Math Help

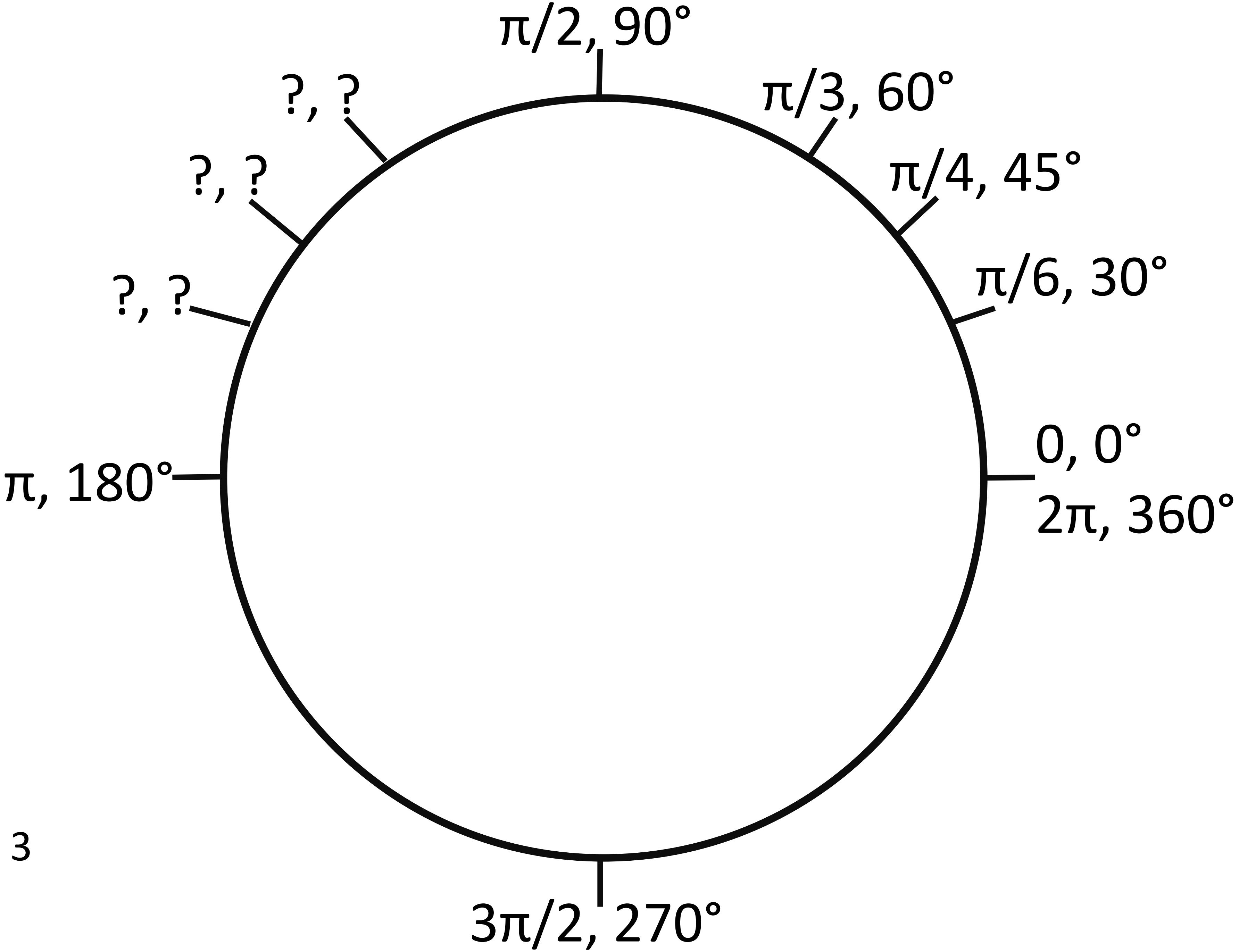
Trigonometry

Topics, Unit 3

- How to describe angles: Radians and degrees
- Arc distance
- Trigonometric functions and their values
- Trigonometric identities
- Inverse trigonometric functions and their identities
- Solving equations involving trigonometric functions

