Abstract

We develop a model of elections in which two candidates poll voters prior to taking policy positions, providing the candidates with private information about the voters’ preferred policies. In the essentially unique equilibrium, candidates who receive moderate signals adopt more extreme platforms than their information suggests, but candidates with more extreme signals may moderate their platforms. We investigate how the statistical properties of the polling technology—signal precision and correlation—affect equilibrium platforms. Although candidates’ platforms diverge in equilibrium, we find that they do not do so as much as voters would like. As a result, the electorate always prefers less correlation in candidate signals, and thus private to public polling. For similar reasons, some noise in the polling technology raises voters’ welfare; this suggests a novel justification for spending caps in election campaigns.
1 Introduction

Since the seminal papers of Hotelling (1929), Downs (1957), and Black (1958), spatial competition models have greatly advanced our understanding of elections and campaigning. The central prediction is the median voter theorem, possibly the most famous result in political economy: Given voters with single-peaked preferences over a unidimensional policy space and two office-motivated candidates who are perfectly informed about voter preferences, both candidates locate at the median voter’s preferred policy in the unique equilibrium. An implication is that there is perfect policy convergence. The key normative insight is that such policy convergence cannot hurt the median voter, nor does it lower the welfare of risk averse voters with other ideologies.\footnote{To elaborate, if voters have symmetric single-peaked preferences and are risk averse, they all prefer the known median policy to facing an election with two differentiated candidates who win with equal probability, because such an election requires that the candidates’ platforms are located symmetrically about the median policy.}

In reality, however, candidates often differentiate their platforms. A host of researchers document empirically that candidates’ platforms diverge significantly from the estimated median voter’s preferred policy, and yet are not too extreme.\footnote{See, for example, the National Election Survey data estimating presidential candidates’ platforms from 1964 to 1972 (Page (1978), chapters 3 and 4) and for the 1984 and 1988 races (Merrill and Grofman (1999) pages 55-56). Budge et al. (2001) compare estimates of the U.S. and British median voters based on survey data (such as the NES and British Election Survey) with estimates of candidates’ platforms derived from speech and writing context analyses. They find clear evidence of divergence from the median policy, and no evidence of extremization. Poole and Rosenthal (1997) obtain similar findings using roll call voting to estimate Congress-persons’ platforms (pages 62-63).} At the same time, platform convergence is not perceived favorably by the popular press, nor by many academic scholars. To wit, it is often argued that there is “not enough choice” between candidates, and that “they are all the same.” Indeed, a manifesto calling for “responsible parties” presented in 1950 by the Committee on Political Parties of the American Political Science Association, which included the most influential political scientists of the day, was based on the premise that office-motivated candidates do not provide the electorate enough choice.\footnote{For example, the opening statement reads, “Popular government of a nation [...] requires political parties that provide the electorate with a proper range of choice between alternatives of action.” (Committee on Political Parties (1950), page 15). Practical proposals are also presented on how differentiated party platforms should be formed and how parties should insure their implementation by elected candidates (pages 50–56). Page (1978), page 21, observes that “[M]any American political scientists, most notably Woodrow Wilson and E.E. Schattschneider, have called for parties to provide the electorate with sufficient choice.” Our italics.}

Because policy convergence is inherent to the basic spatial model, and it is in fact socially optimal in that framework, the model appears insufficiently rich to address these issues.

Our analysis starts with the observation that in practice, political candidates do not know voters’ policy preferences with certainty when selecting platforms. Determining the median voter’s location is a difficult task, especially in the context of a complex political debate. Accordingly, can-
candidates devote substantial resources to gathering information about voters through private polling. Eisinger (2003) finds that since the Roosevelt administration, private polls have been an integral part of the White House modus operandi. Medvic (2001) finds that 46 percent of all spending on U.S. Congressional campaigns in 1990 and 1992 was devoted to the hiring of political consultants, primarily political pollsters. In addition, the major parties provide polling services to their candidates. Of course, private polling information is jealously guarded by candidates and parties. Indeed, Nixon had polls routinely conducted, but did not disclose results even to the Republican National Committee; and F.D. Roosevelt described private polling as his “secret weapon” (Eisinger (2003)).

We develop a model of elections in which candidates receive private polling information about voters’ preferences. Before selecting a platform, each candidate receives a signal drawn from an arbitrary finite set of possible signals. Each candidate updates about the median voter’s preferred policy and the platform of the opponent, then chooses a platform, and the candidate closest to the actual median policy wins. In our model, the median policy is given by \( \theta = \alpha + \beta \), candidates receive signals about \( \beta \), and \( \alpha \) is independently and uniformly distributed. One interpretation of this median policy decomposition is that voters are unwilling or unable to provide pollsters accurate summaries about all of their views, as is suggested, for instance, by the empirical work of Gelman and King (1993). Another interpretation is that candidates learn about the position \( \beta \) initially preferred by the median voter, after which electoral preferences may shift by \( \alpha \), during the electoral campaign.\(^4\) Our construction is completely general with respect to correlation in polling signals, capturing both private and public polling.

In a companion paper, Bernhardt, Duggan, and Squintani (henceforth BDS) (2005), we allow for general distributions of the median policy \( \mu \) and provide a general analysis of the existence and continuity properties of equilibria in the private polling model. BDS (2005) show that when even a small amount of private information is present, the nature of equilibrium policies changes drastically from the median voter theorem: In any pure strategy equilibrium, after receiving a signal, a candidate locates at the median of the posterior distribution over the location of the median voter, where the posterior is conditioned on both candidates receiving that same signal. As a consequence, each candidate locates more extremely than is implied by his private signal.

In this paper, we give a necessary and sufficient condition for this pure strategy equilibrium to exist. As the polling technology becomes finer, so that the number of possible signals increases, this condition becomes more difficult to fulfill. Thus, we find that the strategic incentives of

\(^4\)For example, after platforms have been selected, a weakening economy may change voters’ views about increased fiscal spending; or terrorist attacks may alter voters’ views about civil rights restrictions.
privately-informed candidates lead them to take locations that cannot be precisely predicted, i.e.,
the candidates use mixed strategies. We prove that where the pure strategy equilibrium does not
exist, there is a unique mixed strategy equilibrium in which the locations of the candidates are
ordered with respect to their signals. We give the closed-form solution of this equilibrium and
derive several empirical predictions.

First, we show that candidates with sufficiently moderate signals adopt their pure strategy
equilibrium platforms, locating more extremely than their information suggests, while candidates
who receive more and more extreme signals mix over policy positions, tempering their positions
by more and more toward their prior expectation. Intuitively, this reflects that a politician whose
pollster predicts greater shifts in the median anticipates that she is more likely to compete against
an opponent with a more moderate signal, who will take a more moderate platform. This result
is broadly consistent with the empirical evidence that candidates’ platforms significantly diverge
from the median voter’s preferred policy, and yet are not too extreme.

We then turn to the effect of the statistical properties of the polling technology on equilibrium
platforms. We show that an increase in the precision of the candidates’ signals leads candidates to
locate more extremely, in the sense of first order stochastic dominance. This finding is consistent
with the concurrent trends of platform polarization (see the NES data as reported in Budge et al.
(2001)) and technological improvement in polling. The effect of signal correlation across candidates
(which can be induced by public polling, for example) is ambiguous for candidates with extreme
signals, but it unambiguously moderates the locations following moderate signals.

Our welfare analysis allows us to make sense of the wide-spread unfavorable view of platform
convergence and of the claims that office-motivated candidates do not provide voters with enough
choice. Because the location of the median voter is unknown, candidates cannot perfectly target the
median voter’s preferred policy, so that the platform ultimately implemented is the one closest to
the median voter’s position. Assuming voters’ preferences are correlated with the median voter’s,
the benefit of dispersion in candidate platforms is that it gives voters greater choice, while the cost
is that candidates target the median less accurately. Because candidates care only about winning,
they do not internalize these externalities. As a consequence, candidates may not collectively
provide enough platform dispersion from the standpoint of the electorate.

We consider an environment in which shifts in preferences in the electorate affect all voters in a
common way. We prove that with quadratic utilities, all voters have the same ex ante ordering of
candidate location strategies, so that welfare comparisons are unambiguous. Under simple regular-
ity assumptions, voter welfare increases if candidates extremize their platforms past their already extreme equilibrium locations. This implies that equilibrium locations are socially too moderate for some signal realizations, and we then give conditions under which equilibrium locations are too moderate for all signal realizations. These conditions are satisfied when signals are sufficiently precise or sufficiently correlated. In sum, while candidates’ extremize their platforms relative to their private information, they still do not provide as much platform differentiation as voters would like. Thus, the message of the Downsian model is fully reversed.

Our welfare analysis then proceeds to show that greater signal correlation makes voters worse off: Correlation reduces both the degree by which candidates “extremize” their platforms given their signals, as well as the probability that candidates receive different signals, choose distinct platforms, and thus provide more variety to the electorate. In contrast, the effect of signal precision on welfare is non-monotonic. Increased polling accuracy raises the probability that candidates correctly identify the median voter’s preferred policy, raising the welfare from any one candidate’s platform, but it also raises the probability that the candidates adopt similar platforms, reducing the choice that candidates give to the voters. The net effect is that up to some point, raising precision raises welfare, but too much precision has the opposite effect.

These two final results have implications for public policy. First, the electorate prefers private to public polling, because sharing information raises the correlation between candidates’ information and adversely reduces platform diversity. This finding provides support for polling bans provisions adopted in countries such as Canada, Italy, France, Argentina (see Emery (1994)), support that does not rest on claims that public polling may distort elections because of bandwagon effects or effects on voter participation. Second, because greater precision eventually reduces voter welfare, campaign spending caps that limit resources devoted to polling may raise voter welfare, even when campaign advertising is truly informative and beneficial to the electorate.

A few other papers have considered aspects of elections with privately-informed candidates. Ledyard (1989) was the first to raise the issue of privately-informed candidates and considered examples exploring the effects of the order of candidate position-taking, public polls, and repeated elections. In independent work, Chan (2001) studies a three-signal model that differs structurally from ours in that a valence term, common to all voters but unobserved by the candidates, is attached to each candidate. He shows existence of a pure strategy equilibrium when signals are almost uninformative, but his analysis does not allow for the possibility that pure strategy equilibria fail to exist. Chan also gives an example in which increased signal precision lowers the welfare of the
median voter when signals are almost uninformative and almost perfectly correlated. Ottaviani and Sørensen (2003) numerically characterize a model of financial analysts who receive private signals of a firm’s earnings and simultaneously announce forecasts, with rewards depending on the accuracy of their predictions. The case of two analysts can be interpreted as a model of electoral competition with privately-informed candidates. They show that greater competition increases the strategic bias in forecasts: As the number of forecasters increases, forecasts become more extreme.\(^5\)

2 The Electoral Framework

Two political candidates, A and B, simultaneously choose policy platforms on the real line, where we use \(x\) to denote candidate A’s platform and \(y\) to denote B’s. There is a unique median voter, whose preferred policy position is given by \(\mu\). Candidate A wins the election if his platform is closer to the median voter’s preferred position than candidate B’s, i.e., if \(|x - \mu| < |y - \mu|\), and A loses if he locates further away. If \(|x - \mu| = |y - \mu|\), then the election is decided by a fair coin toss, so that A wins with probability one half.

Candidates do not observe \(\mu\). However, candidates can privately poll voters about their preferred policies. We assume that polling generates signals about the date 1 location of the median voter, given by \(\beta\), and then candidates choose platforms. The election is at date 2. Between dates 1 and 2, the median voter’s preferred platform may shift, so that the median voter’s final preferred position is \(\theta = \alpha + \beta\).\(^6\) For simplicity, we assume that \(\beta\) is a discrete random variable with support on \(b_1 < b_2 < \cdots < b_N\) and that \(\alpha\) is independently and uniformly distributed on \([-a, a]\).

Polling provides candidates private real-valued signals \(i\) and \(j\) about \(\beta\), drawn from a finite set. Signals are drawn prior to shifts in the median voter’s position, and they are therefore independent of \(\alpha\). Let \(P(i, j, b_k)\) denote the joint prior probability of candidate A’s signal \(i\) and candidate B’s signal \(j\) and realization \(b_k\). We let \(P(i, j)\) denote the marginal probability of signals \(i\) and \(j\); we let \(P(i)\) represent the marginal probability of signal \(i\); and we let \(P(b_k)\) denote the marginal probability of \(b_k\). We assume that \(P(i, i) > 0\) for all signals \(i\) and \(P(b_k) > 0\) for all \(k\). Thus, the conditional probabilities \(P(i, j|b_k)\) and \(P(i|j)\) are well-defined. Let \(F_{i,j}(\cdot)\) denote the distribution

\(^5\)More distantly related, Heidhues and Lagerlöf (2001) study a setting where candidates know more than voters about the optimal policies for voters, and where each candidate may precommit to one of two exogenous policy alternatives. Martinelli (2001) analyzes a related setting where platforms are endogenous and voters have private information. If the voters’ information is biased, equilibrium results in less than full convergence even if parties know with certainty the optimal policy.

\(^6\)An alternative interpretation is that \(\alpha\) simply captures an additional source of error inherent in the polling process, e.g., \(\alpha\) may represent a component of policy preferences about which voters are unwilling to divulge information.
of $\mu = \alpha + \beta$ conditional on signal pair $(i, j)$, and let $f_{i,j}$ be the associated density. Note that $f_{i,j}$ is strictly positive on its convex support. We assume that candidates have access to equally informative polling technologies: $P$ and $F$ are symmetric in the sense that $P(i, j) = P(j, i)$ and $F_{i,j} = F_{j,i}$ for all signals $i$ and $j$. We impose no restrictions on the correlation between candidates’ signals, allowing for conditionally-independent and perfectly correlated signals as special cases.

