$\underline{\pi}_\bigstar H\underline{\mathbb{Z}}$ for C_{p^n} and Homological Algebra

Mingcong Zeng

Tuesday 12th September, 2017

Introduction

Introduction

- Computation of $\underline{\pi}_\bigstar H\underline{\mathbb{Z}}$ for C_{p^n}

Introduction

- Computation of $\underline{\pi}_\bigstar H\underline{\mathbb{Z}}$ for C_{p^n}
- Relation to homological algebra of $\underline{\mathbb{Z}}$-modules.

Introduction

- Computation of $\underline{\pi}_\star H\underline{\mathbb{Z}}$ for C_{p^n}
- Relation to homological algebra of $\underline{\mathbb{Z}}$-modules.
- Application in slice spcetral sequences.

Notations

Notations

- $G = C_{p^n}$, cyclic group of order p^n.

Notations

- $G = C_{p^n}$, cyclic group of order p^n.
- $\bigstar$ means indexing in $RO(G)$, the orthogonal representation group of G.

Notations

- $G = C_{p^n}$, cyclic group of order p^n.
- $\bigstar$ means indexing in $RO(G)$, the orthogonal representation group of G.
- $*$ means indexing in $\mathbb{Z}$.

Mackey Functors, I

Mackey Functors, I

- A G-Mackey functor $\underline{M}$ (usually omit G) is an assignment to each obrit G/H, an abelian group $\underline{M}(G/H)$, with structure maps:

Mackey Functors, I

- A G-Mackey functor $\underline{M}$ (usually omit G) is an assignment to each obrit G/H, an abelian group $\underline{M}(G/H)$, with structure maps:
  Given $H' < H < G$, morphisms

Mackey Functors, I

- A G-Mackey functor $\underline{M}$ (usually omit G) is an assignment to each obrit G/H, an abelian group $\underline{M}(G/H)$, with structure maps:
  Given $H' < H < G$, morphisms
  $Res^H_{H'} : \underline{M}(G/H) \to \underline{M}(G/H')$

Mackey Functors, I

- A G-Mackey functor $\underline{M}$ (usually omit G) is an assignment to each obrit G/H, an abelian group $\underline{M}(G/H)$, with structure maps:
  Given $H' < H < G$, morphisms
  $Res^H_{H'} : \underline{M}(G/H) \to \underline{M}(G/H')$
  $Tr^H_{H'} : \underline{M}(G/H') \to \underline{M}(G/H)$

Mackey Functors, I

- A G-Mackey functor $\underline{M}$ (usually omit G) is an assignment to each obrit G/H, an abelian group $\underline{M}(G/H)$, with structure maps:
  Given $H' < H < G$, morphisms
  $Res^H_{H'} : \underline{M}(G/H) \to \underline{M}(G/H')$
  $Tr^H_{H'} : \underline{M}(G/H') \to \underline{M}(G/H)$
  Wyle group action $N_H(G) \times \underline{M}(G/H) \to \underline{M}(G/H)$

Mackey Functors, I

- A G-Mackey functor $\underline{M}$ (usually omit G) is an assignment to each obrit G/H, an abelian group $\underline{M}(G/H)$, with structure maps:
  Given $H' < H < G$, morphisms
  $Res^H_{H'} : \underline{M}(G/H) \to \underline{M}(G/H')$
  $Tr^H_{H'} : \underline{M}(G/H') \to \underline{M}(G/H)$
  Wyle group action $N_H(G) \times \underline{M}(G/H) \to \underline{M}(G/H)$
  With some compatibility condition.

Mackey Functors, II

Mackey Functors, II

- The category $Mack_G$ can also be defined as a functor category.

Mackey Functors, II

- The category $Mack_G$ can also be defined as a functor category.
- By Day convolution this gives a closed symmetric monoidal structure on $Mack_G$, $(Mack_G, \Box, \underline{Hom}, \underline{A})$

Mackey Functors, II

- The category $Mack_G$ can also be defined as a functor category.
- By Day convolution this gives a closed symmetric monoidal structure on $Mack_G$, $(Mack_G, \Box, \underline{Hom}, \underline{A})$
- The tensor product $\Box$ is called "box product", $\underline{Hom}$ stands for the internal Hom functor and the tensor unit $\underline{A}$ is the Burnside Mackey functor.