Angles

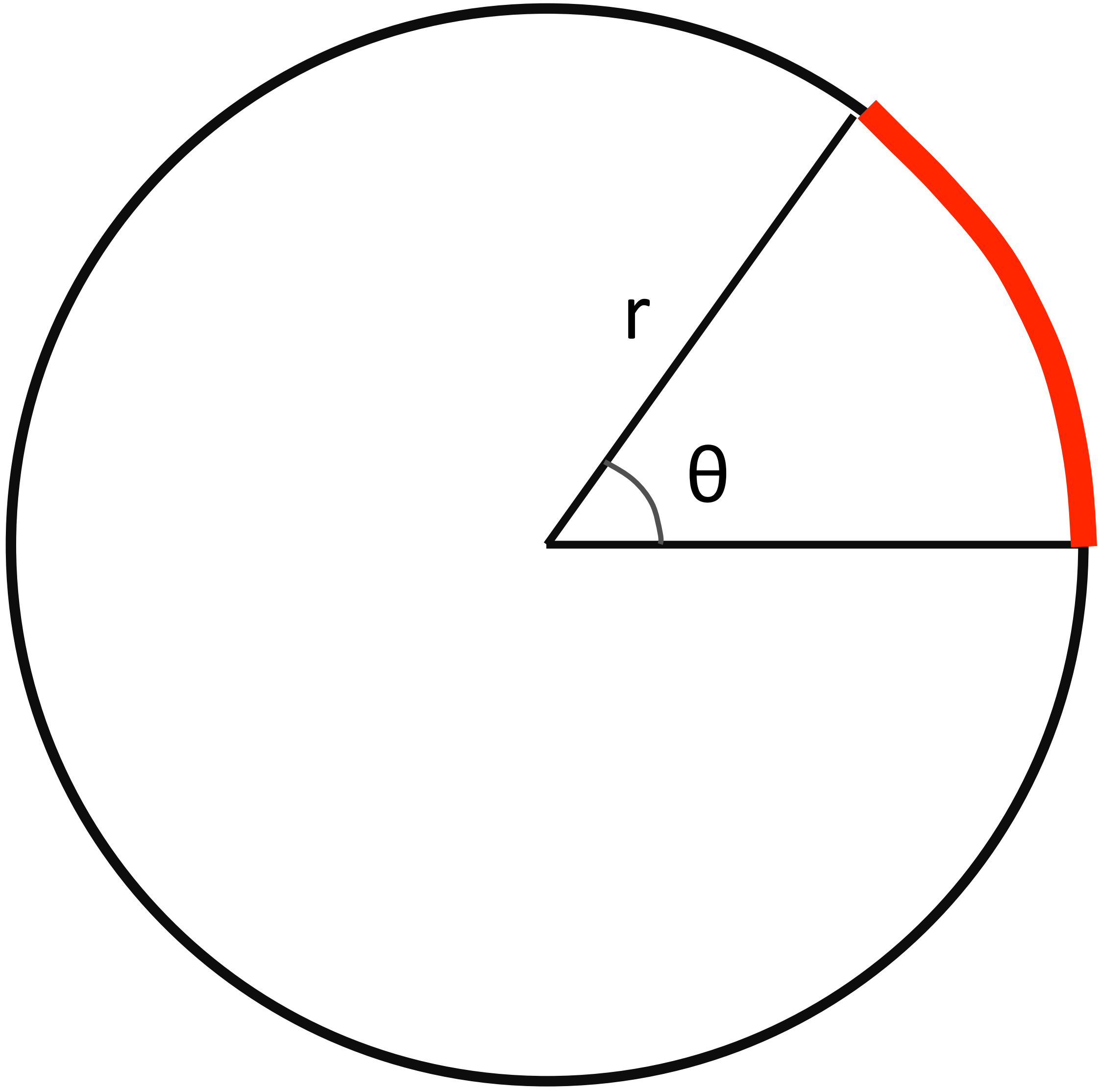
- Unit circle
- Radians
- Degrees



See Webwork Unit 3
#1, 3.