Given signals $i$ and $j$, we let $m_{i,j}$ be the uniquely-defined median of $F_{i,j}$, and we let $m_i$ be the uniquely defined median conditional on signal $i$. Given a subset $K$ of signals, let $m_{i,K}$ be the uniquely-defined median conditional on one candidate receiving signal $i$ and the other receiving a signal in the set $K$. We index signals naturally, so that greater signals imply higher values of the median conditional on like signals, i.e., $i < j$ implies $m_{i,i} < m_{j,j}$, and we let $i + 1$ denote the signal following $i$ in this ordering. We assume that distinct signals determine distinct conditional medians: For all signals $i$ and $j$ with $i < j$ and for every subset $K$ of signals, $m_{i,K} < m_{j,K}$.

To simplify the analysis, we assume that $a > b_N - b_1$, so that the conditional distribution of the median given signals $i$ and $j$ is linear over the sub-interval $[b_N - a, b_1 + a]$:

$$F_{i,j}(z) = \frac{a - m_{i,j} + z}{2a}, \text{ for all } z \in [b_N - a, b_1 + a].$$

(1)

This assumption captures the idea that with positive probability, the unresolved uncertainty in the median voter’s position at the time of polling can swamp policy preferences elicited through polling. Figure 1 depicts the “stacked uniform” shape of the associated density when $\beta$ has three possible values.

Routine algebra reveals that the conditional median $m_{i,K}$ equals the associated conditional expectation of $b$, $E[b|i, K]$. We sometimes entertain the notion that signals are “self-reinforcing,” in the sense that (i) there exists an uninformative signal $i = 0$, i.e., $E[b|i = j = 0] = E[b|i = 0] = E[b]$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure_1}
\caption{The conditional density $f_{i,j}$}
\end{figure}
and (ii) positive signals lead to positive updating, i.e., \( i > 0 \) implies \( m_{i,i} > m_i > E[b] \), and likewise for negative signals.

To illustrate our findings, we maintain a running example with three possible values of \( \beta \) and three corresponding possible signals.

**Example 1:** Suppose there are three equally-likely values of \( b \in \{-1, 0, 1\} \), and three possible signals, \( i \in \{-1, 0, 1\} \). With probability \( q < 1 \), the candidates receive the same signal, and with probability \( 1 - q \) they receive conditionally-independent signals. A signal is correct with probability \( p \in (1/3, 1) \), and with equal probability \( 1 - p \) either of the other signals is drawn. Thus, \( p \) captures the accuracy of the polling signals, and \( q \) is a measure of the correlation between the candidates’ signals. It is a simple matter to check that

\[
P(i, j) = (1 - q)\left(\frac{1 + 2p - 3p^2}{12}\right) \quad \text{for } i \neq j \quad \text{and} \quad P(i, i) = \frac{q}{3} + (1 - q)\left(\frac{1 - 2p + 3p^2}{6}\right).
\]

Given our assumption on \( a \), which here amounts to \( a > 1 \), it is straightforward to solve for the median conditional only on a candidate’s own signal and for the median conditional on both candidates receiving the same signal: We have

\[
m_1 = \frac{3p - 1}{2} \quad \text{and} \quad m_{1,1} = \left(\frac{3p - 1}{2}\right)\left(\frac{p + 1}{3p^2 - 2p + 1} + 1 - q\right),
\]

and due to symmetry, \( m_0 = m_{0,0} = 0 \). Since \( m_1 < m_{1,1} \), signals are self-reinforcing.

If candidate \( A \) locates to the left of \( B \), i.e., \( x < y \), then candidate \( A \) wins when \( \mu < (x + y)/2 \). Conversely, if \( x > y \), then \( A \) wins when \( \mu > (x + y)/2 \). The probability that \( B \) wins is just one minus the probability that \( A \) wins. Thus, the probability that candidate \( A \) wins when \( A \) adopts platform \( x \) following signal \( i \) and \( B \) adopts platform \( y \) following signal \( j \) is

\[
\pi_A(x, y|i, j) = \begin{cases} 
F_{i,j}\left(\frac{x+y}{2}\right) & \text{if } x < y, \\
1 - F_{i,j}\left(\frac{x+y}{2}\right) & \text{if } y < x, \\
\frac{1}{2} & \text{if } x = y.
\end{cases}
\]

We define a Bayesian game between the candidates in which pure strategies for candidates \( A \) and \( B \) are vectors \( X = (x_i) \) and \( Y = (y_j) \), respectively, and the solution concept is Bayesian equilibrium. The equilibrium is symmetric if \( X = Y \). Given pure strategies \( X \) and \( Y \), candidate \( A \)’s interim expected payoff conditional on signal \( i \) and \( A \)’s ex-ante expected payoff are

\[
\Pi_A(X, Y|i) = \sum_j P(j|i)\pi_A(x_i, y_j|i, j) \quad \text{and} \quad \Pi_A(X, Y) = \sum_i P(i)\Pi_A(X, Y|i),
\]
respectively. The ex-ante game is a two-player, constant-sum game: For all \( X \) and \( Y \), \( \Pi_A(X,Y) + \Pi_B(X,Y) = 1 \). Because the game is constant sum, equilibria are interchangeable in the sense that if \((X,Y)\) and \((X',Y')\) are equilibria, then so are \((X,Y')\) and \((X',Y)\).

These concepts extend to mixed strategies, where candidate behavior is described by cumulative distribution functions over platforms. We let \( G_i \) represent the distribution over platforms adopted by a candidate after receiving signal \( i \). A mixed strategy is then a vector \( G = \{G_i\} \) of cumulative distribution functions. A mixed strategy equilibrium is a pair \((G,H)\) of mixed strategies such that candidate \( A \)'s strategy \( G \) maximizes her expected payoff given each signal \( i \), and similarly for candidate \( B \)'s strategy \( H \).

### 3 Equilibrium Analysis

Pure strategy equilibria take a simple form (BDS (2005)): If a pure strategy equilibrium exists, then it is unique, and after receiving a signal, each candidate locates at the median of the distribution of \( \mu \) conditional on both candidates receiving that signal.

**Theorem 1** If \((X,Y)\) is a pure strategy equilibrium, then \( x_i = y_i = m_{i,i} \) for all signals \( i \).

Intuitively, if the candidates were to locate at a platform away from \( m_{i,i} \) following signal \( i \), then either candidate, say \( A \), could exploit this by moving toward \( m_{i,i} \): Candidate \( A \)'s expected payoff would jump up discontinuously given signal realization \( i \) for \( B \), but \( A \)'s payoff would vary continuously with \( A \)'s location given other signal realizations for \( B \). Hence, this slight deviation would raise \( A \)'s payoff, which is impossible in equilibrium. This result is reminiscent of the findings of Milgrom (1981), who shows that in a common-value second-price auction, the equilibrium bid of a type \( \theta \) corresponds to the expected value of the good conditional on both types being equal to \( \theta \). Here, because candidates maximize the probability of winning, the relevant statistic is the median.

Theorem 1 has strong implications when signals are self-reinforcing: A corollary is that candidates tend to take policy positions that are extreme relative to the median of \( \mu \) given only their own information. That is, the platforms of candidates who receive high signals tend to overshoot \( \mu \), while those who receive low signals tend to undershoot.

To facilitate our analysis of the conditions for the existence of the pure strategy equilibrium in Theorem 1, we now impose a simplifying condition on the conditional medians. Specifically, we
assume that $m_{i,j}$ is the average of the two conditional medians $m_{i,i}$ and $m_{j,j}$, a weak restriction in three-signal models such as Example 1, where it is implied by symmetry around zero.

(A1) For all signals $i$ and $j$, we have $m_{i,j} = \frac{m_{i,i} + m_{j,j}}{2}$.

The next result, proved in the appendix, establishes that the pure strategy equilibrium exists if and only if each signal $i$ is a median of the probability distribution $P(\cdot | i)$. That is, the probability conditional on signal $i$ that the other candidate receives a signal less than $i$ is no greater than one half, and similarly for signals greater than $i$. This limits the incentive to move away from the conditional median to compete against a candidate who receives a more moderate signal or more extreme signal. In fact, we use assumption (A1) only to show that this is necessary for existence of the pure strategy equilibrium: Later, we give a stochastic dominance condition under which the inequalities in Theorem 2 deliver existence.

Theorem 2 Under (A1), a necessary and sufficient condition for existence of the only possible pure strategy equilibrium, specified in Theorem 1, is that for all signals $i$,

$$
\sum_{j:j\leq i} P(j|i) \geq \sum_{j:j>i} P(j|i) \quad \text{and} \quad \sum_{j:j<i} P(j|i) \leq \sum_{j:j\geq i} P(j|i). \quad (3)
$$

Condition (3) is weakest when there are just two possible signals, in which case it is satisfied as long as signals are not negatively correlated. The condition holds in our three-signal example as long as $P(1|1) \geq \frac{1}{2}$. It is most restrictive for the greatest and least signals, for which $P(1|i) \geq \frac{1}{2}$ is implied, and its restrictiveness obviously increases with the number of signals. Thus, in elections with finely-detailed polling and hence many possible signals, it becomes important that candidate platforms are not uniquely pinned down by their polling information.

We therefore analyze equilibria of the electoral game when we allow for mixed strategies for the candidates. Such strategies capture the possibility that a candidate’s position following a signal may not be precisely predicted by her opponent. We search for equilibria that satisfy a simple monotonicity condition. Specifically, we assume that following any given signal, if candidates do not locate deterministically, then they locate according to a distribution with connected support; and we assume that these supports are non-overlapping and ordered according to the candidates’ signals. In the following definition, we let $\underline{x}_i$ and $\overline{x}_i$ denote the lower and upper bounds of a candidate’s mixed strategy distribution following signal $i$. When these bounds coincide, the distribution is degenerate on $\underline{x}_i = \overline{x}_i$, and the candidate locates deterministically.
Definition 1 A mixed strategy \( G \) is ordered if:

(a) for all signals \( i \), Supp\((G_i) = [x_i, \overline{x}_i]\)

(b) for all signals \( i \) and \( j \) with \( i < j \), \( x_i \leq x_j \).

To obtain an explicit characterization of ordered mixed strategy equilibria, we impose more structure on the distribution over candidates’ signals. Assumption (A2) below is a stochastic dominance restriction on the conditional distributions of signals. It says that raising one candidate’s signal leads to a first-order stochastic increase in the distribution of his opponent’s signal. Assumption (A3) merely says that the higher is a candidate’s signal, the more likely is his signal to exceed his opponent’s. Both assumptions (A2) and (A3) hold in Example 1.

(A2) For all signals \( i, k \) with \( k < i \),

\[
\sum_{j: j < \ell} P(j|k) \geq \sum_{j: j < \ell} P(j|i), \text{ for all signals } \ell.
\]

(A3) For all signals \( i, k \) with \( k < i \),

\[
\sum_{j: j < i} P(j|i) \geq \sum_{j: j < k} P(j|k) \text{ and } \sum_{j: j \leq i} P(j|i) \geq \sum_{j: j \leq k} P(j|k).
\]

We use these assumptions to characterize the set of signals \( i \) that satisfy the inequalities in (3) of Theorem 2. We define the set \( C \) to consist of the set of signal realizations that satisfy these two inequalities, and we let \( \overline{\sigma} = \max C \) and \( \underline{\sigma} = \min C \). The next proposition establishes the non-emptiness and “centrality” of the set \( C \).

Proposition 1 Under (A2), \( C \) is non-empty. Adding (A3), \( C \) is connected: Given any signal \( i \), we have \( i \in C \) if and only if \( \underline{\sigma} \leq i \leq \overline{\sigma} \).

Under assumption (A1), Theorem 2 implies that if all signals are in \( C \), then the pure strategy equilibrium identified in Theorem 1 exists and is unique. We now prove that this qualitative result extends to the general case where the central set \( C \) may be a proper subset of the signal space. the next result characterizes the unique ordered equilibrium, when it exists. Candidates who receive moderate signals in the central set \( C \) play the pure-strategy \( m_{i,i} \), and hence extremize their platforms. Candidates with more extreme signals, say \( i > \overline{\sigma} \), adopt convex, increasing mixed
strategy densities. We show that the supports of the distributions following extreme signals are adjacent, and with probability one candidates locate more moderately than the conditional medians. Figure 2 depicts the ordered equilibrium.