Mackey Functors, II

- The category $Mack_G$ can also be defined as a functor category.
- By Day convolution this gives a closed symmetric monoidal structure on $Mack_G$, $(Mack_G, \Box, \underline{Hom}, \underline{A})$
- The tensor product $\Box$ is called "box product", $\underline{Hom}$ stands for the internal *Hom* functor and the tensor unit $\underline{A}$ is the Burnside Mackey functor.
- $Mack_G$ is an abelian category with enough projective and injective objects.

Mackey Functors, II

- The category $Mack_G$ can also be defined as a functor category.
- By Day convolution this gives a closed symmetric monoidal structure on $Mack_G$, $(Mack_G, \Box, \underline{Hom}, \underline{A})$
- The tensor product $\Box$ is called "box product", $\underline{Hom}$ stands for the internal *Hom* functor and the tensor unit $\underline{A}$ is the Burnside Mackey functor.
- $Mack_G$ is an abelian category with enough projective and injective objects.
- Therefore we can play with homological algebra!

Mackey Functors, II

- The category $Mack_G$ can also be defined as a functor category.
- By Day convolution this gives a closed symmetric monoidal structure on $Mack_G$, $(Mack_G, \Box, \underline{Hom}, \underline{A})$
- The tensor product $\Box$ is called "box product", $\underline{Hom}$ stands for the internal *Hom* functor and the tensor unit $\underline{A}$ is the Burnside Mackey functor.
- $Mack_G$ is an abelian category with enough projective and injective objects.
- Therefore we can play with homological algebra!
- But it has infinite cohomological dimension...

Mackey Functors, II

- The category $Mack_G$ can also be defined as a functor category.
- By Day convolution this gives a closed symmetric monoidal structure on $Mack_G$, $(Mack_G, \Box, \underline{Hom}, \underline{A})$
- The tensor product $\Box$ is called "box product", $\underline{Hom}$ stands for the internal *Hom* functor and the tensor unit $\underline{A}$ is the Burnside Mackey functor.
- $Mack_G$ is an abelian category with enough projective and injective objects.
- Therefore we can play with homological algebra!
- But it has infinite cohomological dimension...
- $\underline{\mathbb{Z}}$ is the Mackey functor with $\underline{\mathbb{Z}}(G/H) = \mathbb{Z}$ and all restrictions are isomorphism.

G-Spectra, I

G-Spectra, I

- Why do we care about $Mack_G$?

G-Spectra, I

- Why do we care about $Mack_G$?
- Because it plays the role of abelian groups in equivariant stable homotopy theory.

G-Spectra, I

- Why do we care about $Mack_G$?
- Because it plays the role of abelian groups in equivariant stable homotopy theory.
- A G-spectrum X is (not) a spectrum X with G action.

G-Spectra, I

- Why do we care about $Mack_G$?
- Because it plays the role of abelian groups in equivariant stable homotopy theory.
- A G-spectrum X is (not) a spectrum X with G action.
- And it comes with $RO(G)$-graded homotopy Mackey functor $\underline{\pi}_{\bigstar}(X)$, as the fundamental algebraic invariant.(Think about $\pi_*(X)$ in the classical case)

G-spectra, II

G-spectra, II

- Spheres $\to$ Representation Spheres.

G-spectra, II

- Spheres $\to$ Representation Spheres.
- Given a virtual representation $V \in RO(G)$, let S^V be the one-point compactificaiton of V, thus a sphere with G-action.

G-spectra, II

- Spheres $\to$ Representation Spheres.
- Given a virtual representation $V \in RO(G)$, let S^V be the one-point compactificaiton of V, thus a sphere with G-action.
- $\underline{\pi}_V(X)(G/H) := [S^V \wedge_H G_+, X]$ can be made into a Mackey functor.

Eilenberg-Mac Lane Spectra

Eilenberg-Mac Lane Spectra

- Theorem (Lewis, May, McClure)

Eilenberg-Mac Lane Spectra

- Theorem (Lewis, May, McClure)

  Given a Mackey functor $\underline{M}$, there is a unique G-spectrum $H\underline{M}$ up to homotopy, such that the integer graded $\underline{\pi}_*(H\underline{M})$ is concentrated in dimension 0 and it is $\underline{M}$.