Arc distance

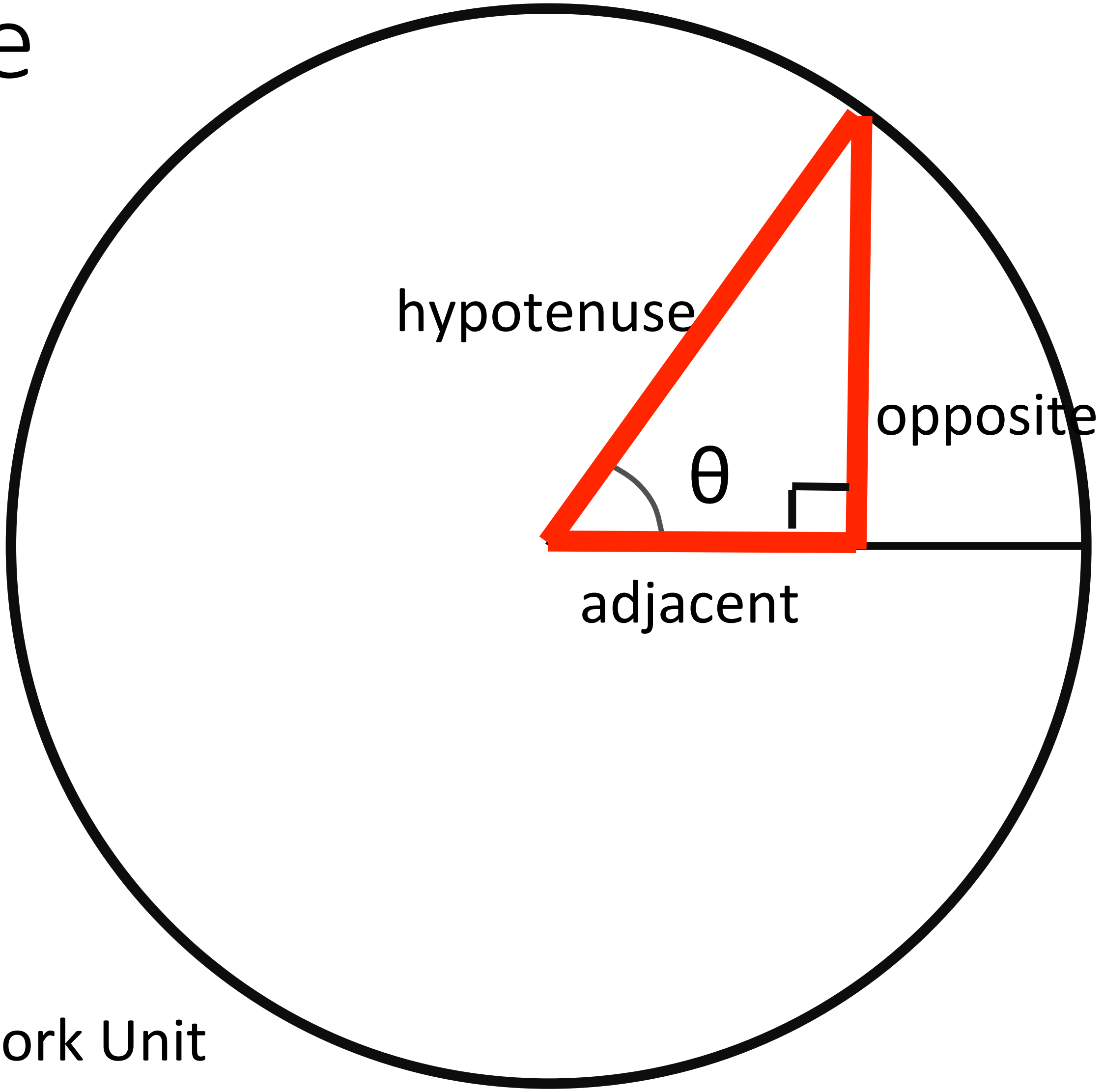
- Definition of circumference
  - $C = 2\pi r$
- Arc = Fraction of unit circle
  - $\frac{Arc}{C} = \frac{\theta}{2\pi}$
  - $Arc = \theta r$



See Webwork
Unit 3 #2.

Turning it into a triangle

- Start with unit circle
- Define trig functions
  - Sine
    - sin(θ) = opposite/hypotenuse
  - Cosine
    - cos(θ) = adjacent/hypotenuse
  - Tangent
    - tan(θ) = opposite/adjacent
    - tan(θ) = sin(θ)/ cos(θ)
  - Cotangent
    - cot(θ) = 1/ tan(θ)
  - Cosecant
    - csc(θ) = 1/ sin(θ)
  - Secant
    - sec(θ) = 1/ cos(θ)



See Webwork Unit 3 #15.