**Theorem 3** Under (A2) and (A3), if there is an ordered equilibrium, then it is unique and has the following symmetric form. For all \(i \in C\), candidates locate at \(m_{i,i}\), and for all \(i > c\), candidates mix according to an increasing, convex density,

\[
g_i(x) = \frac{\Phi_i}{2} \sqrt{\frac{m_{i,i} - x}{(m_{i,i} - x)^3}} > 0
\]

(4)
on the interval \([x_i, x_{i+1}]\) with \(x_i < m_{i,i}\). Here,

\[
\Phi_i = \sum_{j \not< i} \frac{P(j|i) - 1/2}{P(i|i)} > 0 \quad \text{and} \quad x_i = m_{i,i} \left[1 - \left(\frac{\Phi_i}{\Phi_i + 1}\right)^2\right] + x_{i-1} \left(\frac{\Phi_i}{\Phi_i + 1}\right)^2.
\]

(5)
with \(x_{i+1} = m_{i,c}\). These supports are adjacent, in the sense that \(x_{i-1} = x_i\) for all \(i > c\). The associated cumulative distribution function with which candidates mix is given by

\[
G_i(x) = \Phi_i \left[\sqrt{\frac{m_{i,i} - x}{m_{i,i} - x} - 1}\right]
\]

(6)
on the interval \([x_i, x_{i+1}]\). Further, the expected platform of a candidate with signal \(i > c\) is a weighted average of \(x_i\) and \(m_{i,i}\),

\[
E[x_i] = \frac{\Phi_i}{\Phi_i + 1} x_i + \frac{1}{\Phi_i + 1} m_{i,i}.
\]

(7)
An analogous characterization of \(G_i\) holds for \(i < c\).
The proof in the appendix proceeds sequentially. We first prove that an equilibrium mixed strategy cannot put positive probability following signal \( i \) on any platform other than \( m_{i,i} \). Given an interval on which \( G_i \) is continuous, our assumption of non-overlapping supports implies that the candidate’s expected payoff conditional on signal \( i \) is differentiable over that interval. Since the candidate must be indifferent over all positions in the support of his mixed strategy, the second-order condition must hold over the interval, taking the simple form

\[
3g_i(x)f_{i,i}(x) + g_i'(x)(2F_{i,i}(x) - 1) = 0.
\]

We solve this system of ordinary differential equations for a mixed-strategy equilibrium up to the initial condition \( g_i(x_i) \). We use this characterization to show that if \( i \in C \), then the candidate necessarily places probability one on the conditional median \( m_{i,i} \). Following an extreme signal \( i \notin C \), the candidate has an incentive to locate to compete against opponents with more moderate signals, and this implies that a candidate with a more extreme signal mixes according to a non-degenerate distribution, moderating his position relative to the conditional median. We show that there can be no gaps between the supports of the candidates’ mixed strategies. Finally, integration reveals the properties of the equilibrium mixed strategy stated in the theorem.

We next provide conditions under which the unique equilibrium described in Theorem 3 exists.

**(A4)** For all signals \( i \) and \( k \), \( P(k|k) \geq P(k|i) \).

Assumption (A4) holds in Example 1 and simply says a candidate is most likely to receive signal \( k \) when the other candidate also receives \( k \). Adding (A4) to the conditions in Theorem 3, we show that each candidate’s payoffs are single-peaked in \( x_i \) around the support of his mixed strategy, as in Figure 3. Such single-peakedness implies that the candidate’s play best responses with probability one and is obviously a more stringent condition than needed to ensure existence.

**Theorem 4** Assume (A2)–(A4). Then if one candidate adopts the strategy specified in Theorem 3, the other candidate’s payoff following each signal \( i \) is weakly single-peaked in his location \( x_i \) and maximized by all \( x_i \in [\underline{x}_i, \bar{x}_i] \).

In the proof of Theorem 4, assumptions (A3) and (A4) are used only to address signals \( i \notin C \). It follows that if (3) holds for all signals, then (A2) delivers existence of the pure strategy equilibrium; assumption (A1) is then not needed in Theorem 2. Together, Theorems 3 and 4 yield the following key corollary.
expected payoff following $\tau + 2$

$$m_\tau \quad m_{0,0} \quad m_{\tau,\tau} \quad m_{\tau+1,\tau+1} \quad m_{\tau+2,\tau+2}$$

Figure 3: Weak single-peakedness

**Corollary 1** Under \((A2)-(A4)\), there exists a unique ordered mixed strategy equilibrium, and its closed form is specified in Theorem 3.

We conclude this section with a brief note on the robustness of pure-strategy equilibrium, in the light of our mixed strategy characterization result. Note that $\Phi_i \downarrow 0$ is equivalent to $\sum_{j:j \geq i} P(j|i) \uparrow 1/2$, so that condition (3) in Theorem 2 for candidates to adopt their pure strategy location of $m_{i,i}$ becomes close to being satisfied. As $\Phi_i$ goes to zero, inspection of the closed-form solution for the equilibrium mixed strategy in Theorem 3 reveals that candidates place almost all probability close to their pure strategy location of $m_{i,i}$: Given any $x < m_{i,i}$, the expression for $G_i(x)$ in equation (6) goes to zero. This continuity result illustrates the robustness of the pure strategy equilibrium with respect to small deviations from the inequalities in Theorem 2.

### 4 Empirical Implications

In this section, we exploit the closed-form solution for $G_i$ in Theorem 3 to develop a qualitative understanding of how candidates use their private information when choosing platforms. We first determine how the statistical properties of the candidates’ polling technology—the correlation and precision of their signals—affect the platform choices of candidates. We then examine the propensity of candidates to extremize their location as a function of their signals.

**Correlation.** To address the impact of correlation, we generalize Example 1 as follows. Assume that with probability $q$ both candidates receive the same signal drawn from $P(\cdot|b)$ for each realization of $b$, and with probability $1 - q$ candidates receive conditionally-independent signals drawn
from the same $P(\cdot | b)$ distributions. We further assume that if one candidate receives a given positive signal, then the median conditional on the other candidate receiving that same positive signal exceeds the median conditional on the other candidate not receiving that signal. We denote the latter conditional median by $m_{i,-i}$.

(A5) There is a signal $0 \in C$ such that $m_{i,i} > m_{i,-i}$ for all signals $i > 0$ and $m_{i,i} < m_{i,-i}$ for all signals $i < 0$.

The next result summarizes the effects of increased signal correlation, as measured by $q$, on the equilibrium strategies of the candidates.

**Proposition 2** An increase in signal correlation $q$ decreases $\Phi_i$ for all signals $i > \tau$, and under (A5) it decreases $m_{i,i}$ for all positive signals:

$$\frac{d\Phi_i}{dq} < 0 \text{ for } i > \tau \quad \text{and} \quad \frac{dm_{i,i}}{dq} < 0 \text{ for } i > 0.$$

An analogous characterization holds for negative signals. Furthermore, the central set $C$ is weakly increasing in $q$.

Proposition 2 has immediate implications via Theorem 3 for the equilibrium location of candidates. When correlation increases, every signal $i \in C$ belonging to the central set remains in that set, and candidate locations following such signals become more moderate. For signals $i > \tau$ outside the central set, a decrease in $\Phi_i$ leads candidates to take more extreme platforms, but the decrease in conditional medians following positive signals shifts the lower bounds of supports, $x_i$, in toward the central signals. Thus, increasing correlation generates countervailing impacts on candidate location following extreme signals. To gain insight into which effect may dominate, we return to Example 1. There the pure strategy equilibrium exists for $P(1|1) \geq 0.5$, and increasing $q$ reduces $m_{1,1}$, causing candidates to moderate their positions. However, for $P(1|1) < 0.5$, the equilibrium is in mixed strategies, and using the expression in (7) of Theorem 3, we can show that $\frac{dE|x_i|}{dq} > 0$. That is, raising correlation causes the expected platform to become more extreme if the equilibrium is in mixed strategies, but to become less extreme if the equilibrium is in pure strategies.

**Precision.** Under reasonable structural assumptions, increasing signal precision implies that when candidate $A$ receives signal $i$, candidate $B$ is also more likely to receive this signal, i.e., $P(i|i)$ rises with precision as candidates are more likely to get the right signal. Similar reasoning suggests that
increasing precision should cause \( \sum_{j:j<i} P(j|i) \) to fall. Also, increasing precision raises the “content” of a positive signal, which should cause \( m_{i,i} \) to rise for \( i > 0 \). In Example 1, differentiation reveals that these conditions hold as we increase the precision parameter \( p \). As the meaning of increasing signal precision is not defined unambiguously in a model with more than three signal realizations, we consider changes in the specification of our model that are consistent with the broad implications of increasing signal precision. To simplify our analysis here, we assume that there exists a zero signal \( i = 0 \) and that the zero signal is central, i.e., \( 0 \in \mathbb{C} \).

**Definition 2** A model is subject to a transformation consistent with an increase in signal precision if for all signals \( i \geq 0 \), \( P(i|i) \) and \( m_{i,i} \) weakly increase, while \( \sum_{j:j<i} P(j|i) \) decreases; and symmetrically for \( i \leq 0 \).

The next result shows that increasing precision causes candidates to locate more extremely in equilibrium, in the sense of first order stochastic dominance.

**Proposition 3** Under \( (A2)-(A3) \), if a model is subject to a transformation consistent with an increase in signal precision, then candidates locate more extremely in equilibrium: For all signals \( i > 0 \), \( \Delta G_i(x) \leq 0 \) for all \( x \), strictly so for all \( x \) in the support of \( G_i \) in the initial specification of the model. An analogous condition holds for all signals \( i < 0 \).

The proof follows from inspection of equation (6) in Theorem 3, and it highlights an essential difference between an increase in correlation and an increase in precision: Both decrease \( \Phi_i \), but increased correlation shifts conditional medians in toward \( m_{0,0} \), producing a moderating effect.

Whereas Proposition 3 uses an “absolute” measure of extremization, one may also be interested in the probability that a candidate chooses a platform more extreme than his estimate of the median voter’s position. That is, one may want to know whether raising signal precision raises the probability that the candidate’s platform exceeds \( m_i \) following a positive signal \( i > 0 \). Proposition 3 shows that raising precision increases the probability of locating more extremely; but, raising precision also raises \( m_i \). This leads us to perform a numerical analysis of Example 1. We find that the probability a candidate chooses a platform that is more moderate than his information suggests, i.e., \( x_1 \in [0, m_i] \), is bounded from above by \( \frac{1}{2}(\sqrt{2} - 1) \approx 0.207 \), achieving this bound when signals are uninformative, i.e., \( p \downarrow \frac{1}{3} \), and signals are uncorrelated, i.e., \( q = 0 \). Thus, a candidate is always very likely to extremize his location in the sense that he locates more extremely than his forecast,
given his signal, of the median voter’s position. Further, raising the signal accuracy $p$ always raises the probability that a candidate locates more extremely than his information suggests.

**Signal magnitude.** We now derive a key characterization result: Candidates with more moderate signals expect to locate more extremely relative to their information than do candidates with more extreme signals. Specifically, we provide simple conditions under which candidates with extreme signals $i > \bar{c}$ expect to locate further away from $m_{i,i}$ as $i$ increases.

First note that intuitively the coefficients $\Phi_i$ in the characterization of Theorem 3 increase in $i$ for $i > \bar{c}$. In fact, under (A3), $\sum_{j,j \leq i} P(j|i)$ rises with $i$: Increasing a candidate’s signal raises the probability that his signal is at least as high as his opponent’s. Hence, as a candidate’s signal grows more extreme, $\Phi_i$ rises as long as the conditional probability $P(i|i)$ that the other candidate receives the same signal does not rise too sharply with $i$ (and one might think that $P(i|i)$ should fall with $i$). When $\Phi_i$ is increasing in $i > \bar{c}$, it follows immediately that $G_i (m_{i,i} - x)$ rises with $i > \bar{c}$. Hence, to prove that candidates with more extreme signals expect to locate more moderately relative to their information, we just need to show that $m_{i,i} - \bar{x}_i$ is strictly increasing in $i$. To do this, we use the difference equation (5) describing the relationship between $\bar{x}_{i+1}$ and $\bar{x}_i$ to solve for
\[
m_{i+1,i+1} - \bar{x}_{i+1} = m_{i+1,i+1} - m_{i,i} + \left(\frac{\Phi_{i+1}}{\Phi_{i+1} + 1}\right)^2 (m_{i,i} - \bar{x}_i), \quad \text{for } i \geq \bar{c}.
\]
Note that $\bar{x}_{\bar{c}} = m_{\bar{c},\bar{c}} = \bar{x}_{\bar{c}+1}$, so that $(m_{\bar{c}+1,\bar{c}+1} - \bar{x}_{\bar{c}+1}) - (m_{\bar{c},\bar{c}} - \bar{x}_{\bar{c}}) = m_{\bar{c}+1,\bar{c}+1} - m_{\bar{c},\bar{c}} > 0$.

Continuing inductively,
\[
(m_{i+1,i+1} - \bar{x}_{i+1}) - (m_{i,i} - \bar{x}_i)
\]
\[
= \left[ m_{i+1,i+1} - m_{i,i} \right] - \left[ m_{i,i} - m_{i-1,i-1} \right] + \left( \frac{\Phi_{i+1}}{\Phi_{i+1} + 1} \right)^2 (m_{i,i} - \bar{x}_i) - \left( \frac{\Phi_i}{\Phi_i + 1} \right)^2 (m_{i-1,i-1} - \bar{x}_{i-1})
\]
\[
\geq \left[ m_{i+1,i+1} - m_{i,i} \right] - \left[ m_{i,i} - m_{i-1,i-1} \right] + \left( \frac{\Phi_{i+1}}{\Phi_{i+1} + 1} \right)^2 \left[ (m_{i,i} - \bar{x}_i) - (m_{i-1,i-1} - \bar{x}_{i-1}) \right]
\]
\[
> \left[ m_{i+1,i+1} - m_{i,i} \right] - \left[ m_{i,i} - m_{i-1,i-1} \right],
\]
where the first inequality follows because $\Phi_{i+1} \geq \Phi_i$, and the second inequality follows from the induction hypothesis. We have shown that when $[m_{i+1,i+1} - m_{i,i}]$ is constant, $m_{i,i} - \bar{x}_i$ is strictly increasing with significant slack.

