Private Polling in Elections and Voter Welfare

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Starting point

• How do candidates use private information generated from polling?

• How does private information affect voter welfare?

• Our answers shed light on issues of candidate separation:
  * Why candidates separate?
  * How much candidates separate?
  * Do candidates separate enough?
Outline of talk

• literature

• model

• equilibrium analysis

• voter welfare
Private polling literature

- Ledyard (1989)
- Chan (2001)
- Ottaviani and Sorensen (2002)
- Bernhardt, Duggan, and Squintani (2005)
Separation literature

• Downs: MVT ⇒ No separation.

• Probabilistic voting with office-motivated candidates: No separation.

• Probabilistic voting with policy-motivated candidates: Separation through preferences.

• Candidate with valence advantage: Separation through mixing.

• Private polling: Separation through information.
Elections

- Two candidates, $A$ and $B$.

- Candidates simultaneously adopt policy platforms, $x$ and $y$, which lie in the real line.

- Voters have symmetric utilities, and the median voter is uniquely defined with ideal point $\mu$.

- Candidate $A$ wins if $|x - \mu| < |y - \mu|$, $A$ loses if the reverse holds, and $A$ ties if $|x - \mu| = |y - \mu|$ (a coin flip decides the winner).

- Each candidate seeks to maximize the probability of winning.
Information

- Median voter’s location has two components: $\mu = \alpha + \beta$, where
  - $\alpha$ is uniformly distributed on $[-a, a]$,
  - $\beta$ is discrete, with finite support $b_1 < \cdots < b_N$.

- Candidates $A$ and $B$ receive real-valued signals $i$ and $j$ about $\beta$, while $\alpha$ is independently distributed.

- Stories:
  - a component of preferences is not revealed by polling,
  - polling occurs first, then voter preferences are subject to shocks, then the election is held.
Information (cont.)

• Let $P(b, i, j)$ be the probability of $(b, i, j)$, with usual notation for marginals and conditionals: e.g., $P(i, j)$, $P(j|i)$.

• Self-possibility: $P(i, i) > 0$.

• Symmetry: $P(i, j) = P(j, i)$ and $F_{i,j} = F_{j,i}$.

• Arbitrary correlation is possible. Perfect correlation is Bayesian version of probabilistic voting model.
• Let $F_{i,j}$ be the distribution of the median voter conditional on signals $i$ and $j$, with density $f_{i,j}$.

• Let $m_{i,j}$ denote the median of $F_{i,j}$. Note that $m_{i,j} = E[b|i, j]$.

• WLOG: $i < j$ implies $m_{i,i} < m_{j,j}$.

• Ordered signals: $i < j$ implies $m_{i,K} < m_{j,K}$.

• Sufficient uncertainty: $a > b_N - b_1$. 

Information (cont.)
• A three “state” picture:

\[ P(b_3 | i, j) \]
\[ P(b_2 | i, j) \]
\[ P(b_1 | i, j) \]

\([b_1 - a, b_1 + a]\]

• In general,

\[ F_{i,j}(z) = \frac{a - m_{i,j} + z}{2a} \]

for all \( z \in [b_N - a, b_1 + a] \).
The game

- Pure strategies: $X = (x_i)$ and $Y = (y_j)$.

- Mixed strategies: $G = (G_i)$ and $H = (H_j)$.

- Ex post payoff is probability of winning:
  For $A$, it is

$$\Pi_A(X, Y | i, j)$$

$$= \begin{cases} 
F_{i,j} \left( \frac{x_i + y_j}{2} \right) & \text{if } x_i < y_j \\
1 - F_{i,j} \left( \frac{x_i + y_j}{2} \right) & \text{if } y_j < x_i \\
\frac{1}{2} & \text{if } x_i = y_j.
\end{cases}$$
The game (cont.)

• Ex ante payoff is

\[ \Pi_A(X, Y) = \sum_{i,j} P(i, j) \Pi_A(X, Y|i, j). \]

• Note: This is a two-player, symmetric, zero-sum game.

• Solution: Bayesian equilibrium.
Bernhardt, Duggan, and Squintani (2005)

- If \((X, Y)\) is a pure strategy equilibrium, then \(x_i = y_i = m_{i,i}\) for all signals \(i\).

- Adding an arbitrarily small amount of private information to the Downsian model can lead to non-existence of pure strategy equilibrium.

- Mixed strategy equilibria always exist.