Changing the angle

- Start with unit circle
- Define trig functions
  - Sine
    - sin(θ) = opposite/hypotenuse
  - Cosine
    - cos(θ) = adjacent/hypotenuse
  - Tangent
    - tan(θ) = opposite/adjacent
    - tan(θ) = sin(θ)/ cos(θ)
  - Cotangent
    - cot(θ) = 1/ tan(θ)
  - Cosecant
    - csc(θ) = 1/ sin(θ)
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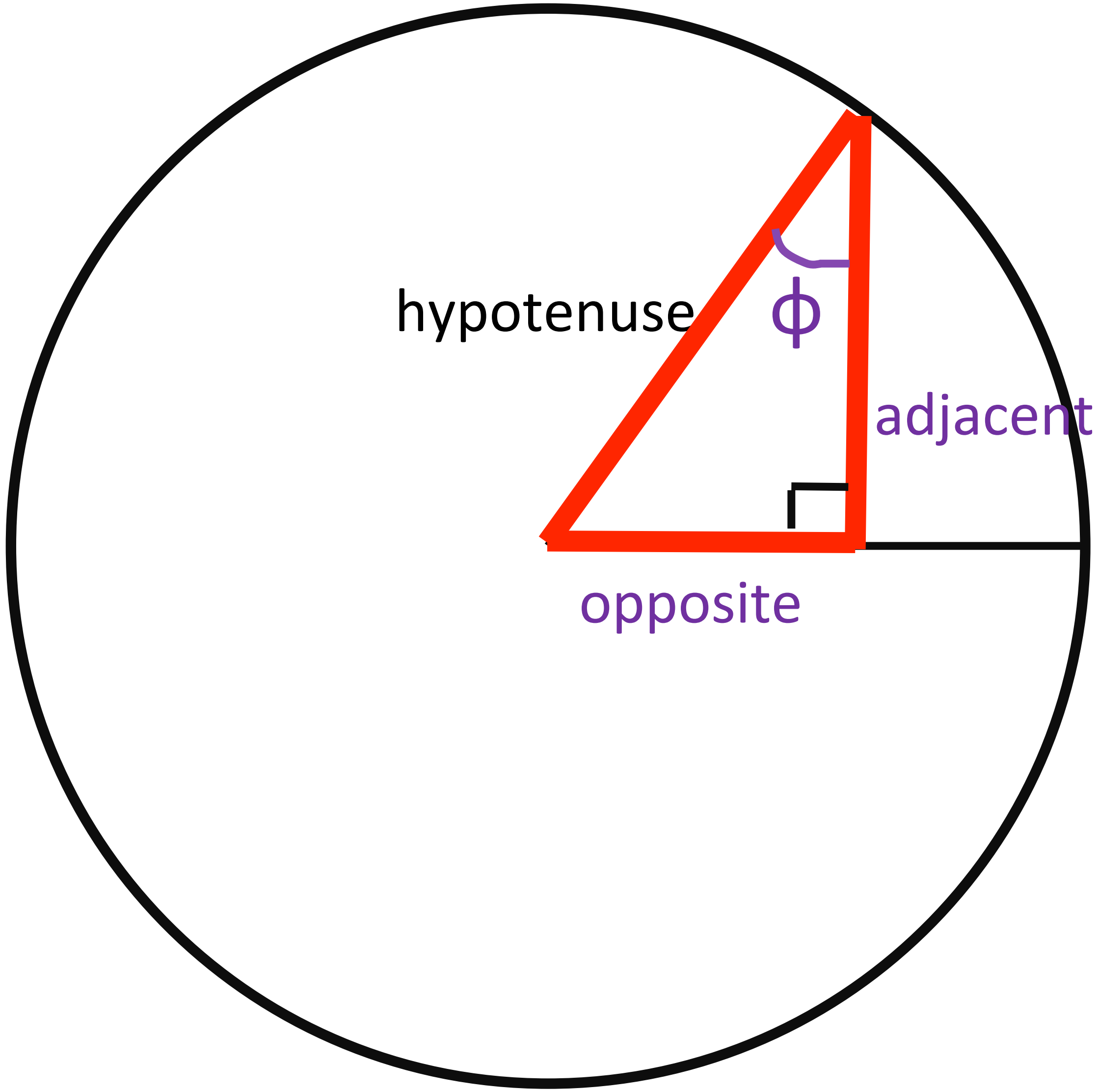
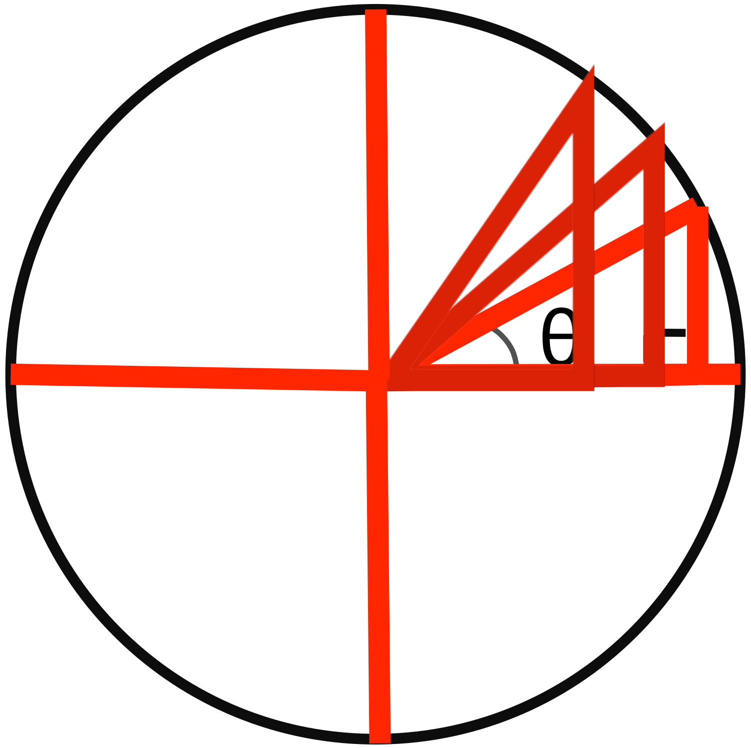


Some landmark values of trig functions

Radians	Degrees	Sin	Cos	Tan
0	0°	0	1	0
π/6	30°	$^1/_2$	$^{\sqrt{3}}/_2$	$^1/_{\sqrt{3}}$
π/4	45°	$^{\sqrt{2}}/_2$	$^{\sqrt{2}}/_2$	?
π/3	60°	?	?	$\sqrt{3}$
π/2	90°	?	?	Undefined
π	180°	?	?	?
3π/2	270°	?	?	?
2π	360°	?	?	?



Trig identities – Half angle formulas

- $\sin(A/2) = [(1 - \cos(A))/2]^{1/2}$
- $\cos(A/2) = [(1 + \cos(A))/2]^{1/2}$
- $\tan(A/2) = ?$
  - Can you simplify this into a form without a radical?
- *Test it out*
  - Find $\sin(\pi/6)$, $\cos(\pi/6)$, and $\tan(\pi/6)$ by evaluating this functions for $A = \pi/3$. Does your answer make sense?

See WebworkUnit 3 #8, 12.