**Proposition 4** Suppose that for all signals $i > \bar{c}$, $\Phi_{i+1} \geq \Phi_i$. Under (A2)-(A4), there exists $\delta > 0$ such that if $[m_{j+1,j+1} - m_{j,j}] - [m_{j,j} - m_{j-1,j-1}] > -\delta$ for all $j \geq \bar{c}$, then $m_{i,i} - E[x_i]$ is strictly increasing in $i$ for $i \geq \bar{c}$. An analogous result holds for $i \leq \bar{c}$. 

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Thus, a gross sufficient condition for candidates with increasingly extreme signals to locate increasingly moderately relative to their information is that the distance between successive conditional medians, \([m_{i+1,i+1} - m_{i,i}]\), not fall too quickly with \(i \geq \bar{c}\).

5 Voter Welfare

We now investigate the properties of socially optimal platforms and the consequences of equilibrium platform choices for voter welfare. We adopt the standpoint of a social planner who does not have an informational advantage over the candidates in the electoral game, and whose role is simply to choose strategies for the two candidates to maximize voter welfare. To maintain consistency with equilibrium choices, we restrict the social planner to symmetric and monotone strategies. That is, the social planner selects the same strategy \(X\) for the two candidates, and \(x_i > x_j\) for signals \(i > j\).

While “overshooting” by candidates causes them to bias their location away from the expected location of the median voter, it also leads to greater expected separation between the candidates and therefore to greater choice for voters. We show that under simple regularity conditions, the latter effect dominates: Voters prefer that candidates take even more extreme positions than they do in equilibrium. We then investigate how the statistical properties of the polling technology affect voter welfare. In particular, we show that increased correlation in candidate signals reduces voter welfare; and we illustrate how the optimal amount of noise in the polling technology from the perspective of voters is affected by the correlation in signals and the amount of uncertainty about the median voter’s location.

We address these issues in a symmetric setting in which \(i \in \{-K, ..., -1, 0, 1, ..., K\}\) and \(\beta \in \{-B, ..., -1, 0, 1, ..., B\}\), where \(P(b) = P(-b)\) and \(P(i, j|b) = P(-i, -j|-b)\). We capture the two-signal model as the limiting case where \(P(0, 0)\) goes to zero. Note that the zero signal is central, i.e., \(0 \in C\), and that \(m_{i,i} = -m_{-i,-i}\) for all signals \(i\). We assume that the distribution of voter preferences is fixed up to the shift parameter \(\mu\), and that voters have quadratic utilities. Voter \(v\)’s preferred policy, \(\theta_v\), is therefore defined relative to the median voter’s preferred policy \(\mu\): \(\theta_v = \mu + \delta_v\), where \(\delta_v\) represents the position of \(v\)’s ideal point relative to \(\mu\), and a change in \(\mu\) simply shifts the distribution of voter ideal points. A voter with ideal point \(\theta\) receives utility \(u(\theta, z) = -(\theta - z)^2\) from policy outcome \(z\), and each voter votes for the candidate whose platform is closest to his preferred policy. We let \(W_{\delta_v}(X)\) represent the utility that voter \(v\) expects to receive if candidates use strategy \(X\).
We have shown that equilibrium pure strategies are symmetric around zero, in the sense that \( x_i = -x_{-i} \) for all signals. The next result implies that all voters have identical preferences over candidate strategies that are symmetric around zero.

**Proposition 5** For all strategies \( X \) symmetric around zero, voter \( \delta_v \)'s expected utility from \( X \) is a fixed amount \( \delta_v^2 \) less than the expected utility of the median voter:

\[
W_{\delta_v}(X) = -\delta_v^2 + W_0(X).
\]

A consequence of Proposition 5 is that without loss of generality we may focus on the median voter's welfare:

\[
W_0(X) = -\sum_b \sum_i P(b, i, i) \int_{-a}^{a} \frac{(\alpha + b - x_i)^2}{2a} d\alpha
\]

In what follows, we therefore drop the subscript on \( W(\cdot) \).

We now establish the strict concavity of the welfare function, which allows us to characterize the social optimum via first-order conditions. The proof proceeds by decomposing the Hessian of \( W(\cdot) \) into the sum of two matrices, where one matrix diagonal with negative entries and the other is a symmetric matrix in which the diagonal entry in each row \( i \) is equal to minus one times the sum of the other entries in that row. We then prove that this latter matrix is negative semi-definite.

**Proposition 6** The welfare function \( W(\cdot) \) is strictly concave.

As a result, the welfare function \( W(\cdot) \) has a unique maximizer, which we denote \( X^* \). By our symmetry assumptions, \( X^* \) is symmetric around zero, with \( x_0^* = m_{0,0} \). Thus, candidates locate at the social optimum following the zero signal in equilibrium, and we only need to consider the social optimality of candidate locations following signal realizations \( i \neq 0 \).

We first prove that in coarse information environments with at most three signals such as Example 1, candidates do not locate as extremely as voters would like. The proof follows from our general analysis below, which shows that in such environments

\[
\frac{\partial W}{\partial x_1}(M) = \frac{1}{a} \sum_{j:j<1} P(1, j) \left[ (a + m_{j,1} - m_{1,1})^2 + \sigma_{j,1}^2 - \left( \frac{m_{j,j} - m_{1,1}}{2} \right)^2 \right] > 0,
\]
where $\sigma_{j,k}^2$ is the variance of $\beta$ conditional on signals $j$ and $k$. To see the inequality, note that the term corresponding to $j = -1$ is positive. Under our maintained assumptions, we have $a > m_{1,1} - m_{-1,-1}$, so that the term corresponding to $j = 0$ is also positive. It follows that social welfare would be raised by a marginal increase in the candidates’ platform following signal $i = 1$. Concavity and symmetry around zero then immediately imply that candidates locate too moderately following signals $i = 1$ and $i = -1$ relative to the social optimum, i.e., $m_{1,1} < x_1^*$ and $x_{-1}^* < m_{-1,-1}$.

The following theorem summarizes these observations.

**Theorem 5** Assume that information is coarse, so there are at most three signals. When the pure strategy equilibrium exists, candidates locate too moderately relative to the social optimum following signals $i = 1, -1$. Adding (A2) and (A3), candidates locate too moderately relative to the social optimum with probability one following every signal $i \neq 0$ in the ordered mixed strategy equilibrium.

Extending our analysis to arbitrary numbers of signal realizations requires more structure. We first consider the local properties of social welfare near the vector of conditional medians, $M = (m_{i,i})$. Specifically, the following lemma characterizes the partial derivatives of welfare with respect to candidate location following an arbitrary signals $i > 0$.

**Lemma 1** Given signal $k > 0$, the partial derivative of welfare at the vector of medians is

$$\frac{\partial W}{\partial x_k}(M) = \frac{1}{a} \sum_{j:j < k} P(k,j) \left[ (a + m_{j,k} - m_{k,k})^2 + \sigma_{j,k}^2 - \left( \frac{m_{j,j} - m_{k,k}}{2} \right)^2 \right] - \frac{1}{a} \sum_{j:j > k} P(k,j) \left[ (a - m_{j,k} + m_{k,k})^2 + \sigma_{j,k}^2 - \left( \frac{m_{j,j} - m_{k,k}}{2} \right)^2 \right].$$

Lemma 1 allows us to sign the partials of the welfare function, which indicate the voters’ preferences for more moderate or extreme candidate platforms. Given this result, the following condition is necessary and sufficient for positive partial derivatives of the welfare function with respect to candidate locations following positive signals, generalizing our result for the three-signal model.

**(A6)** For all signals $k > 0$,

$$\sum_{j:j < k} P(j|k)[(a + m_{j,k} - m_{k,k})^2 + \sigma_{j,k}^2 - \left( \frac{m_{j,j} - m_{k,k}}{2} \right)^2] > \sum_{j:j > k} P(j|k)[(a - m_{j,k} + m_{k,k})^2 + \sigma_{j,k}^2 - \left( \frac{m_{j,j} - m_{k,k}}{2} \right)^2].$$
Adding assumption (A1) for the sake of simplicity, the inequality in (A6) reduces to:

\[ \sum_{j:j<k} P(j|k)[a(a - (m_{k,k} - m_{j,j})) + \sigma^2_{j,k}] > \sum_{j:j>k} P(j|k)[a(a - (m_{j,j} - m_{k,k})) + \sigma^2_{j,k}]. \] (9)

To interpret inequality (9), consider a positive signal \( k > 0 \). This inequality always holds if there are no more than three signals, and it holds for the most extreme signal \( k = K \). More generally, it is clear that inequality (9) holds for a sufficiently large, as long as

\[ \sum_{j:j<k} P(j|k) > \sum_{j:j>k} P(j|k), \] (10)

which is not too restrictive as there are \( 2k \) more signals to the left of \( k \) than there are to the right.

Even when \( a \) is not arbitrarily large, inequality (9) holds under reasonable conditions. First, observe that the probability the opposing candidate receives a more moderate signal than \( k \) plausibly exceeds the probability that he receives an equally more extreme signal, i.e., \( P(k - \ell|k) > P(k + \ell|k) \) for all \( \ell \) with \( 0 < \ell \leq K - k \), strengthening (10). Note that \( a > 2B > \max_{j,i} (m_{i,i} - m_{j,j}) \). Comparing the two sides of inequality (9) term by term, \( k - \ell \) with \( k + \ell \), the inequality therefore holds if \( \sigma^2_{j,k} \) weakly decreases in \( j \) and \( m_{j+1,j+1} - m_{j,j} \) weakly increases in \( j > 0 \).

**Theorem 6** Assume that for all signals \( k > 0 \), and all \( \ell \) with \( 0 < \ell \leq K - k \), we have \( P(k - \ell|k) > P(i + \ell|k) \), that \( \sigma^2_{j,k} \) weakly decreases in \( j \) for each fixed \( k \), and that \( m_{j+1,j+1} - m_{j,j} \) weakly increases in \( j > 0 \). Under (A1), for all signals \( i > 0 \), voter welfare increases if candidates extremize their platforms past the conditional medians: \( \frac{\partial W}{\partial x_i}(M) > 0 \). An analogous result holds for \( i < 0 \).

Effectively, voters gain more from marginally increasing separation in platforms when the opposing candidate receives a signal \( j < k \) than they lose from the reduced separation when the opposing candidate receives a signal \( j > k > 0 \) and from the reduced absolute accuracy of the candidates’ platform. The significance of this result lies in the fact that for all signals \( i \), the associated pure-strategy equilibrium location \( x_i \) coincides with the conditional median \( m_{i,i} \), and, under (A2) and (A3), \( m_{i,i} \) exceeds the upper bound \( \pi_i \) of the support of the mixed strategy distribution \( G_i \).

We now turn to a global analysis of voter welfare with an arbitrary number of signal realizations. While Theorem 6 together with Proposition 6 implies that equilibrium platforms are too moderate relative to the global optimum for at least two signal realizations, this local welfare result provides limited insight into the prevalence of this phenomenon. With more structure on polling technologies, however, equilibrium locations are too moderate for all signals.
(A7) For all signals $i > 0$,

$$\frac{P(j|i)}{P(-j|i)} > \frac{m_{i,i} + m_{j,j}}{m_{j,j} - m_{i,i}} \quad \text{for all } j = i + 1, \ldots, K,$$

and

$$\frac{P(j|i)}{P(-j|i)} > \frac{3m_{i,i} + m_{j,j}}{m_{i,i} - m_{j,j}} \quad \text{for all } j = 1, \ldots, i - 1.$$

The first inequality in Assumption (A7) says that if one candidate receives a positive signal, then the probability the other candidate receives a positive signal $j$ further from zero is sufficiently greater than the probability of receiving the even more distant signal $-j$. The second inequality slightly strengthens this requirement for moderate signals. Both inequalities are always (vacuously) satisfied in a model with three signals. More generally, these conditions are satisfied if candidate signals are sufficiently correlated or sufficiently precise, as in high-profile elections in which candidates have substantial resources to devote to accurate polling.

**Theorem 7** Under (A6) and (A7), the conditional medians are more moderate than the socially optimal platforms. That is, $m_{i,i} < x_i^*$ for all signals $i > 0$ and $x_i^* < m_{i,i}$ for all signals $i < 0$.

Theorem 7 does not follow directly from our previous results. Rather, we must exploit the deeper structure of $W$: We use the quadratic form of the partial derivative with respect to $x_i$, when viewed as a function of candidate positions following other signals. Assumption (A7) is a grossly sufficient condition for the result that $x_i^* > m_{i,i}$ for all positive signals. It ensures that the cross partials of $W(\cdot)$ are well-behaved even under the “worst case scenario” in which $x_j = m_{i,i}$ for positive signals $j < i$ and $x_j = m_{j,j}$ for signals $j > i$. One can derive alternative sufficient conditions with a suitable strengthening of (A3), when $a$ is sufficiently large. In particular, one can show that under these conditions, $x_k^* - m_{k,k}$ is increasing in $k$ for $k > 0$. Combining this result with the fact that $x_1^* > m_{1,1}$ (for signal 1, the worst case scenario is simply the vector of conditional medians, and the result is implied by Lemma 1), then yields that the conditional medians are more moderate than the socially optimal platforms.

We now relate Theorem 7 to the equilibrium analysis of Section 3, giving us a result paralleling Theorem 5 for the model with an arbitrary number of possible signal realizations.

**Corollary 2** Assume (A6) and (A7). When the pure strategy equilibrium exists, candidates locate too moderately relative to the social optimum following every signal $i \neq 0$. Furthermore, adding
\((A2)\) and \((A3)\), candidates locate too moderately relative to the social optimum with probability one following every signal \(i \neq 0\) in the mixed strategy ordered equilibrium.

