Unit 1 Part 1

Order of Operations

Factoring

Fractions

Order of Operations

For more information and practice see

http://www.purplemath.com/modules/orderops.htm

Introduction

- Evaluate the expression: 2+3*4 = ?
- Some might say 20. Others will say 14.
- Answer depends on whether you add or multiply first.
- Are there rules for the order?
- YES!

Operation Order

- Remember: PEMDAS

1) Parenthesis
2) Exponents
3) Multiplication

   Division
4) Addition

   Subtraction

What is 2+3*4 Really?

- Are there parenthesis?
  - No
- Are there exponents?
  - No
- Is there multiplication/division?
  - YES! 3*4 = 12
    - Equation Becomes 2+12
- Finally add/subtract
    - 2+12 = 14!

A More Complicated Example

$$(3(3 + 2)^2 – 10/2) – 25$$

- Parenthesis
  - Notice 2 sets (3+2) and $(3(3 + 2)^2 – 10/2)$
  - Must do PEMDAS for each set of parenthesis then solve equation as a whole
  - Start with parenthesis that contain no other sets of parenthesis
  - 3 + 2 = 5 so equation becomes $(3(5)^2 – 10/2) – 25$
- Exponents
  - $5^2 = 25$ so equation becomes (3(25) – 10/2) – 25
- Multiplication/Division
  - 3*25 = 75 and 10/2 = 5 so equation becomes (75 – 5) – 25
- Addition/Subtraction
  - (75 – 5) = 70 so equation becomes 70 – 25
  - 70 – 25 = 45
- Answer: 45

Factoring

For more information on factoring see:

http://www.purplemath.com/modules/factquad.htm

What is Factoring

- m is a factor of n if n/m has remainder 0
  - Ex) 4 is a factor of 12 because 12/4 = 3 with no remainder
    - *Note that by definition, 3 is also a factor of 12*
- Theorem 1
  - If any of the factors of an expression are 0, the expression itself must be 0
    - x = 4*0*(-2) implies that x = 0
- Theorem 2
  - If each monomial in a polynomial shares a common factor, that factor can be taken pulled out of each term
    - 4x – 2 = 2*2*x – 2 = 2(2x – 1)

Solving Equations with Factoring

- Lets look at an example
  - $ax^2 + bx = 0$
- We can use the previous theorems to solve the equation.
- Note that $x^2 = x*x$
  - – Each term has common factor of x
    - $x(ax + b) = 0$

Understanding Multiple Solutions

- Remember, if **either** of the factors of a term are 0, the entire term is 0
- So, we must set both factors equal to zero and solve both equations to find all possible values of x
    - $x(ax + b) = 0$
    - $x = 0$ and $ax+b = 0$
    - So $x = 0$ and $x = -b/a$

Example

- $4x^2 = 8x$
- Standard form
  - $4x^2 - 8x = 0$
- Factor
  - $4x(x - 2) = 0$
- Separate and solve
  - $4x = 0$ and $x - 2 = 0$
  - $x = 0$ and $x = 2$
- Check
  - Plug each value into the original equation and solve
  - $4(\mathbf{0})^2 = 8(\mathbf{0})$ and $4(\mathbf{2})^2 = 8(\mathbf{2})$
  - $0 = 0$ and $16 = 16$ so they are right!

Even More Complicated Problem

- Neither a, b, or c are zero
    - $ax^2 + bx + c = 0$
- Cannot solve directly
- Cannot pull a common factor out of each term
- How can we solve for x now?
- First lets explore multiplication of polynomials

Multiplying Polynomials

For more information see

http://www.purplemath.com/modules/polymult.htm

F.O.I.L.