Trig identities – Addition formulas

- $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
- $\cos(A \pm B) = \cos(A)\cos(B) \pm [-\sin(A)\sin(B)]$
- $\tan(A \pm B) = ?$
  - Can you simplify this into a form using only tan(A) and tan(B)?
- *Test it out*
  - Find sin(π/2), cos(π/2), and tan(π/2) by adding π/3 and π/6. Does your answer make sense?

See Webwork Unit 3 #9, 12,
13, 20.

Trig identities – Double-angle formulas

- Can you derive these using the addition formulas?
- sin(2A) = ?
- cos(2A) = ?
- tan(2A) = ?
- *Test it out*
    - Find sin(π/3), cos(π/3), and tan(π/3) using these functions for π/6. Does your answer make sense?

Trig identities – Product to sum formulas

- sin(A)sin(B) = ½[cos(A-B) – cos(A+B)]
- cos(A)cos(B) = ½[cos(A-B) + cos(A+B)]
- sin(A)cos(B) = ½[sin(A-B) + sin(A+B)]
- *Test it out*
  - What values could you use to test this?
  - Try A = B = π/4

See Webwork Unit 3 #14.

Trig identities – Things that equal 1

- $\sin^2(A) + \cos^2(A) = 1$
- $\sec^2(A) - \tan^2(A) = 1$
- $\csc^2(A) - \cot^2(A) = 1$
- Can you start with the first identity and derive the other two?

See Webwork Unit 3 #11, 16.

Inverse trig functions

- $\sin^{-1}[\sin(\theta)] = \theta$
  - What type of number is $\sin(\theta)$?
  - When would you do this -> $\sin^{-1}(\theta)$? **NEVER!**
- $\cos^{-1}[\cos(\theta)] = \theta$
- $\tan^{-1}[\tan(\theta)] = \theta$
- $\sin^{-1}[\cos(\theta)]$ = something ugly

See Webwork Unit 3 #5, 6, 7.

Pulling it all together: Solving equations

- $1 - \sin^2(x) = 1/4$
- $\cos^2(x) = 1/4$
- $\cos(x) = 1/2$
- $x = \cos^{-1}(1/2)$
- $x = \pi/3$
  - *But is it true for any other values?*
- $x = \pi/3 + 2\pi m$ *or* $5\pi/3 + 2\pi n$, where m and n are integers

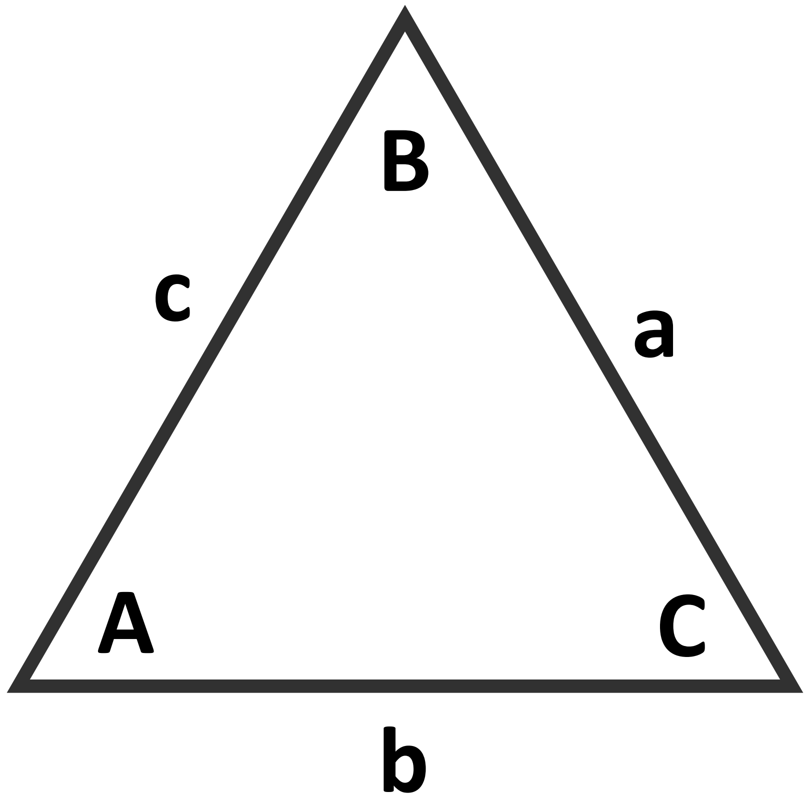
See Webwork Unit 3 #17, 18, 19.

Topics, Unit 4

- Notation for angles and sides of triangles
- Pythagorean theorem
- Review sine, cosine, tangent
- Law of Sines and Law of Cosines
- Trigonometry to find triangle sizes and angles
- Some tricks for using trigonometry

Common notation for triangles

- Angles: A,B,C
- Sides: a,b,c
  - Letters match opposite angles.