Finally, we turn to the statistical properties of the polling technology. As in Section 4, we address the impact of correlation by assuming that conditional on the realization \(b\), candidates receive the same signal with probability \(q\), and with probability \(1 - q\) they receive conditionally-independent signals. For simplicity, we focus on equilibrium pure strategies, so that we can exploit the result in Lemma 1 that voters would be made better off if candidates located more extremely than the conditional medians.

**Theorem 8** Under \((A5)\) and \((A6)\), an increase in signal correlation \(q\) reduces voter welfare: 
\[
\frac{dW}{dq}(M) < 0, \quad \text{where we suppress the dependence of the vector } M \text{ of conditional medians on } q.
\]

From Proposition 2, we know that an increase in correlation shifts the conditional medians inward, decreasing candidate separation. By assumption \((A6)\), this indirect effect of increasing correlation leads to a decrease in voter welfare. The proof then consists of showing that the direct effect of increasing correlation also reduces voter welfare. Theorem 8 bears the fundamental implication that voters prefer private polling by candidates to public polling.

The effect of increasing signal precision on voter welfare is more subtle. It is immediate that perfectly precise polling dominates contentless polling. We now prove that the impact of signal precision on welfare is not monotonic: While increasing precision up to some level raises welfare, eventually further increases in precision lower welfare. To prove this, we show that welfare is raised if rather than having perfectly precise polling signals, signals are slightly noisy. We say that signals are perfectly precise if for all \(b\) and for signals \(i = j = b\), \(P(i, j|b) = P(i|b) = 1\).\(^7\) When signals are perfectly precise, both candidates receive the “correct” signal and choose the same equilibrium platform, which equals the realization of \(\beta\).

We now prove that a small reduction in precision makes all voters better off, as it introduces the possibility that candidates receive different signals and hence differentiate their platforms, thereby providing voters more choice. In particular, we suppose that candidates receive conditionally-independent signals, and that the probability a candidate’s signals is correct is only \(1 - \epsilon\). Inspection of the proof reveals that the result extends as long as when candidates receive the wrong signal, their signals are not perfectly correlated. Thus, we prove that welfare increases if we introduce signal noise.

\(^7\)Accordingly, we assume that \(B \leq K\), so each realization of \(\beta\) may be associated with a unique element of \(K\).
such that the probability that both candidates receive “incorrect” signals is infinitesimal relative to the probability that one candidate receives the “correct” signal and one receives the “incorrect” one.

**Theorem 9** If signals are perfectly precise, then a sufficiently small reduction in signal precision increases social welfare in equilibrium.

While Theorem 9 reveals that voters are better off if polling is not too accurate, it does not provide insights into the optimal amount of noise to introduce to the polling technology from the perspective of voters, and how that optimal noise varies with the parameters describing the economy. To glean such insights, we consider a binary variant of Example 1 in which there are two equally-likely values of $b \in \{-1, 1\}$ and two possible signals, $i \in \{-1, 1\}$. As in Example 1, signals are correct with probability $p$, and conditional on the realization $b$, candidates receive the same signal with probability $q$, and with probability $1 - q$ they receive conditionally-independent signals.

Increasing the signal quality $p$ raises the probability that both candidates receive the “correct” signal, but it also increases the probability that candidates receive the same signal, and hence do not offer voters variety.

Figure 4 presents level sets of voter welfare $W$ as functions of $(p, a)$ when $q = 0$, and level sets of $W$ in $(p, q)$ for $a = 2$. The figure illustrates two general features of this binary setting. First, the value of precisely targeting $\beta$ decreases with $a$. This reflects that as $a$ rises, there is more uncertainty about the median voter’s preferred platform, which raises the value of increased platform choice. As a result, the optimal precision decreases in $a$: Voters want less accurate polls, so that candidates are less likely to receive the same signal, and thus provide greater choice. Second, the optimal signal precision rises with the signal correlation $q$. Intuitively, when signal correlation is
increased, the candidates are more likely to receive the same signal, raising the value of targeting the median voter more accurately.

6 Conclusion

This paper shows how private polling radically alters the nature of the strategies of office-motivated candidates, overturning the apparently robust result of platform convergence. Specifically, in any pure strategy equilibrium, candidates’ platforms over-emphasize their private information: Candidates locate at the median given that both receive the same signal. When candidates are not sufficiently likely to receive the same signal, equilibrium is characterized by mixed strategies. In the mixed strategy equilibrium, candidates who receive moderate signals adopt more extreme platforms than their information suggests, while candidates who receive more extreme signals moderate their platforms relative to their pollsters’ advice.

Contrary to existing models of elections, some platform differentiation always increases voters’ welfare. Although candidates differentiate their platforms in equilibrium, voters would prefer that candidates extremize their positions by even more. From the perspective of voters, this paper finds that there is an optimal amount of noise in the polling technology. That is, the marginal social value of better information for candidates about voters becomes negative, once polls are sufficiently accurate. This suggests a rationale for campaign spending limits—such limits reduce expenditures on polling, thereby reducing the precision of candidate’s signals, and possibly raising voter welfare. So too this suggests that voters may want to give dishonest answers to political pollsters in order to add noise to their polling technology. Finally, the electorate prefers private to public polling, because the increased signal correlation due to public polling reduces the diversity of platforms that candidates provide voters.

Our analysis suggests fruitful directions for future research. First, because the strategic value of better information is always positive for candidates, it is conceptually straightforward to endogenize the choice of costly polling technologies by candidates. Second, it would be worthwhile to determine how outcomes are affected when candidates have ideological preferences, and to endogenize contributions by ideologically-motivated lobbies to fund polling by candidates. Finally, as Ledyard (1989) observes, it would be useful to uncover how equilibrium outcomes are affected when candidates sequentially choose platforms, so the second candidate can see where the first locates, and hence can unravel the latter’s signal, before locating.
7 Appendix

Proof of Theorem 2: Let \((X, Y)\) denote the pure strategy profile in which \(x_i = y_i = m_{i,i}\) for all signals \(i\). This is an equilibrium if and only if for all signals \(i\), candidate \(A\) is not willing to deviate and play \(z \neq m_{i,i}\) following signal \(i\). Since deviations outside \([\min m_{j,j}, \max m_{j,j}]\) are strictly dominated by either \(\min m_{j,j}\) or \(\max m_{j,j}\), this amounts to the requirement that for all \(k > i\) and for all \(z \in (m_{k-1,k-1}, m_{k,k}]\),

\[
0 \leq \sum_{j : j < i} P(j|i) \left[ F_{i,j} \left( \frac{z + m_{j,j}}{2} \right) - F_{i,j} \left( \frac{m_{i,i} + m_{j,j}}{2} \right) \right] + P(i|i) \left[ F_{i,i} \left( \frac{z + m_{i,i}}{2} \right) - F_{i,i} \left( \frac{m_{i,i} + m_{i,i}}{2} \right) \right] + \sum_{j : i < j < k} P(j|i) \left[ F_{i,j} \left( \frac{m_{i,i} + m_{j,j}}{2} \right) - F_{i,j} \left( \frac{z + m_{j,j}}{2} \right) \right] + \sum_{j : j \leq k} P(j|i) \left[ F_{i,j} \left( \frac{m_{i,i} + m_{j,j}}{2} \right) - F_{i,j} \left( \frac{z + m_{j,j}}{2} \right) \right];
\]

and that for all \(k < i\) and for all \(z \in [m_{k,k}, m_{k+1,k+1}]\),

\[
0 \leq \sum_{j : j \leq k} P(j|i) \left[ F_{i,j} \left( \frac{z + m_{j,j}}{2} \right) - F_{i,j} \left( \frac{m_{i,i} + m_{j,j}}{2} \right) \right] + \sum_{j : i < j < k} P(j|i) \left[ 1 - F_{i,j} \left( \frac{m_{i,i} + m_{j,j}}{2} \right) - F_{i,j} \left( \frac{z + m_{j,j}}{2} \right) \right] + P(i|i) \left[ F_{i,i} \left( \frac{m_{i,i} + m_{i,i}}{2} \right) - F_{i,i} \left( \frac{z + m_{i,i}}{2} \right) \right] + \sum_{j : j \leq i} P(j|i) \left[ F_{i,j} \left( \frac{m_{i,i} + m_{j,j}}{2} \right) - F_{i,j} \left( \frac{z + m_{j,j}}{2} \right) \right].
\]

By (A1), it follows that \(F_{i,j} \left( \frac{z + m_{j,j}}{2} \right) = F_{i,i} \left( \frac{z + m_{i,i}}{2} \right)\) and \(F_{i,j} \left( \frac{m_{i,i} + m_{j,j}}{2} \right) = \frac{1}{2}\). Using condition (1), we therefore have

\[
F_{i,j} \left( \frac{z + m_{j,j}}{2} \right) - F_{i,j} \left( \frac{m_{i,i} + m_{j,j}}{2} \right) = \frac{z - m_{i,i}}{4a}
\]

for all signals \(i\) and \(j\), and condition (11) reduces to

\[
0 \leq \left[ \frac{z - m_{i,i}}{4a} \right] \left[ \sum_{j : j \leq i} P(j|i) - \sum_{j : i < j} P(j|i) \right].
\]

Since \(z > m_{i,i}\), this reduces to the first inequality in (3). Analogously, condition (12) simplifies to

\[
0 \leq \left[ \frac{z - m_{i,i}}{4a} \right] \left[ \sum_{j : j \leq k} P(j|i) - \sum_{j : j < i} P(j|i) \right],
\]

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and since \( z < m_{i,i} \), this reduces to the second inequality in (3).

**Proof of Proposition 1**: Let \( k \) be the highest signal such that \( \sum_{j:j>k} P(j|k) > \frac{1}{2} \). (If such a \( k \) does not exist, then the minimum signal \( \min i \) satisfies the inequalities.) Clearly, \( k \) is less than the maximum signals: \( k < \max i \). We claim that signal \( k+1 \) satisfies (3). First, note that

\[
\sum_{j:j\leq k} P(j|k+1) \leq \sum_{j:j\leq k} P(j|k) \leq \frac{1}{2},
\]

where the first inequality above follows from (A2) and the second follows from the definition of \( k \). Then (15) implies

\[
\sum_{j:j<k+1} P(j|k+1) \leq \sum_{j:j\geq k+1} P(j|k+1).
\]

Finally, from \( k+1 > k \) and the assumption that \( k \) is the highest signal such that \( \sum_{j:j>k} P(j|k) > \frac{1}{2} \), we have \( \sum_{j:j>k+1} P(j|k+1) \leq \frac{1}{2} \), i.e.,

\[
\sum_{j:j\geq k+1} P(j|k+1) \leq \sum_{j:j\leq k+1} P(j|k+1).
\]

Therefore, \( k+1 \) satisfies (3), and \( C \neq \emptyset \). We now establish that, adding (A3), \( C \) is connected. Let \( \ell \) be the lowest signal such that \( \sum_{j:j<\ell} P(j|\ell) > \frac{1}{2} \). Repeating the argument above, we have \( \ell - 1 \) satisfies (3). By construction, \( k+1 \leq \ell - 1 \). Take any \( i \) such that \( k+1 < i < \ell - 1 \). If \( i \notin C \), then we may assume without loss of generality that \( \sum_{j:j>i} P(j|i) > \frac{1}{2} \). Then the second part of (A3) implies that \( \sum_{j:j>k+1} P(j|k+1) > \frac{1}{2} \), a contradiction. Finally, consider \( i \) such that \( i < k+1 \) or \( i > \ell - 1 \), and without loss of generality assume the former. If \( \sum_{j:j>i} P(j|i) \leq \frac{1}{2} \), then \( i < k \). Then the second part of (A3) implies that \( \sum_{j:j>k} P(j|k) \leq \frac{1}{2} \), a contradiction. Hence, \( C = \{ j \mid k+1 \leq j \leq \ell - 1 \} \) is connected.

**Proof of Theorem 3**: Let \((G, H)\) be an ordered equilibrium. We first assume the equilibrium is symmetric, so that \( G = H \). Then \( x \) is a differentiable point of a candidate’s expected payoff, conditional on signal \( i \), if and only if the following holds: for all signals \( j \), if \( G_j \) puts positive probability on \( x \), then \( x = m_{i,j} \). At every point of differentiability \( x \in \text{Supp}(G_i) \), the derivative of the candidate’s expected payoff function at \( x \), conditional on signal \( i \), with respect to \( x_i \) is:

\[
\sum_{j: m_{j,j} < x} P(j|i) \left[ -f_{i,j} \left( \frac{x + m_{j,j}}{2} \right) \frac{(G_j(m_{j,j}) - G_j(m_{j,j})^-)}{2} \right] + \sum_{j: x < m_{j,j}} P(j|i) \left[ f_{i,j} \left( \frac{x + m_{j,j}}{2} \right) \frac{(G_j(m_{j,j}) - G_j(m_{j,j})^-)}{2} \right] + \sum_j P(j|i) \left[ \int_{-\infty}^{x} -f_{i,j} \left( \frac{x + z}{2} \right) \frac{g_j(z)}{2} dz + (1 - F_{i,j}(x))g_j(x) \right] + \int_{x}^{\infty} f_{i,j} \left( \frac{x + z}{2} \right) \frac{g_j(z)}{2} dz - F_{i,j}(x)g_j(x) \right],
\]

(16)
where \( g_i \) is the density of \( G_i \), wherever it is defined. BDS (2005) show generally that the supports of equilibrium mixed strategies are bounded by the left- and right-most conditional medians.