- In models “close” to Downsian, mixed strategy equilibria “converge” to the median voter’s ideal point.
Pure strategy equilibrium

- Let $i = 0$ be such that $m_{0,0}$ equals the unconditional median. Given $i > 0$, we plausibly have $m_i < m_{i,i}$.

- With two or three signals, non-zero signals are always “self-reinforcing,” in this sense.

- Thus, candidate’s with positive signals tend to overshoot the median voter.

- (A1) Average medians:
  $$m_{i,j} = \frac{m_{i,i} + m_{j,j}}{2}.$$
Pure strategy equilibrium (cont.)

- **Theorem 1.** Under (A1), a necessary and sufficient condition for existence of the only possible pure strategy equilibrium is that

\[
\sum_{j : j \leq i} P(j|i) \geq \frac{1}{2}
\]

and

\[
\sum_{j : j \geq i} P(j|i) \geq \frac{1}{2}
\]

for all signals \( i \).

- This always holds in the two-signal model, assuming signals are not negatively correlated.

- For the extremal signals, existence requires \( P(i|i) \geq \frac{1}{2} \). This is restrictive when the number of signals is large.
Mixed strategy equilibrium

• We now suppose candidates locate in a non-deterministic way.

• (A2) Stochastic dominance: For all signals $i$ and $k$ with $k < i$,

\[ \sum_{j:j<k} P(j|k) \geq \sum_{j:j<k} P(j|i) \quad \text{for all } \ell. \]

• (A3) Increasing signals: For all signals $i$ and $k$ with $k < i$,

\[ \sum_{j:j<i} P(j|i) \geq \sum_{j:j<k} P(j|k) \]

and

\[ \sum_{j:j\leq i} P(j|i) \geq \sum_{j:j\leq k} P(j|k). \]
Mixed strategy equilibrium (cont.)

- Let $C$ be the set of signals $i$ satisfying the condition of Theorem 1. Let

- **Proposition 1.** Under (A2) and (A3), the set $C$ is non-empty and connected.

- We refer to signals in $C$ as “moderate.”

- A mixed strategy $G$ is **ordered** if:
  
  * for all $i$, $\text{Supp}G_i = [x_i, \bar{x}_i]$,
  
  * for all $i$ and $j$ with $i < j$, $\bar{x}_i \leq x_j$. 
Necessary conditions

• **Theorem 2.** Under (A2) and (A3), if there is an ordered equilibrium, then it is unique and has the following symmetric form.

  * For all $i \in C$, candidates locate at $m_{i,i}$.

  * For all $i > C$, candidates mix according to the increasing, convex density

    \[
    g_i(x) = \frac{\Phi_i}{2} \sqrt[3]{\frac{m_{i,i} - x_i}{(m_{i,i} - x)^3}}.
    \]

  * Candidates with extreme signals moderate relative to pure strategy equilibrium locations: $\bar{x}_i < m_{i,i}$.

  * Supports are adjacent: for all $i > C$, $\bar{x}_{i-1} = \bar{x}_i$. 
Necessary conditions (cont.)

- A picture of equilibrium strategies:

\[
\begin{align*}
\text{A picture of equilibrium strategies:} & \\
\end{align*}
\]
Sufficient conditions

- **(A4) Self-likelihood:** For all signals $i$ and $k$, $P(k|k) \geq P(k|i)$.

- **Theorem 3.** Under (A2)–(A4), if one candidate adopts the strategy specified in Theorem 2, then following each signal $i$, the other candidate’s payoff is weakly single-peaked in his location $x_i$ and maximized by all $x_i \in [x_i, \bar{x}_i]$.

- **Corollary 1.** Under (A2)–(A4), there exists a unique ordered equilibrium, as specified above.

- **Note:** These are maxmin strategies.
Sufficient conditions (cont.)

- Picture of expected payoffs:
Implications

- The expected location following signal $i > C$ is
  \[ E[x_i] = \frac{\Phi_i}{1 + \Phi_i} x_i + \frac{1}{1 + \Phi_i} m_{i,i}. \]

- Increasing the conditional medians $m_{i,i}$ for $i > C$ leads to a first order stochastic increase in the equilibrium distributions.

- Reducing
  \[ \Phi_i = \frac{\sum_{j:j\leq i} P(j|i) - \frac{1}{2}}{P(i|i)} \]
  for $i > C$ also leads to a first order stochastic increase.
Voter welfare

- Assume the distribution of ideal points in the electorate is fixed, up to parameter $\mu$.