Eilenberg-Mac Lane Spectra

- Theorem (Lewis, May, McClure)

  Given a Mackey functor $\underline{M}$, there is a unique G-spectrum $H\underline{M}$ up to homotopy, such that the integer graded $\underline{\pi}_*(H\underline{M})$ is concentrated in dimension 0 and it is $\underline{M}$.

  - But how about the $RO(G)$-grading?

Eilenberg-Mac Lane Spectra

- Theorem (Lewis, May, McClure)

  Given a Mackey functor $\underline{M}$, there is a unique G-spectrum $H\underline{M}$ up to homotopy, such that the integer graded $\underline{\pi}_*(H\underline{M})$ is concentrated in dimension 0 and it is $\underline{M}$.

  - But how about the $RO(G)$-grading?
  - It is coming!

$RO(G)$-graded homotopy of $H\underline{\mathbb{Z}}$

$RO(G)$-graded homotopy of $H\underline{\mathbb{Z}}$

- Now, we focus on $H\underline{\mathbb{Z}}$.

$RO(G)$-graded homotopy of $H\underline{\mathbb{Z}}$

- Now, we focus on $H\underline{\mathbb{Z}}$.
- How about a little bit more about $\underline{\mathbb{Z}}$?

$RO(G)$-graded homotopy of $H\underline{\mathbb{Z}}$

- Now, we focus on $H\underline{\mathbb{Z}}$.
- How about a little bit more about $\underline{\mathbb{Z}}$?
- $\underline{\mathbb{Z}}$ is a monoid, and $\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$ is an $RO(G)$-graded $\underline{\mathbb{Z}}$-module.

$G = C_p$

$G = C_p$

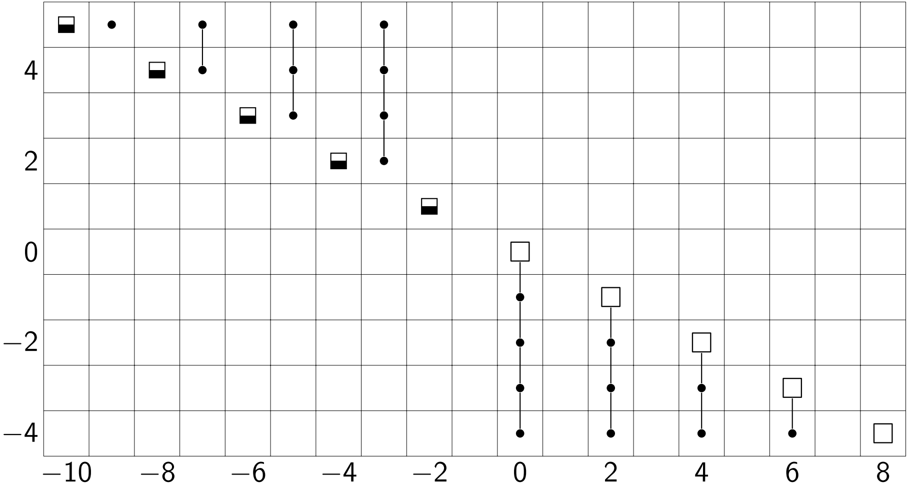
- $G = C_p$ and $p > 2$, $RO(G) = \mathbb{Z}\langle 1, \lambda \rangle$, where 1 is the trivial representation and λ is the $\mathbb{R}^2$-representation given by rotation by $\frac{2\pi}{p}$.

$G = C_p$

- $G = C_p$ and $p > 2$, $RO(G) = \mathbb{Z}\langle 1, \lambda\rangle$, where 1 is the trivial representation and λ is the $\mathbb{R}^2$-representation given by rotation by $\frac{2\pi}{p}$.
- $\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$ is...

$\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$

$\underline{\pi}_\star(H\underline{\mathbb{Z}})$



Main Theorem

Main Theorem

- Theorem (Z.)

  $G = C_{p^n}$, $\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$ with its multiplicative structure can be computed by Tate diagram.

Main Theorem

- Theorem (Z.)