- $(ax + b)(cx + d)$
- First – multiply the first terms in each
    - $ax*cx = acx^2$
- Outer
    - $ax*d = adx$
- Inner
    - $b*cx = bcx$
- Last
    - $b*d = bd$
- Combine all terms
    - $acx^2 + adx + bcx + bd = acx^2 + (ad + bc)x + bd$

Example

- $(4x + 2)(x - 4)$
- First
    - $4x*x = 4x^2$
- Outer
    - $4x*(-4) = -16x$
- Inner
    - $2*x = 2x$
- Last
    - $2*(-4) = -8$
- Answer
    - $4x^2 - 16x + 2x - 8 = 4x^2 - 14x - 8$

Factoring Polynomials

- FOIL took two polynomials and multiplied them together to form a new polynomial
- With some careful thought, we can “un-FOIL” a polynomial into two factors.
- Then we can solve equations by using the previous factoring theorems

Basic Factoring

- Given $ax^2 + bx + c = 0$
- Form we want $(kx + L)(mx + n) = 0$
- Solve for x as before
- $Kx + L = 0$ and $mx + n = 0$
- $x = -L/k$ and $x = -n/m$

Steps

- Given $1x^2 + bx + c = 0$
- Want $(kx + L)(mx + n) = 0$
- Notice what happens if we FOIL $(kx + L)(mx + n) = 0$
  - $(km)x^2 + (Lm + kn)x + nL = 0$
- Now compare with original equation
  - $km = 1$; $Lm + kn = b$; $L*n = c$
- Let $k = m = 1$
  - Implies $L + n = b$ and $L*n = c$
- Choose L and n by "guess and check" so that they satisfy the two equations above
- Final form $\mathbf{(x + L)(x + n) = ax^2 + bx + c = 0}$
  - Where $Ln = c$ and $L + n = b$

Example

- $x^2 - 4x - 12 = 0$
- Which two numbers add to -4 and multiply to -12?
- Factors of 12 are *1,12*; *2,6*; *3,4*
    - *One number will be negative in each pair*
- -6 + 2 = -4 and -6*2 = -12 Bingo!
- Factored form $(x - 6)(x + 2) = 0$
- Solve
    - $x = 6$ and $x = -2$
- Plug in and Check
    - $(6)^2 - 4(6) - 12 = 0$ and $(-2)^2 - 4(-2) - 12 = 0$
    - 0 = 0 and 0 = 0
    - The answers check!

Fractions

For more information see:

http://www.purplemath.com/modules/fraction5.htm

http://www.purplemath.com/modules/fraction.htm

Multiplying Fractions

- Multiply numerators
- Multiply denominators
- Reduce If possible

$$\frac{1}{2} * \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$$

Adding Fractions

- Find common denominator
- Add numerators
- Reduce if possible

$$\frac{a}{b}+\frac{c}{d}=\frac{a}{b}\left(\frac{d}{d}\right)+\frac{c}{d}\left(\frac{b}{b}\right)=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$$

$$\frac{1}{2}+\frac{2}{5}=\frac{1}{2}\left(\frac{5}{5}\right)+\frac{2}{5}\left(\frac{2}{2}\right)=\frac{5}{10}+\frac{4}{10}=\frac{9}{10}$$

Reducing Fractions With Variables

- Factor numerator and denominator
- Cancel like terms if possible to reduce

$$\frac{3x-9}{x^2-6x+9}$$

$$=\frac{3(x-3)}{(x-3)(x-3)}$$

$$=\frac{3}{(x-3)}$$

Adding Fractions With Variables

- Factor numerator and denominator
- Find common denominator
- Add numerator
- Reduce

$$\frac{3x-9}{x^2-6x+9}+\frac{x}{x-3}$$

$$=\frac{3x-9}{(x-3)(x-3)}+\frac{x}{x-3}\left(\frac{x-3}{x-3}\right)$$

$$=\frac{3x-9}{(x-3)(x-3)}+\frac{x^2-3x}{(x-3)(x-3)}$$

$$=\frac{x^2-9}{(x-3)(x-3)}=\frac{(x+3)(x-3)}{(x-3)(x-3)}=\frac{x+3}{x-3}$$

Rationalizing Denominator

- Multiply numerator and denominator by conjugate of denominator
- http://www.purplemath.com/modules/radicals5.htm

$$\frac{x+\sqrt{3}}{x-\sqrt{2}} = \frac{x+\sqrt{3}}{x-\sqrt{2}}\left(\frac{x+\sqrt{2}}{x+\sqrt{2}}\right) = \frac{x^2+x\sqrt{3}+x\sqrt{2}+\sqrt{6}}{x^2-4}$$