Our premise of non-overlapping supports, and the fact that \( f_{i,j} \) is constant at \( \frac{1}{2a} \) over the relevant range under our assumptions, allows us to simplify (16) greatly. If \( g_k \) is defined at \( x \in \left[ z_k, \bar{z}_k \right] \), then the derivative of the candidate’s expected payoff, conditional on signal \( i \), with respect to \( x_i \) is

\[
\frac{1}{4a} \left[ - \sum_{j:j<k} P(j|i) + \sum_{j:j>k} P(j|i) + P(k|i) \left[ 1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x) \right] \right].
\]  

(17)

If \( x \in \left( \bar{z}_{k-1}, \bar{z}_k \right) \), then the interior term in brackets simplifies, and the derivative becomes

\[
\frac{1}{4a} \left[ - \sum_{j:j<k} P(j|i) + \sum_{j:j \geq k} P(j|i) \right].
\]

(18)

Clearly, the candidate must be indifferent among all locations in any interval in the support of \( G_i \), and the first order condition must hold at all points of differentiability on such an interval.

**Step 1:** For all \( k \) and all \( z \), if \( G_k \) puts positive probability on \( z \), then \( z = m_{k,k} \). Condition (C4*) of BDS (2005) is satisfied in our model, and this step therefore follows from their Theorem 7.

**Step 2:** If \( G_i \) is continuous in an interval \( \left[ \underline{z}_i, \hat{x} \right) \), then given \( g_i(\underline{z}_i) \), the density \( g_i \) on \( \left[ \underline{z}_i, \hat{x} \right) \) is characterized by the candidates’ second-order condition. Since the candidate’s expected payoff is constant over the interval, it must in particular be linear over this interval, so the second-order condition must be satisfied with equality. Differentiating (17), we have

\[
P(i|i)[-3g_i(x) + 4(m_{i,i} - x)g_i'(x)] = 0,
\]

for all \( x \in (\underline{z}_i, \hat{x}) \). Since the candidate chooses the platform \( \underline{z}_i \) with zero probability, we include it in the interval as well, yielding a differential equation in \( g_i \) that is easily solved. We find that

\[
g_i(x) = g_i(\underline{z}_i) \left( \frac{m_{i,i} - x}{m_{i,i} - \underline{z}_i} \right)^{\frac{2}{3}}
\]

(19)

for all \( x \in [\underline{z}_i, \hat{x}) \), with associated distribution

\[
G_i(x) = g_i(\underline{z}_i)(m_{i,i} - x)^{3/2} \left( \frac{2}{\sqrt{m_{i,i} - \underline{z}_i}} - \frac{2}{\sqrt{m_{i,i} - x}} \right).
\]

(20)

Thus, the second-order condition pins down the density \( g_i \) and distribution \( G_i \) on \( [\underline{z}_i, \hat{x}) \) up to the initial condition \( g_i(\underline{z}_i) \).

**Step 3:** For all \( i \in C \), \( G_i \) is the point mass on \( m_{i,i} \). Suppose not, so \( \underline{z}_i < m_{i,i} \). As the argument is symmetric, suppose without loss of generality \( \underline{z}_i < m_{i,i} \). By Step 1, \( G_i \) is continuous on \( [\underline{z}_i, m_{i,i}) \).

The first-order condition can be written as

\[
g_i(x) = \frac{\sum_{j:j<i} P(j|i) + P(i|i)(1 - 2G_i(x)) - \sum_{j:j<i} P(j|i)}{4P(i|i)(m_{i,i} - x)}.
\]

(21)
Note that \( m_{i,i} \notin [\xi_i, \pi_i] \), for otherwise by (20) would imply that \( G \) takes values greater than one in a neighborhood of \( m_{i,i} \). Therefore, \( \pi_i < m_{i,i} \), and \( G \) is continuous on the entire interval \([\xi_i, \pi_i]\).

Substituting \( x = \pi_i \) in (21) and using \( i \in C \), we have

\[
g_i(x) = \frac{\sum_{j < i} P(j|i) - \sum_{j \geq i} P(j|i)}{4P(i|i)(m_{i,i} - x)} < 0,
\]
a contradiction.

**Step 4:** For all \( i > \tau \), \( G \) is continuous. First, suppose that \( \xi_i = \pi_i \); then from Step 1, \( G \) puts probability one on \( m_{i,i} \). We claim that for all \( j < i \), \( \pi_j < m_{i,i} \). If \( j \in C \), then this follows from Step 3 and \( m_{j,j} < m_{i,i} \). If \( j > \tau \) and \( \pi_j = m_{i,i} \), then by equation (21), \( g_j \) is negative in a neighborhood of \( \pi_j \), a contradiction, establishing the claim. Consider any \( z < m_{i,i} \) such that \( \pi_j < z \) for all \( j < i \).

The derivative of the candidate’s expected payoff, conditional on \( i \), at \( z \) is given by (18), which is negative, as \( i > \tau \). Therefore, a sufficiently small move from \( m_{i,i} \) to \( z < m_{i,i} \) raises the candidate’s expected payoff, a contradiction. Therefore, \( \pi_i < \pi_i \). As in Step 3, we must have \( m_{i,i} \notin [\xi_i, \pi_i] \), and then \( G \) is continuous.

**Step 5:** For all \( i > \tau \), supports are adjacent, in the sense that \( \pi_{i-1} = \xi_i \). Suppose \( \pi_{i-1} < \xi_i \).

As in Step 4, the derivative in the interval \([\pi_{i-1}, \xi_i]\) is given by (18), which is negative. As above, a sufficiently small move from \( \xi_i \) to \( z < \xi_i \) raises the candidate’s expected payoff, a contradiction.

**Step 6:** For all \( i > \tau \), the distribution \( G_i \) and its increasing, convex density \( g_i \) are given by (4) and (6); the upper bound of the support of \( G \) is strictly less than \( m_{i,i} \); the lower bounds of the supports are as in (5); and the expected value of \( x_i \) is given by (7). The parameters \( \xi_i \) and \( g_i(\xi_i) \) are determined by the first-order condition, which yields

\[
g_i(\xi_i) = \sum_{j < i} P(j|i) - \sum_{j \geq i} P(j|i) = \frac{\Phi_i}{2} \frac{1}{(m_{i,i} - \xi_i)}.
\]
when evaluated at \( \xi_i \). Substituting for \( g_i(\xi_i) \) in (19) and (20) yields the following expressions for the density \( g_i \) and distribution \( G_i \) on the support \([\xi_i, \pi_i]\):

\[
g_i(x) = \Phi_i \left[ \frac{(m_{i,i} - \xi_i)^{\frac{1}{2}}}{(m_{i,i} - x)^{\frac{1}{2}}} \right] \quad \text{and} \quad G_i(x) = \Phi_i \left[ \frac{m_{i,i} - x}{m_{i,i} - \xi_i} - 1 \right].
\]

The condition \( G_i(x) = 1 \) determines the upper bound \( \pi_i \) of the support, which by Step 5 coincides with \( \xi_{i+1} \). Note that for \( \xi_i < m_{i,i} \), a solution to \( G_i(x) = 1 \) does indeed exist for all \( i > \tau \), since \( \frac{\Phi_i}{2} \sqrt{m_{i,i} - x} \) goes to infinity as \( x \) increases to \( m_{i,i} \). By Steps 3 and 5, \( \xi_{i+1} = m_{\pi_i} \). Therefore, solving \( G_i(\xi_{i+1}) = 1 \), the lower bounds are pinned down recursively by difference equation (5), with the initial condition \( \xi_{\tau+1} = m_{\pi_{\tau}} \). These observations, with an induction argument starting with \( \xi_{\tau+1} = m_{\pi_{\tau}} \), yield \( \pi_i < m_{i,i} \) for all signals \( i \). The expectation (7) is derived simply by integrating. That \( g_i \) is increasing and convex is apparent from the functional form of the density.

Finally, we argue that there are no non-symmetric ordered equilibria \((G, H)\), where \( G \neq H \). Given any ordered equilibrium \((G, H)\), because the electoral game is symmetric and zero-sum, the
strategy pair \((H, G)\) is also an equilibrium. By interchangeability, \((G, G)\) is a symmetric equilibrium, so \(G\) is uniquely characterized above, as is \(H\) by an analogous argument.

**Proof of Theorem 4:** Assume each candidate uses the mixed strategy \(G\), defined in Theorem 3, and consider any signal \(i > \pi\). By construction, the candidate’s expected payoff conditional on receiving signal \(i\) is constant on \([x_i, \pi]\). We show that the candidate’s expected payoff falls as we move \(x_i\) further to the left of \(x_i\) or further to the right of \(\pi\). First, take \(k\) such that \(k < i\) and \(k \notin C\). We must show that

\[
- \sum_{j:j<k} P(j|i) + \sum_{j:j>k} P(j|i) + \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)\right] \geq 0. 
\]

(22)

By construction, the candidate’s expected payoff following signal \(k\) is constant over his support, so at \(x \in (x_i, \pi)\) we have

\[
\frac{\sum_{j:j>k} P(j|k) - \sum_{j:j<k} P(j|k)}{P(k|k)} + \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)\right] = 0. 
\]

(23)

By (A2) and (A4), we have

\[
\frac{\sum_{j:j>k} P(j|i) - \sum_{j:j<k} P(j|i)}{P(k|i)} \geq \frac{\sum_{j:j>k} P(j|k) - \sum_{j:j<k} P(j|k)}{P(k|k)}. 
\]

Since \(m_{i,k} > m_{k,k}\), we have

\[
1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x) > 1 - 2G_k(x) + 4(m_{k,k} - x)g_k(x),
\]

which implies that the left-hand side of (22) exceeds the left-hand side of (23), as required.

Now take \(k \in C\). Note that the candidate’s expected payoff conditional on signal \(i\) is discontinuous at \(m_{k,k}\): Letting \(\Pi(x_i|G, i)\) denote the expected payoff conditional on \(i\) from locating at \(x_i\) when the other candidate uses the mixed strategy \(G\), we have

\[
\lim_{w \uparrow m_{k,k}} \Pi_A(w|G, i) - \Pi_A(m_{k,k}|G, i) = P(k|i) \left[1 - F_{i,k}(m_{k,k}) - \frac{1}{2}\right] \geq 0,
\]

where the inequality follows from \(m_{k,k} < m_{i,k}\). Similarly, \(\Pi_A(m_{k,k}|G, i) - \lim_{w \uparrow m_{k,k}} \Pi_A(w|G, i) \geq 0\), so the candidate’s payoff function is non-decreasing at \(m_{k,k}\). Over the interval \((\pi, m_{k,k})\), the derivative of the candidate’s payoff function conditional on signal \(i\) is proportional to

\[
\sum_{j:j \geq k} P(j|i) - \sum_{j:j < k} P(j|i) \geq \sum_{j:j \geq k} P(j|k) - \sum_{j:j < k} P(j|k) \geq 0,
\]

where the first inequality follows from (A2) and the second from the definition of \(k \in C\).

Now take \(k > i\). We must show that

\[
- \sum_{j:j<k} P(j|i) + \sum_{j:j>k} P(j|i) + P(k|i) \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)\right] \geq 0.
\]

(24)
is non-positive. By \((A2)\), we have

\[
- \sum_{j:j<k} P(j|k) + \sum_{j:j>k} P(j|k) \geq - \sum_{j:j<k} P(j|i) + \sum_{j:j>k} P(j|i).
\]

(25)

Because (23) holds at \(x \in (\bar{x}_k, \bar{x}_k)\) and \(k > \bar{c}\), the left-hand side above is negative, which implies that \(1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)\) is positive. By \((A4)\) and \(m_{k,k} > m_{i,k}\), we have

\[
P(k|k) \left[1 - 2G_k(x) + 4(m_{k,k} - x)g_k(x)\right] > P(k|i) \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)\right].
\]

(26)

Together, (23), (25), and (26) imply that (24) is negative, as required.

Proof of Proposition 2: To see that \(\frac{d\Phi_i}{dq} < 0\) for \(i > \bar{c}\), write \(\Phi_i\) as \(\Phi_i = \left(\sum_{j:i<j} P(j|i) - P(i|i)\right)\). Note that

\[
P(i,i) = \sum_b [qP(i|b) + (1-q)P(i|b)^2]P(b)
\]

\[
\frac{dP(i,i)}{dq} = \sum_b [1 - P(i|b)]P(i|b)P(b)
\]

\[
P(j,i) = \sum_b [(1 - q)P(i|b)P(j|b)]P(b) \quad \text{for} \quad j \neq i
\]

\[
\frac{dP(j,i)}{dq} = -\sum_b P(i|b)P(j|b)P(b) \quad \text{for} \quad j \neq i.
\]

Therefore, using the fact that \(P(i)\) is independent of \(q\), we have

\[
\frac{\Phi_i}{dq} \propto P(i,i) \left[ - \sum_{j:j<i} P(i|b)P(j|b)P(b) \right] - \left[ \sum_{j:j<i} P(j,i) \right] - \frac{P(i)}{2} \left[ \sum_b [1 - P(i|b)]P(i|b)P(b) \right] < 0,
\]

where the inequality follows from \(i > \bar{c}\), which implies \(\sum_{j:j<i} P(j|i) > \frac{1}{2}\). To see that \(\frac{dm_{i,i}}{dq} < 0\), note that

\[
m_{i,i}(q) = E[b|i,i,q]
\]

\[
= \sum_b \left[ qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b) \right] \left[ \sum_{b'} [qP(i|b')P(b') + (1-q)P(i|b')P(i|b')P(b')] \right].
\]

Therefore, for all \(i > 0\), we have

\[
\frac{dm_{i,i}}{dq} \propto \sum_b \left[ P(i|b)P(b) - P(i|b)P(i|b)P(b) \right] \sum_{b'} \left[ qP(i|b')P(b') + (1-q)P(i|b')P(i|b')P(b') \right]
\]

\[
- \sum_{b'} \left[ P(i|b')P(b') - P(i|b')P(i|b')P(b') \right] \sum_b \left[ qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b) \right]
\]

\[
\sum_b \left( P(i|b)P(b) - P(i|b)P(i|b)P(b) \right) - \sum_{b'} \left[ qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b) \right]
\]

\[
\sum_{b} \left( P(i|b)P(b) - P(i|b)P(i|b)P(b) \right) - \sum_{b'} \left[ qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b) \right]
\]

\[
\sum_{b} \left( P(i|b)P(b) - P(i|b)P(i|b)P(b) \right) - \sum_{b'} \left[ qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b) \right]
\]
\[
\begin{align*}
= & \sum_b b [1 - P(i|b)] P(i|b) P(b) - \sum_b b[q P(i|b) + (1 - q) P(i|b) P(i|b)] P(b) \\
= & \bar{m}_{i,-i} - m_{i,i} \\
< & 0,
\end{align*}
\]
as required. That \( C \) is weakly increasing in \( q \) follows from the fact that \( \frac{dP(i,j)}{dq} < 0 \) for all \( j \neq i \).