Pythagorean Theorem

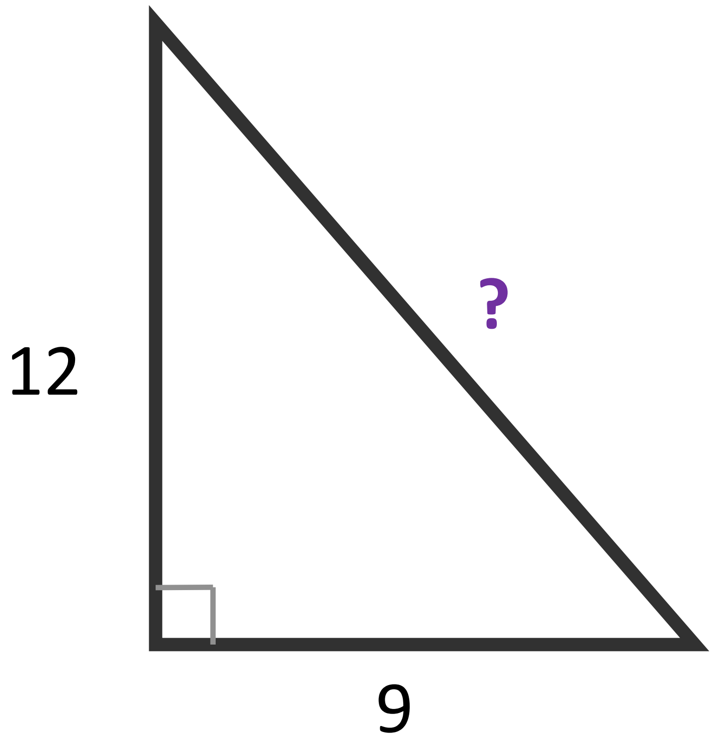
- Conditions
  - Triangle
  - One of the angles (we' ll call it C) must be π/2 or 90°

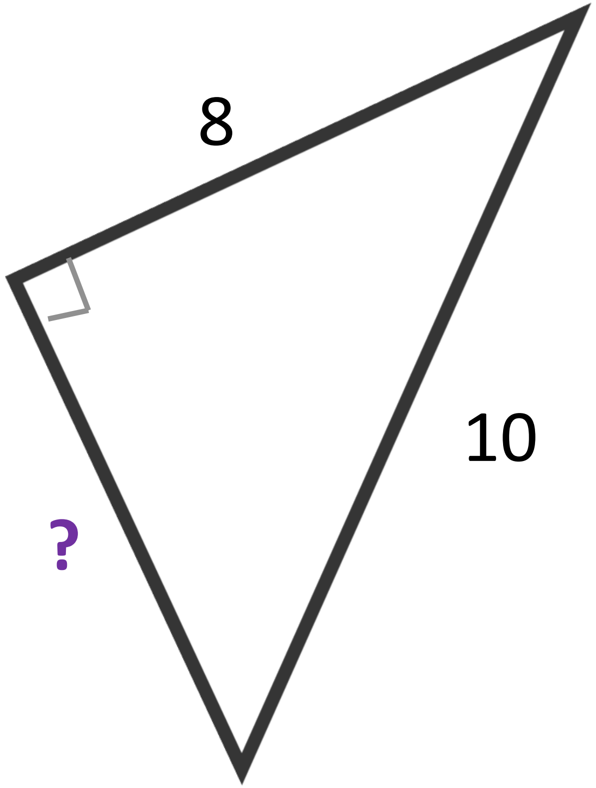
$$a^2 + b^2 = c^2$$

- Applications
  - If you have two sides, then you can find the third.

Using the Pythagorean Theorem

- Find the length of the missing side for each triangle (Drawings not to scale).





See Webwork #14, 22.

Review of trig functions

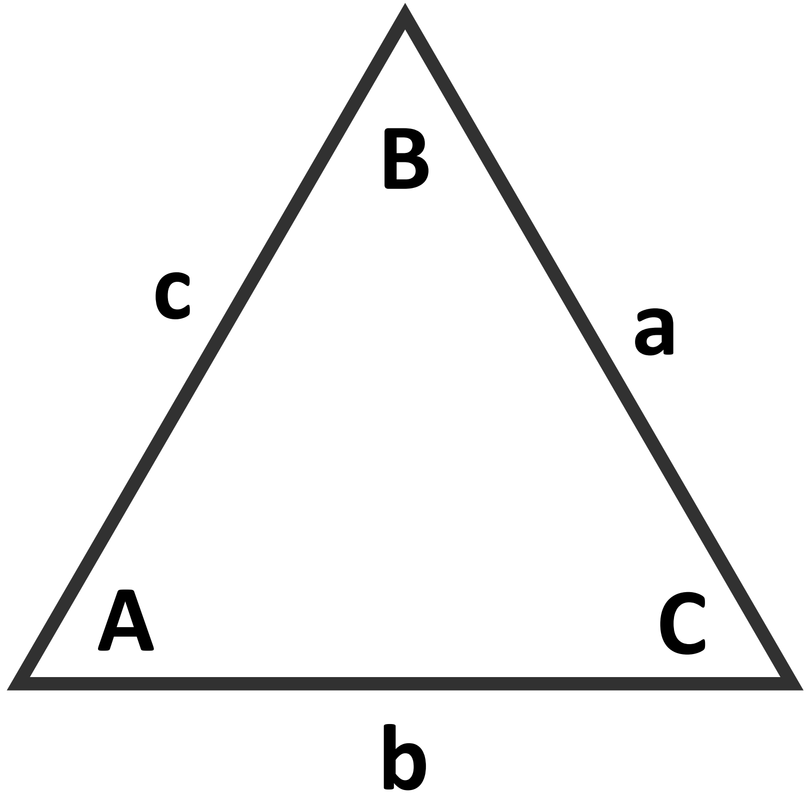
- Sine
  - $\sin(\theta)$ = opposite/hypotenuse
- Cosine
  - $\cos(\theta)$ = adjacent/hypotenuse
- Tangent
  - $\tan(\theta)$ = opposite/adjacent
  - $\tan(\theta) = \sin(\theta)/ \cos(\theta)$