- Quadratic utility: For each voter $v$,
  \[ u_v(z) = -(\mu + \delta_v - z)^2. \]

- Symmetry around zero:
  \begin{align*}
  * & \quad i = -K, \ldots, -1, 0, 1, \ldots, K \\
  * & \quad b = -B, \ldots, -1, 0, 1, \ldots, B \\
  * & \quad P(b) = P(-b) \\
  * & \quad P(i, j|b) = P(-i, -j| -b).
  \end{align*}
Voter welfare (cont.)

• Suppose a social planner can require the candidates to locate according to a strategy $X = (x_i)$, where $X$ is monotone in signal.

• Let $W_\delta(X)$ denote the ex ante welfare for the voter who is $\delta$ from $\mu$ when candidates locate according to $X$.

• Say that $X$ satisfies zero-symmetry if $x_i = -x_{-i}$ for all signals $i$. 

Voter welfare (cont.)

• **Proposition 2.** Voter \( v \)'s welfare is a fixed amount less than the welfare of the median voter:

\[
W_{\delta_v}(X) = -\delta_v^2 + W_0(X)
\]

for all zero-symmetric strategies.

• Thus, we focus on the median voter’s welfare \( W \) without loss of generality.

• **Proposition 3.** The welfare function \( W \) is strictly concave.
Optimal separation

- Separation by candidates can be socially beneficial, because it leads to more choice for voters.

- **Theorem 5.** Assume (A1). For all signals $i > 0$,

\[
\frac{\partial W}{\partial x_i}(M) > 0
\]

if:

- $P(\cdot | i)$ exhibits “inertia,” i.e.,

\[
P(i - h | i) > P(i + h | i),
\]

- there is sufficient uncertainty about the location of the median voter.

- The result always holds in the model with two or three signals.
Optimal separation (cont.)

- Define the unique social welfare optimum \( X^\ast \) by

\[
W(X^\ast) = \max W(X).
\]

- (A5) Strong single-peakedness: For all distinct signals \( i, j > 0 \),

\[
\frac{P(j|i)}{P(-j|i)} > \frac{3m_{i,i} + m_{j,j}}{|m_{i,i} - m_{j,j}|}.
\]

- This condition always holds in the model with two or three signals.
Theorem 6. Assume (A5) and for all signals $i > 0$,

$$\frac{\partial W}{\partial x_i}(M) > 0.$$ 

Then the socially optimal platforms of the candidates are more extreme than the conditional medians, i.e., $x_i^* > m_{i,i}$ for all $i > 0$.

Corollary 2. Adding (A2) and (A3), the candidates locate too moderately in equilibrium relative to the social optimum with probability one.
Effects of signal correlation

• Assume that with probability $q$ both candidates receive the same signal drawn from $P(\cdot|b)$, and with probability $1 - q$ they receive conditionally independent signals drawn from $P(\cdot|b)$.

• (A6) Strong signals: For all signals $i > 0$, $m_{i,i} > m_{i,-i}$.

• This always holds in the model with two or three signals.
Effects of signal correlation (cont.)

• **Theorem 7.** Assume (A6) and for all signals $i > 0$,

$$\frac{\partial W}{\partial x_i}(M) > 0.$$  

When the pure strategy equilibrium exists, higher correlation $q$ leads to lower voter welfare:

$$\frac{dW}{dq}(M) < 0.$$  

• Thus, private polling is preferred to public.

• **Conjecture.** The same holds for the mixed strategy equilibrium.
Effects of signal precision

• The effects of signal precision on welfare are more subtle: Greater precision allows candidates to better target the median voter, but they also are more likely to get the same signal (and locate together).

• Consider the two-signal model: \( b = -1, 1 \) and \( i = -1, 1 \). Denote \( q \) as above, and let \( p \) be the probability of a correct signal.

• Then the socially optimal level of precision is above zero and below one.

• If signal precision depends positively on campaign spending, this suggest one argument for spending caps in electoral campaigns.
Conclusion

• Candidates separate because they condition their platforms on private information.

• In pure strategy equilibrium, candidates tend to overshoot the median voter.

• In mixed strategy equilibrium, the same is true for candidates with moderate signals. Candidates with more extreme signals moderate their platforms.

• We give theoretical foundations for the claim that candidates do not separate enough.

• Correlation is bad, because it reduces separation.

• Some precision is good, but not too much.