 1/n" for project sizes less likely to hold even when costs are nearly fully taken out of a common pool.
3. *Total Spending*: Under full cost sharing, total spending is always increasing in the number of districts. When costs are only partially subsidized by the central government, however, the impact of legislature size on spending depends on congestion, the "publicness" of the good, and the number of legislators one starts with. Since the "law of 1/n" is most typically thought of in terms of aggregate spending, this is perhaps the most important of our results.
4. *Empirical Implications*: Total spending is a misleading measure of the impact of districting on project efficiency. The reason is that the number of districts influences both (a) the projects legislators select and (b) the number of projects. Both have an impact on spending, with sometimes countervailing impacts. The theoretical portion of this paper offers a new way to understand the extant empirical literature analyzing the "law of 1/n." Our model allows for both positive and negative effects of legislature size on spending, both of which are found in the literature.

The "law of 1/n" has had a significant impact in political science and economics. The analysis presented here explores the robustness of this "law" and offers a foundation for continued study on a wide variety of topics in American and comparative politics, including legislature size, political fragmentation, the number of parties in government, and bicameralism. This will surely not be the last word on the topic, of course. The common pool problem manifests itself in many political environments, and students of politics will continue to refine our understanding of this phenomenon for many years to come.

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References

Baqir, Reza. 2002. "Districting and Government Overspending." *Journal of Political Economy* 110 (6): 1318–59.

Baron, David P. 1991. "Majoritarian Incentives, Pork Barrel Programs, and Procedural Control." *American Journal of Political Science* 35 (1): 57–90.

Bawn, Kathleen, and Frances Rosenbluth. 2006. "Short versus Long Coalitions: Electoral Accountability and the Size of the Public Sector." *American Journal of Political Science* 50 (2): 251–65.

Bradbury, John Charles, and E. Frank Stephenson. 2003a. "The Law of 1/n: A New Test." Working Paper: University of the South and Berry College.

Bradbury, John Charles, and E. Frank Stephenson. 2003b. "Local Government Structure and Public Expenditures." *Public Choice* 115 (1–2): 185–98.

Bradbury, John Charles, and W. Mark Crain. 2001. "Legislative Organization and Government Spending." *Journal of Public Economics* 82 (3): 309–25.

Chen, Jowei, and Neil Malhotra. 2007. "The Law of k/n: The Effect of Chamber Size on Government Spending in Bicameral Legislatures." *American Political Science Review* 101 (4): 657–76.

Crain, W. Mark. 1999. "Districts, Diversity, and Fiscal Biases: Evidence from the American States." *Journal of Law and Economics* 42 (2): 675–98.

De Figueiredo, Rui J.P., Jr. 2003. "Budget Institutions and Political Insulation: Why States Adopt the Item Veto." *Journal of Public Economics* 87 (12): 2677–2701.

DelRossi, Alison F., and Robert P. Inman. 1999. "Changing the Price of Pork: The Impact of Local Cost Sharing on Legislators' Demands for Distributive Public Goods." *Journal of Public Economics* 71 (2): 247–73.

Franzese, Jr., Robert J. 2002. *Macroeconomic Policies of Developed Democracies*. New York: Cambridge University Press.

Gilligan, Thomas W., and John G. Matsusaka. 1995. "Deviations From Constituent Interests: The Role of Legislative Structure and Political Parties in the States." *Economic Inquiry* 33 (3): 383–401.

Gilligan, Thomas W., and John G. Matsusaka. 2001. "Fiscal Policy, Legislature Size, and Political Parties: Evidence from the First Half of the Twentieth Century." *National Tax Journal* 54 (1): 57–82.

Hines, Jr., James R., and Richard H. Thaler. 1995. "Anomalies: The Flypaper Effect." *Journal of Economic Perspectives* 9 (4): 217–26.

Inman, Robert P., and Michael A. Fitts. 1990. "Political Institutions and Fiscal Policy: Evidence from the U.S. Historical Record." *Journal of Law, Economics, and Organization* 6 (special issue): 79–132.

Jordahl, Henrik, and Che-Yuan Liang. 2006. "Merged Municipalities, Higher Debt: On Free-riding and the Common Pool Problem in Politics." IFN Working Paper No. 679.

Knight, Brian. 2006. "Common Tax Pool Problems in Federal Systems." In *Democratic Constitutional Design and Policy*, ed. Roger D. Congelton and Birgitta Swedenborg. Cambridge: MIT Press, 229–47.

Lledo, Victor. 2003. "Electoral Systems, Legislative Fragmentation, and Public Spending: A Comparative Analysis of Brazilian States." Paper presented at the 2003 meeting of the Latin American Studies Association.

Mukherjee, Bumba. 2003. "Political Parties and the Size of Government in Multiparty Legislatures: Examining Cross-Country and Panel Data Evidence." *Comparative Political Studies* 36 (6): 699–728.

Pettersson-Lidbom, Per. 2004. "Does the Size of the Legislature Affect the Size of Government? Evidence from Two Natural Experiments." Working paper. Stockholm University.

Primo, David M. 2006. "Stop Us Before We Spend Again: Institutional Constraints on Government Spending." *Economics and Politics* 18 (3): 269–312.

Primo, David M., and James M. Snyder, Jr. 2007. "Party Strength, the Personal Vote, and Government Spending." Working Paper. MIT and University of Rochester.

Ricciuti, Roberto. 2004a. "Legislatures and Government Spending: Evidence from Democratic Countries." Working Paper 20/2004. International Centre for Economic Research.

Ricciuti, Roberto. 2004b. "The Role of Organized Interest Groups in Policy Making." In *Trading Interests: Legislature Size, Constituency Size and Government Spending in a Panel of Countries*, ed. Debora Di Gioacchino, Sergio Ginebri, and Laura Sabani. New York: Palgrave Macmillan, 315–36.

Rodden, Jonathan A. 2006. *Hamilton's Paradox: The Promise and Peril of Fiscal Federalism*. New York: Cambridge University Press.

Scartascini, Carlos G., and W. Mark Crain. 2001. "The Size and Composition of Government Spending in Multi-Party Systems." Paper presented at the annual meeting of the Public Choice Society.

Snyder, Jr., James M., and Michiko Ueda. 2007. "Do Multimember Districts Lead to Free-Riding?" *Legislative Studies Quarterly* 32 (4): 649–79.

Tullock, Gordon. 1982. "The Political Economy of Benefits and Costs: A Neoclassical Approach to Distributive Politics–Comment." *Journal of Political Economy* 90 (4): 824–26.

Weingast, Barry R. 1979. "A Rational Choice Perspective on Congressional Norms." *American Journal of Political Science* 23 (2): 245–62.

Weingast, Barry R., Kenneth A. Shepsle, and Christopher Johnsen. 1981. "The Political Economy of Benefits and Costs: A Neoclassical Approach to Distributive Politics." *Journal of Political Economy* 89 (4): 642–64.

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