See Webwork Unit 4 #6, 9, 10.

Law of sines

- $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b} = \dfrac{\sin(C)}{c}$

- *"As the angle increases, so does the length of the opposite side."*
- Does not have to be a right triangle.

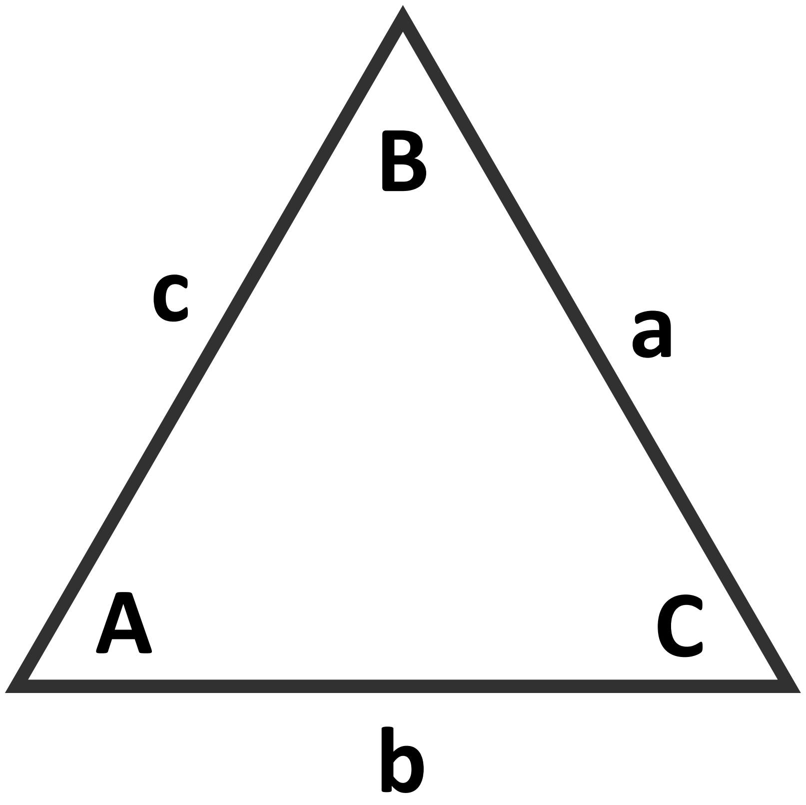


See Webwork Unit 4 #2, 4.

Law of cosines

- $c^2 = a^2 + b^2 - 2ab * \cos(C)$

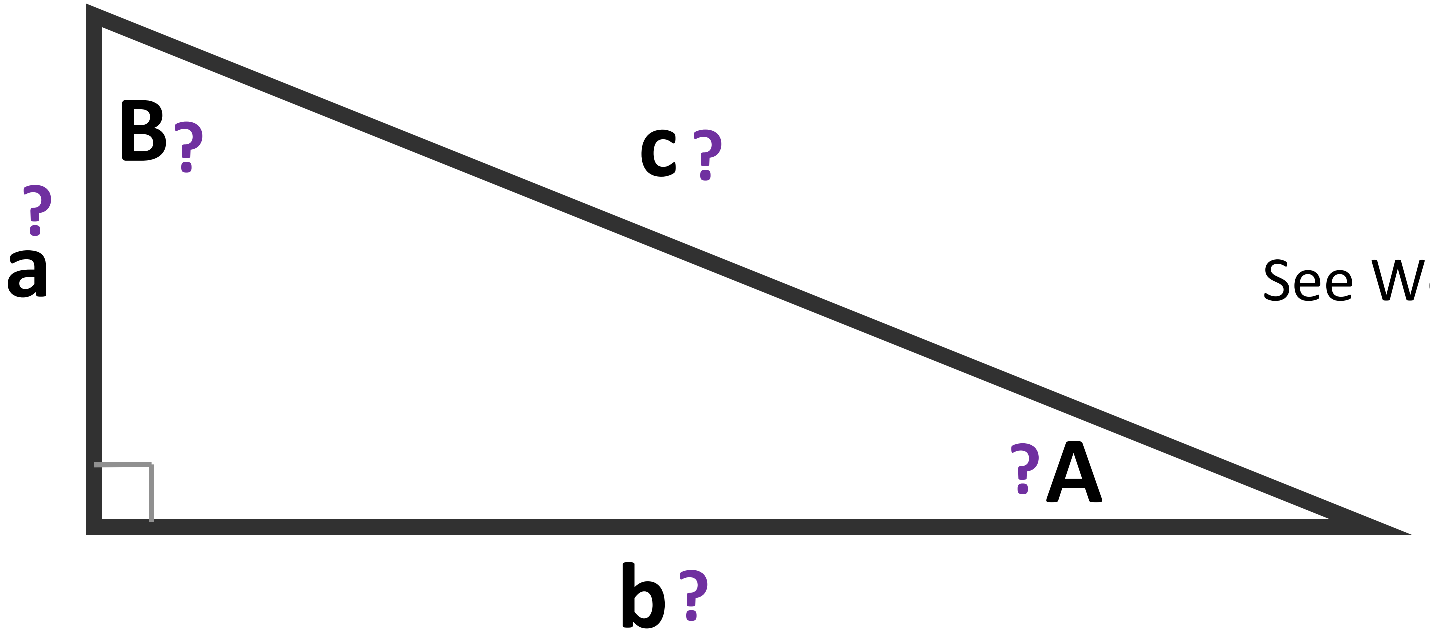
- Does not have to be a right triangle.
- …but what happens when $C = \pi/2$?