**Proof of Proposition 3:** If a model is subject to a transformation consistent with an increase in signal precision, then the zero signal remains an element of \( C \) after the transformation. In fact, every signal that belongs to \( C \) before the transformation remains in the central set, say \( C' \), after the transformation. Given any signal \( i > 0 \) such that \( i \in C \), the shift in equilibrium platform is therefore just \( \Delta m_{i,i} > 0 \), as required. Given any signal \( i > 0 \) such that \( i \in C' \setminus C \), the equilibrium mixed strategy before the transformation is described in Theorem 3. This mixed strategy puts probability one on platforms less than \( m_{i,i} \), and the equilibrium platform after the transformation puts probability one on the new conditional median, say \( m'_{i,i} \). Since \( m'_{i,i} > m_{i,i} \), the claim of the proposition is fulfilled. Let \( \bar{c} \) be the greatest element of the new set \( C' \). It is evident that for signals \( i > \bar{c} \), the transformation decreases \( \Phi_i \). Moreover, the foregoing implies that the lower bound of the candidates’ support following signal \( \bar{c} + 1 \) after the transformation, denoted \( \bar{x}_{\bar{c}+1} \), weakly exceeds the lower bound before the transformation, which is \( \bar{x}_{\bar{c}+1} \). Therefore, expression (6) in Theorem 3 implies that the transformation leads to a stochastic improvement in the candidates’ equilibrium distribution following signal \( \bar{c} + 1 \). This further increases the lower bound \( \bar{x}_{\bar{c}+2} \) and leads to a stochastic improvement following \( \bar{c} + 2 \). An induction argument based on these observations yields the proposition.

**Proof of Proposition 5:** To calculate the welfare of a voter with relative ideal point \( \delta_v \), note that for any \( b \), if both candidates receive a signal \( i \), then they both locate at \( x_i \), yielding voter \( \delta_v \) expected utility of \( (1/a) \int_a^\infty u(\alpha + b + \delta_v, x_i) d\alpha \). If candidates receive different signals \( i \) and \( j \), with \( i < j \), one locates at \( x_i \), while the other locates at \( x_j \), and the candidate closest to the median voter wins the election. That is, the candidate at \( x_i \) wins if \( \mu < [x_i + x_j]/2 \), or equivalently if \( \alpha < [x_i + x_j]/2 - b \); and the candidate at \( x_j \) wins if \( \mu > [x_i + x_j]/2 \), or equivalently if \( \alpha > [x_i + x_j]/2 - b \). Thus, voter \( \delta_v \)’s expected utility is
\[
\frac{1}{2a} \int_{-a}^{[x_i + x_j]/2 - b} u(\alpha + b + \delta_v, x_i) d\alpha + \frac{1}{2a} \int_{[x_i + x_j]/2 - b}^{a} u(\alpha + b + \delta_v, x_j) d\alpha. \tag{27}
\]
Therefore,
\[
W_{\delta_v}(X) = \sum_b \sum_i P(b, i, i) \frac{1}{2a} \int_{-a}^{a} u(\alpha + b + \delta_v, x_i) d\alpha \\
+ \sum_b \sum_i \sum_{j > i} P(b, i, j) \left[ \frac{1}{2a} \int_{-a}^{[x_i + x_j]/2 - b} u(\alpha + b + \delta_v, x_i) d\alpha + \frac{1}{2a} \int_{[x_i + x_j]/2 - b}^{a} u(\alpha + b + \delta_v, x_j) d\alpha \right]. \tag{28}
\]
where we note that in the expression for $W$, $\delta_v$ establishes that the voter’s welfare is just $\delta_v$ less than the welfare of the median voter ($\delta_v = 0$).

Proof of Proposition 6: Differentiating the expression (28) for voter welfare, with $\delta_v = 0$, we have

$$\frac{\partial W}{\partial x_k}(X) = \sum_b P(k, b) \left[ \frac{1}{2a} \int_{-a}^{a} (b + \alpha - x_k) d\alpha + \frac{1}{2a} \int_{-a}^{a} u(\alpha) d\alpha \right].$$

For any $b$ and any pair $i, j$ with $i < j$, we aggregate the term (27) with the corresponding term for $b' = -b$ and $i' = -i$, $j' = -j$, so that $j' < i'$, so as to obtain:

$$-\frac{1}{2a} \int_{-a}^{a} (b + \delta_v - x_i)^2 d\alpha - \frac{1}{2a} \int_{-x_i - x_j}^{a} (b + \delta_v - x_j)^2 d\alpha$$

$$-\frac{1}{2a} \int_{-x_i - x_j}^{a} (\alpha - b + \delta_v + x_j)^2 d\alpha - \frac{1}{2a} \int_{-a}^{a} (\alpha - b + \delta_v + x_j)^2 d\alpha$$

$$= -\delta_v^2 - 2\delta_v (b - x_i) \left[ a + \frac{x_i + x_j}{2} - b \right] \frac{1}{2a} - 2\delta_v (b - x_j) \left[ a + \frac{x_i + x_j}{2} + b \right] \frac{1}{2a}$$

$$-\delta_v^2 - 2\delta_v (b - x_i) \left[ a - \frac{x_i + x_j}{2} + b \right] \frac{1}{2a} - 2\delta_v (b + x_j) \left[ a - \frac{x_i + x_j}{2} + b + a \right] \frac{1}{2a}$$

$$-\frac{1}{2a} \int_{-a}^{a} (\alpha - b - x_i)^2 d\alpha - \frac{1}{2a} \int_{-x_i - x_j}^{a} (\alpha - b - x_j)^2 d\alpha$$

$$-\frac{1}{2a} \int_{-x_i - x_j}^{a} (\alpha - b - x_j)^2 d\alpha - \frac{1}{2a} \int_{-a}^{a} (\alpha - b - x_j)^2 d\alpha$$

$$= -\delta_v^2 - 2\delta_v (b - x_i) \left[ a + \frac{x_i + x_j}{2} - b \right] \frac{1}{2a} - 2\delta_v (b - x_j) \left[ a + \frac{x_i + x_j}{2} + b \right] \frac{1}{2a}$$

$$-\delta_v^2 - 2\delta_v (b - x_i) \left[ a - \frac{x_i + x_j}{2} + b \right] \frac{1}{2a} - 2\delta_v (b + x_j) \left[ a - \frac{x_i + x_j}{2} + b + a \right] \frac{1}{2a}$$

where we used $\int_{-a}^{a} \alpha d\alpha = 0$. The case for pairs $i = j$ follows from analogous manipulations. This establishes that the voter’s welfare is just $\delta_v^2$ less than the welfare of the median voter ($\delta_v = 0$).
\[ + \frac{1}{a} \sum_{j \neq k} \sum_{b} P(k, j, b) \left( \frac{(x_j - x_k)}{2} \right)^2 - (b - a - x_k)^2 \].

Viewing the summands above as quadratic functions of the random variable \( b \), we use mean-variance analysis and the fact that \( E[b|j, k] = m_{j,k} \) to derive

\[
\frac{\partial W}{\partial x_k}(X) = \frac{1}{2a} P(k, k) \left[ (m_{k,k} + a - x_k)^2 - (m_{k,k} - a - x_k)^2 \right] + \frac{1}{a} \sum_{j \neq k} P(k, j) \left[ (m_{j,k} + a - x_k)^2 - \left( \frac{x_j - x_k}{2} \right)^2 + \sigma_{j,k}^2 \right] + \frac{1}{a} \sum_{j \neq k} P(k, j) \left[ \left( \frac{x_j - x_k}{2} \right)^2 - (m_{j,k} - a - x_k)^2 - \sigma_{j,k}^2 \right],
\]

where \( \sigma_{j,k}^2 \) is the variance of \( b \) conditional on signals \( j \) and \( k \). The cross partial with respect to \( x_i \) and \( x_j \) with \( j \neq i \) is then

\[
\frac{\partial^2 W}{\partial x_i \partial x_j}(X) = P(i, j) \frac{|x_i - x_j|}{2a},
\]

where we use monotonicity of strategies in signal. The second partial with respect to \( x_i \) is

\[
\frac{\partial^2 W}{\partial^2 x_i}(X) = \frac{1}{2a} P(i, i) \left[-2(a + m_{i,i} - x_i) + 2(m_{i,i} - x_i - a)\right] + \frac{1}{a} \sum_{j \neq i} P(i, j) \left[-2(a + m_{i,j} - x_i) + \frac{x_j - x_i}{2} \right] - \frac{1}{a} \sum_{j > i} P(i, j) \left[-2(a + m_{i,j} - x_i) + \frac{x_j - x_i}{2} \right] - \frac{1}{a} \sum_{j < i} P(i, j) \left[-2(a + m_{i,j} - x_i) + \frac{x_j - x_i}{2} \right].
\]

Therefore, we may decompose the Hessian of \( W \) into two matrices, \( H = D + E \), where \( E \) is a symmetric matrix such that

\[
e_{i,j} = P(i, j) \frac{|x_j - x_i|}{2} \text{ for } i \neq j \text{ and } e_{i,i} = -\sum_{j \neq i} P(i, j) \frac{|x_j - x_i|}{2},
\]

and \( D \) is a diagonal matrix such that

\[
d_{i,i} = \frac{1}{aP(i)} \left[-2a + \sum_{j < i} P(j|i)(m_{i,j} - x_i) - \sum_{j > i} P(j|i)(m_{i,j} - x_i) \right].
\]

Because \( x_i \) is bounded above by \( B \), it follows that

\[
d_{i,i} \leq \frac{1}{aP(i)} \cdot [-2a + B + B] < 0,
\]

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Furthermore, it has a unique maximizer, \( \hat{\phi} \). For simplicity, we denote \( F \) or \( \hat{\phi} \).

Proof of Theorem 7: Recall that
\[
\partial \hat{W} / \partial x_i (\hat{X}) = \partial W (\phi(x_1, \ldots, x_K)) / \partial x_i (\phi(\hat{X}^*)).
\]
and define the symmetrized welfare function \( \hat{W} / \partial x_i (\hat{X}) = \partial W (\phi(x_1, \ldots, x_K)). \)

For simplicity, we denote \( K \)-tuples by \( \hat{X} = (x_1, \ldots, x_K) \); in particular, the \( K \)-tuple of conditional medians is \( \hat{M} = (m_{1,1}, \ldots, m_{K,K}) \). By Proposition 6 and linearity of \( \phi \), \( \hat{W} \) is strictly concave. Furthermore, it has a unique maximizer, \( \hat{X}^* \), and by the above argument it follows that \( X^* = \phi(\hat{X}^*) \).