  $G = C_{p^n}$, $\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$ with its multiplicative structure can be computed by Tate diagram.

  - But what does it look like?

$G = C_{p^2}$

$G = C_{p^2}$

- When $G = C_{p^2}$, $RO(G) = \mathbb{Z}\langle 1, \lambda_1, \lambda_0 \rangle$. Where λ_i is the representation by rotating by $\frac{2\pi}{p^{2-i}}$ on $\mathbb{R}^2$.

$G = C_{p^2}$

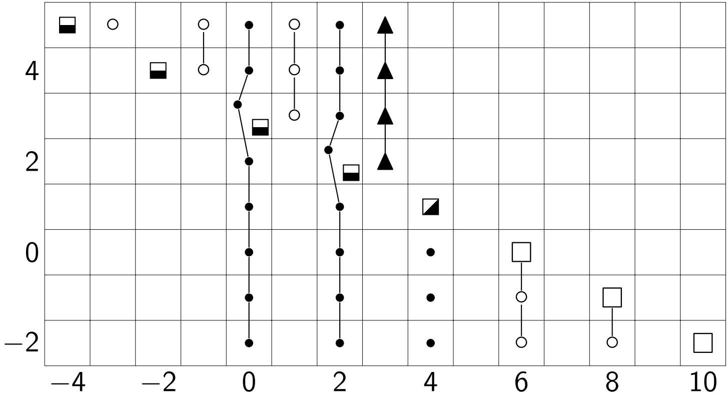
- When $G = C_{p^2}$, $RO(G) = \mathbb{Z}\langle 1, \lambda_1, \lambda_0 \rangle$. Where λ_i is the representation by rotating by $\frac{2\pi}{p^{2-i}}$ on $\mathbb{R}^2$.
- Therefore we need a 3D projector to present the full result!

$G = C_{p^2}$

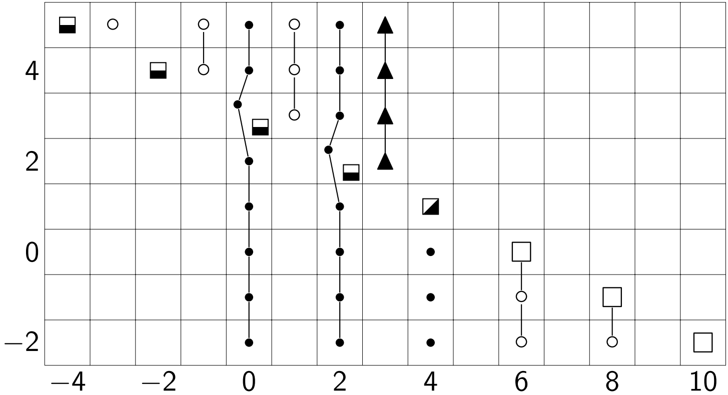
- When $G = C_{p^2}$, $RO(G) = \mathbb{Z}\langle 1, \lambda_1, \lambda_0 \rangle$. Where λ_i is the representation by rotating by $\frac{2\pi}{p^{2-i}}$ on $\mathbb{R}^2$.
- Therefore we need a 3D projector to present the full result!
- How about a 2D slice of it?

$\underline{\pi}_{m\lambda_0-3\lambda_1+i}(H\underline{\mathbb{Z}})$

$\underline{\pi}_{m\lambda_0-3\lambda_1+i}(H\underline{\mathbb{Z}})$



$\underline{\pi}_{m\lambda_0-3\lambda_1+i}(H\underline{\mathbb{Z}})$



We see ∞-towers of multiplication and division by element a_{λ_0}.