See Webwork Unit 4
#20.

Using all of our tools

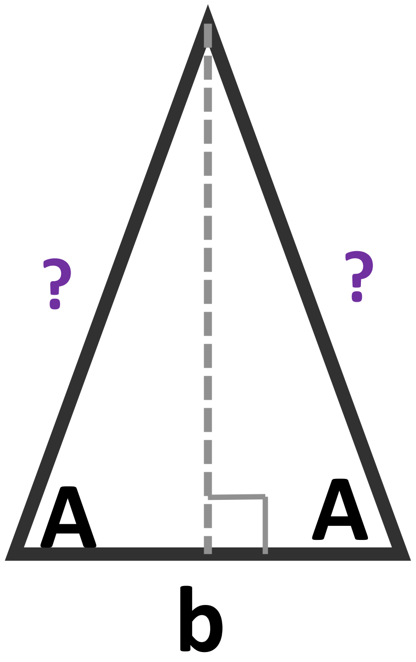
- Finding the missing side or angle
- If you have 3, you usually have the other 3
  - Especially if one is a 90* angle

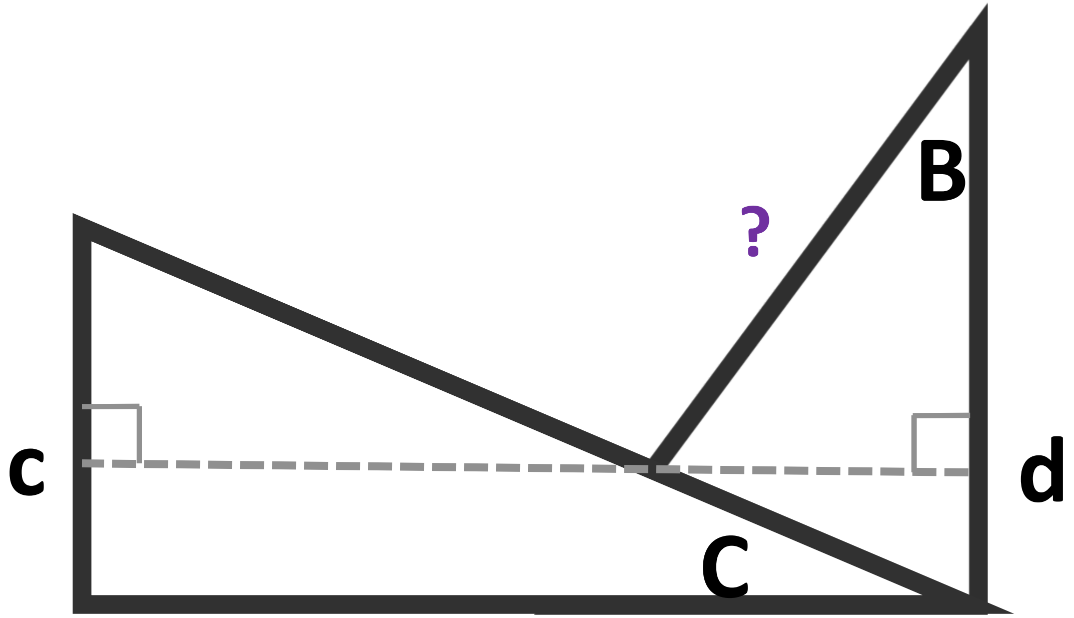


See Webwork #7, 24.

Some more tricks

- If you don’t have a right triangle, make one





See Webwork Unit 4 #3, 17, 23.

Good trig site with problems and solutions

- http://www.analyzemath.com/Trigonometry