Note that, by the chain rule, for all signals \( i > 0 \) and all \( \hat{X} \), we have
\[
\partial \hat{W} / \partial x_i (\hat{X}) = \partial W / \partial x_i (\phi(\hat{X})) = \partial W / \partial x_i (\phi(\hat{X})).
\]

Furthermore, for all \( j \neq i \), we have
\[
\partial^2 \hat{W} / \partial x_i \partial x_j (\hat{X}) = 2 \left[ \partial^2 W / \partial x_i \partial x_j (\phi(\hat{X})) - \partial^2 W / \partial x_i \partial x_j (\phi(\hat{X})) \right]
\]
\[
= \frac{1}{a} \left[ |P(i,j)| \left| x_i - x_j \right| - P(i,-j)(x_i + x_j) \right],
\]
where the second equality uses (30). Consider the welfare maximization problem with the additional constraint that candidates locate at or above the conditional medians corresponding to all signals:

\[
\max_{\hat{X}} \hat{W}(\hat{X}) \quad \text{s.t. } x_i \geq m_{i,i} \text{ for all } i > 0. \tag{33}
\]

Because the domain of this problem is convex, it has a unique solution, say $\hat{X}'$. We seek to show that at this solution, the constraints in (33) are slack, i.e., $x'_i > m_{i,i}$ for all $i > 0$. Then we have $x_i = x'_i > m_{i,i}$ for all $i > 0$, as required. To this end, suppose $x'_i = m_{i,i}$ for some $i > 0$, where without loss of generality we take $i$ to be the lowest such signal. Therefore, we must have

\[
\frac{\partial \hat{W}}{\partial x_i}(\hat{X}') \leq 0. \tag{34}
\]

Consider any $\hat{X}$ such that $x_j = x'_j$ for all $j \leq i$ and $x_j \geq m_{j,j}$ for all $j > i$. Given signal $j > i$, from (32), we see that

\[
\frac{\partial^2 \hat{W}}{\partial x_i \partial x_j}(\hat{X}) > 0 \tag{35}
\]

if and only if

\[
\frac{P(i,j)}{P(i,-j)} > \frac{m_{i,i} + x_j}{x_j - m_{i,i}}.
\]

Differentiating the right-hand side of the latter inequality, we see that it is decreasing in $x_j$, and so it is maximized over $x_j \geq m_{j,j}$ at $x_j = m_{j,j}$. Thus, (35) follows from assumption (A7). Now define $\hat{X}''$ so that $x''_j = x'_j$ for all $j \leq i$ and $x''_j = m_{j,j}$ for all $j > i$, in effect just decreasing candidate positions following signals $j > i$ to their conditional medians. Then (34) and (35) imply

\[
\frac{\partial \hat{W}}{\partial x_i}(\hat{X}'') \leq 0. \tag{36}
\]

This contradicts assumption (A6) immediately if $i = 1$, so assume $i > 1$, and note that $x''_j > m_{j,j}$ for all signals $j < i$. Let $I \subseteq \{1, \ldots, i-1\}$ be any subset of signals less than $i$, and define $\hat{X}^I$ so that

\[
x'_j = \begin{cases} m_{i,i} & \text{if } j < i \text{ and } j \in I \\ m_{j,j} & \text{else.} \end{cases}
\]

That is, for the subset of signals in $I$, we move candidate positions up to $m_{i,i}$; after all other signal realizations, we position candidates at their conditional medians. Note that $\hat{X}''$ is a convex combination of such vectors:

$$\hat{X}'' \in \text{conv}\{\hat{X}^I \mid I \subseteq \{1, \ldots, i-1\}\}.$$ 

Note also that $\hat{X}^0 = \hat{M}$, and that, by assumption (A6), $\frac{\partial \hat{W}}{\partial x_i}(\hat{X}^0) > 0$. More generally, we have

\[
\frac{\partial \hat{W}}{\partial x_i}(\hat{X}^I) - \frac{\partial \hat{W}}{\partial x_i}(\hat{X}^0)
\]

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where the inequality follows from assumption (A7). Therefore, \( \frac{\partial W}{\partial x_i}(\hat{X}^I) > 0 \) for all subsets \( I \). The expression (29) shows that the partial derivative of \( W \) with respect to \( x_i \) is a concave (weighted) quadratic function of positions \((x_1, \ldots, x_{i-1})\), and therefore the set

\[
Z = \left\{ (x_1, \ldots, x_{i-1}, m_{i,i}, \ldots, m_{K,K}) | \frac{\partial W}{\partial x_i}(x_1, \ldots, x_{i-1}, m_{i,i}, \ldots, m_{K,K}) > 0 \right\}
\]

is convex. In particular, since \( \hat{X}^I \in Z \) for all \( I \), and since \( \hat{X}'' \) is a convex combination of the \( \hat{X}^I \), we have \( \hat{X}'' \in Z \), i.e., \( \frac{\partial W}{\partial x_i}(\hat{X}'') > 0 \), contradicting (36). We conclude that \( x_i' > m_{i,i} \) for all signals \( i > 0 \), as required.

**Proof of Theorem 8:** We write \( W(X; q) \) to bring out the dependence of welfare on correlation given locations \( X \), and we write \( m_{i,i}(q) \) to bring out the dependence of the conditional median on correlation. The effect of an increase in correlation \( q \) may be decomposed as follows:

\[
\frac{dW}{dq}(M; q) = \frac{\partial W}{\partial q}(M; q) + \sum_i \frac{\partial W}{\partial x_i}(M; q) \frac{dm_{i,i}}{dq}(q)
\]

\[
= \frac{\partial W}{\partial q}(M; q) + \sum_{i=1}^K \left[ \frac{\partial W}{\partial x_i}(M; q) \frac{dm_{i,i}}{dq}(q) + \frac{\partial W}{\partial x_{-i}}(M; q) \frac{dm_{i,-i}}{dq}(q) \right]
\]

\[
= \frac{\partial W}{\partial q}(M; q) + 2 \sum_{i=1}^K \frac{\partial W}{\partial x_i}(M; q) \frac{dm_{i,i}}{dq}(q),
\]

where the third equality follows from symmetry about zero. By Lemma 1 and assumption (A6), we know that \( \frac{\partial W}{\partial x_i}(M; q) > 0 \) for all signals \( i > 0 \). And Proposition 2 delivers \( \frac{dm_{i,i}}{dq}(q) < 0 \) for all \( i > 0 \). To see that \( \frac{dW}{dq}(M; q) < 0 \), note that

\[
W(M; q) = -\sum_b \sum_i |qP(i|b) + (1-q) P(i|b) P(i|b)| P(b) \int_a^b \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha
\]

\[
- \sum_b \sum_{i,j<i} (1-q) P(i|b) P(j|b) P(b) \left[ \int_{-a}^{[m_{ii}+m_{jj}]/2-b} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \right]
\]

\[
+ \int_{[m_{ii}+m_{jj}]/2-b}^a \frac{(\alpha + b - m_{ij})^2}{2a} d\alpha
\]

\[
- \sum_b \sum_{i,j>i} (1-q) P(i|b) P(j|b) P(b) \left[ \int_{-a}^{[m_{ii}+m_{jj}]/2-b} \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha \right]
\]

\[
+ \int_{[m_{ii}+m_{jj}]/2-b}^a \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha.
\]
Therefore, 
\[
\frac{\partial W}{\partial q}(M; q) = - \sum_{b} \sum_{i} P(i|b)(1 - P(i|b))P(b) \int_{-a}^{a} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \\
+ \sum_{b} \sum_{i} \sum_{j > i} P(i|b)P(j|b)P(b) \left[ \int_{-a}^{[m_{ii} + m_{jj}]/2 - b} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \\
+ \int_{[m_{ii} + m_{jj}]/2 - b}^{a} \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha \right] \\
+ \sum_{b} \sum_{i} \sum_{j < i} P(i|b)P(j|b)P(b) \left[ \int_{-a}^{[m_{ii} + m_{jj}]/2 - b} \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha \\
+ \int_{[m_{ii} + m_{jj}]/2 - b}^{a} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \right].
\]

Given signals \(i\) and \(j > i\), note that \(\frac{m_{ii} + m_{jj}}{2} - b < \alpha\) implies \((\alpha + b - m_{jj})^2 > (\alpha + b - m_{ii})^2\). Thus, we have
\[
- \int_{-a}^{a} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \leq - \int_{-a}^{[m_{ii} + m_{jj}]/2 - b} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha - \int_{[m_{ii} + m_{jj}]/2 - b}^{a} \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha.
\]
Similarly, given signals \(i\) and \(j < i\), \(\alpha < \frac{m_{ii} + m_{jj}}{2} - b\) implies \((\alpha + b - m_{jj})^2 < (\alpha + b - m_{ii})^2\). Thus, we have
\[
- \int_{-a}^{a} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \leq - \int_{-a}^{[m_{ii} + m_{jj}]/2 - b} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha - \int_{[m_{ii} + m_{jj}]/2 - b}^{a} \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha.
\]
Finally, we conclude that \(\frac{\partial W}{\partial q}(M; q) \leq 0\), which delivers the desired result. 

**Proof of Theorem 9:** For \(\epsilon > 0\) sufficiently small, it is straightforward to show that all equilibria are in pure strategies: For example, suppose \(z^\ast = \inf \text{supp}\, G_i^\ast \leq m_{ii}^\ast\) in equilibrium for arbitrarily small \(\epsilon\); then platforms close to \(z^\ast\) lose with probability close to 1 – \(\epsilon\), but the probability of winning when locating at \(m_{ii}^\ast\) is close to \(\frac{1 - \epsilon}{2}\). Then Theorem 1 implies that \(x_i = m_{ii}^\ast\) for all signals \(i\). Further, for \(b = i\), we have \(m_{ii}^\ast \rightarrow m_{ii} = b\) as \(\epsilon\) goes to zero. For signals \(i\) and \(j\) and realization \(b\), define
\[
W(i, j|b) = \frac{1}{2a} \int_{-a}^{[m_{ii} + m_{jj}]/2} (\alpha + b - m_{ii})^2 d\alpha - \frac{1}{2a} \int_{[m_{ii} + m_{jj}]/2}^{a} (\alpha + b - m_{jj})^2 d\alpha
\]
for \(i < j\), with a similar definition for \(i > j\), and
\[
W(i, i|b) = - \frac{1}{2a} \int_{-a}^{a} (\alpha + b - m_{ii})^2 d\alpha,
\]
when the candidates receive the same signals. We use the same conventions for the notation \(W^\epsilon(i, j|b)\), substituting \(m_{ii}^\ast\) and \(m_{jj}^\ast\) where appropriate. By assumption, we have
\[
\lim_{\epsilon \rightarrow 0} \frac{\sum_{j \neq b} P^\epsilon(j, b|b)}{\epsilon} = 1 \quad \text{and} \quad \lim_{\epsilon \rightarrow 0} \frac{\sum_{i, j \neq b} P^\epsilon(i, j|b)}{\epsilon} = 0.
\]
Then welfare conditional on realization \( b \) under perfect precision is \( W(b) = W(b, b|b) \), and with slightly noisy signals it is

\[
W^\epsilon(b) = P^\epsilon(b, b|b)W^\epsilon(b, b|b) + \sum_{j \neq b} P^\epsilon(j, b|b)W^\epsilon(j, b|b) + \sum_{i, j \neq b} P^\epsilon(i, j|b)W^\epsilon(i, j|b).
\]

Therefore,

\[
\frac{W^\epsilon(b) - W(b)}{\epsilon} \geq \frac{P^\epsilon(b, b|b)}{\epsilon}[W^\epsilon(b, b|b) - W(b, b|b)] + \sum_{j \neq b} \frac{P^\epsilon(j, b|b)}{\epsilon}\epsilon_{\min}[W^\epsilon(j, b|b) - W(b, b|b)] + D^\epsilon,
\]

where \( D^\epsilon \) goes to zero with \( \epsilon \). Using the fact that \( m_{b,b} = b \), note that

\[
W^\epsilon(b, b|b) - W(b, b|b) = -\frac{1}{2}\frac{2}{a} \int_{-a}^{a} (\alpha + b - m_{b,b}^\epsilon)^2 d\alpha + \frac{1}{2a} \int_{-a}^{a} \alpha^2 d\alpha
\]

\[
= -\frac{1}{2}\frac{2}{a} \int_{-a}^{a} (\alpha + b - m_{b,b}^\epsilon)^2 - (\alpha + b - m_{b,b})^2 d\alpha
\]

\[
= -\frac{1}{2}\frac{2}{a} \int_{-a}^{a} [- (m_{b,b} - m_{b,b}^\epsilon)(2b + 2\alpha - m_{b,b} - m_{b,b}^\epsilon)] d\alpha
\]

\[
= (m_{b,b} - m_{b,b}^\epsilon)(2b - m_{b,b} - m_{b,b}^\epsilon).
\]

Further,

\[
m_{b,b} - m_{b,b}^\epsilon = b - \frac{\sum_{b'} b'P^\epsilon(b, b|b') P(b')}{\sum_{b''} P^\epsilon(b, b|b'') P(b'')}
\]

\[
= b - \frac{bP^\epsilon(b, b|b) + \sum_{b' \neq b} b'P^\epsilon(b, b|b')}{P^\epsilon(b, b|b) + \sum_{b'' \neq b} P^\epsilon(b, b|b'')}.
\]

Hence,

\[
\frac{m_{b,b} - m_{b,b}^\epsilon}{\epsilon} \leq \frac{b}{\epsilon} \frac{\sum_{b'' \neq b} P^\epsilon(b, b|b'')}{P^\epsilon(b, b|b)}
\]

which goes to zero with \( \epsilon \) by assumption. Finally, without loss of generality, let \( \min_{j \neq b}[W^\epsilon(j, b|b) - W(b, b|b)] \) be achieved at \( k > b \) as \( \epsilon \) goes to zero. Then

\[
\frac{\min_{j \neq b}[W^\epsilon(j, b|b) - W(b, b|b)]}{\epsilon} = \int_{-a}^{\left[m_{b,b}^\epsilon + \sqrt{m_{b,b}^\epsilon + m_{k,b}^\epsilon}\right]/2} (\alpha + b - m_{b,b}^\epsilon)^2 - \alpha^2 d\alpha + \int_{\left[m_{b,b}^\epsilon + \sqrt{m_{b,b}^\epsilon + m_{k,b}^\epsilon}\right]/2}^{a} (\alpha + b - m_{b,b}^\epsilon)^2 - \alpha^2 d\alpha
\]

has limit

\[
\int_{[k+b]/2}^{a} (\alpha + b - k)^2 - \alpha^2 d\alpha > 0.
\]

Therefore,

\[
\lim_{\epsilon \to 0} \frac{W^\epsilon(b) - W(b)}{\epsilon} \geq \int_{[k+b]/2}^{a} (\alpha + b - k)^2 - \alpha^2 d\alpha > 0.
\]

Since this is true for all \( b \), the result follows.
References


