

# Unit 1 Part 2

Properties of Equations  
Single Variable

# Properties

For more information on properties, see

- <http://www.purplemath.com/modules/numbprop.htm>
- <http://www.purplemath.com/modules/negative4.htm>

# Algebra Properties

- Commutativity
  - $a+b = b+a$
- Associativity
  - $a+(b+c) = (a+b)+c$
- Distribution
  - $a(b+c) = ab + cd$

# Exponential Properties

- $a^0 = 1$
- $a^1 = a$
- $a^x * a^y = a^{x+y}$
- $(a^x)^y = a^{xy}$
- $a^{-x} = 1/a^x$
- $a^{1/2} = \sqrt{a}$
- $a^x = (-a)^x$  if  $x$  is even
- $\ln(e^x) = x$

# Single Variable Equations

- For more information see
- <http://www.purplemath.com/modules/evaluate.htm>
- <http://www.purplemath.com/modules/quadform.htm>

# Steps to Solving First Order Equations

- Clear Parenthesis
- Clear Exponents
- Combine Like Terms
- Clear Constants
- Clear Coefficient
- Check by Substitution

# Clear Parenthesis

- Distributive Property

- Ex.  $2(x - 2) + 5 + x = 4^2$

- Multiply each term in parenthesis by the coefficient

- $2(x-2) = (2*x) - (2*2) = 2x - 4$

- Remaining Equation:  $2x - 4 + 5 + x = 4^2$

# Clear Exponent

- Simplify terms with exponents
- $2x - 4 + 5 + x = 4^2$
- $4^2 = 16$
  
- Remaining equation
- $2x - 4 + 5 + x = 16$



# Combine Like Terms

- Group constant terms together
  - $2x - 4 + 5 + x = 16 \Rightarrow 2x + 1 + x = 16$
- Group variable terms together
  - $2x + 1 + x = 16 \Rightarrow 3x + 1 = 16$

# Clear Constant

- Manipulate equation so variables and constants are on opposite sides of the equation
  - $3x + 1 = 16$  is the same as
  - $3x + 1 - 1 = 16 - 1$
- Recombine Terms
  - $3x = 15$

# Clear Coefficient

- Divide both sides of the equation by the coefficient in front of the variable
  - $3x = 15$
  - $3x/3 = 15/3$
  - $x = 5$

# Substitute and check

- Plug the value for  $x$  back into the original equation and solve using order of operations
  - $2(x - 2) + 5 + x = 4^2$
  - $2((5) - 2) + 5 + (5) = 4^2$
  - $2(3) + 5 + 5 = 4^2$
  - $6 + 5 + 5 = 4^2$
  - $16 = 4^2$
  - $16 = 16$
  - So 5 is the correct answer!

# Second Order Equations

- Standard Form  $ax^2 + bx + c = 0$
- $a, b, c$  are constants
- Combine terms to get equation into standard form

# Simple Problems

- Case:  $b = 0$
- $ax^2 + (\mathbf{0})x + c = 0$
- $ax^2 + c = 0$
- Solve for  $x^2$ , then take square root of both sides to solve for  $x$
- Two answers
  - Positive **and** negative root
    - $(-x)^2 = x^2$

# Example

- $x^2 - 9 = 0$
- $x^2 = 9$
- $x = 3$  and  $-3$

# Factoring

- As discussed in Unit 1 Part 1, factoring can also be used to solve higher-order single-variable equations
- What if the expression cannot be factored?



# Quadratic Formula

- Given  $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Just plug in the coefficients and solve!

# Example

- $x^2 + 6x - 3 = 0$
- Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{48}}{2}$$

$$x = -3 - 2\sqrt{3} \quad x = -3 + 2\sqrt{3}$$