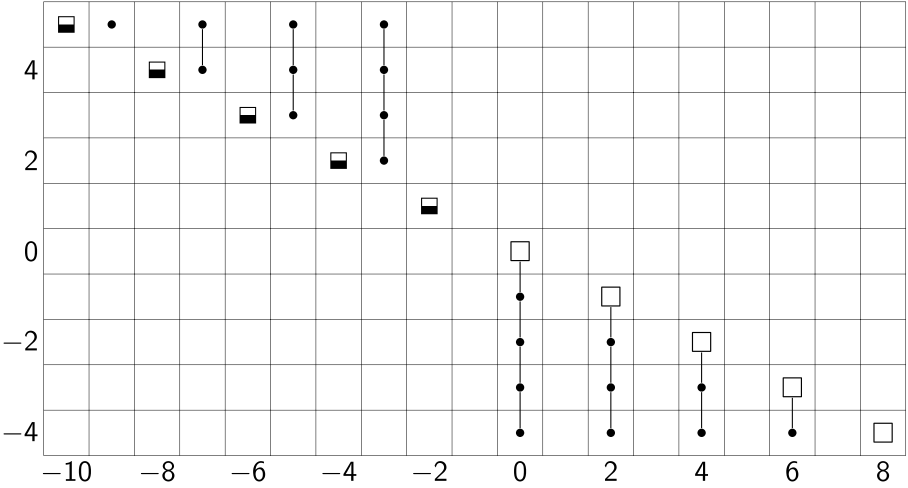
Duality in $\underline{\pi}_{\bigstar}(H\underline{\mathbb{Z}})$

Duality in $\underline{\pi}_\star(H\underline{\mathbb{Z}})$

Look at the figure for $G = C_p$ again:

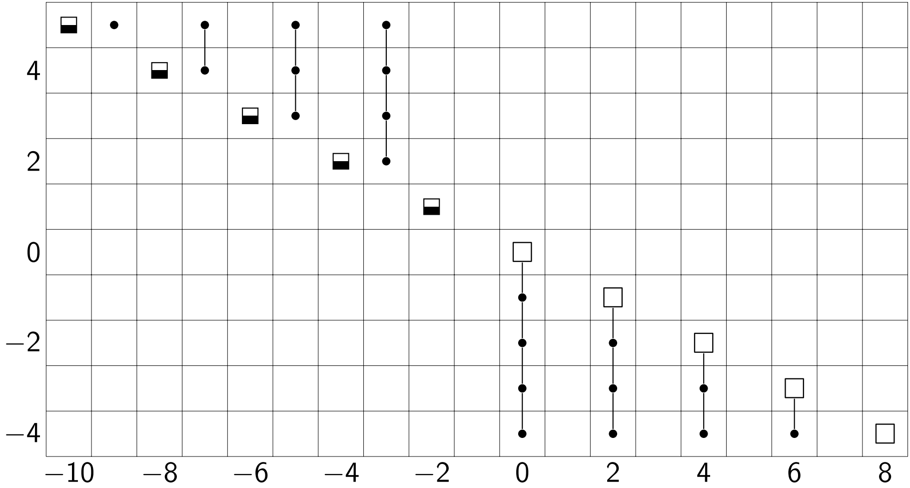
Duality in $\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$

Look at the figure for $G = C_p$ again:



Duality in $\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$

Look at the figure for $G = C_p$ again:



You see duality with a strange shift.

Duality, II

Duality, II

- There are two dualities inside.

Duality, II

- There are two dualities inside.
- The equivariant Anderson duality give rise to a universal coefficient exact sequence, computing $\underline{\pi}_{-V-*}(I_{\mathbb{Z}}(H\underline{\mathbb{Z}}))$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$.

Duality, II

- There are two dualities inside.
- The equivariant Anderson duality give rise to a universal coefficient exact sequence, computing $\underline{\pi}_{-V-*}(I_{\mathbb{Z}}(H\underline{\mathbb{Z}}))$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$. But what is the Anderson dual $I_{\mathbb{Z}}(H\underline{\mathbb{Z}})$?

Duality, II

- There are two dualities inside.
- The equivariant Anderson duality give rise to a universal coefficient exact sequence, computing $\underline{\pi}_{-V-*}(I_{\mathbb{Z}}(H\underline{\mathbb{Z}}))$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$. But what is the Anderson dual $I_{\mathbb{Z}}(H\underline{\mathbb{Z}})$?

- Lemma (The Miracle)

  If $G = C_{p^n}$, $I_{\mathbb{Z}}(H\underline{\mathbb{Z}}) \simeq H\underline{\mathbb{Z}}^* \simeq S^{2-\lambda_0} \wedge H\underline{\mathbb{Z}}$

Duality, II

- There are two dualities inside.
- The equivariant Anderson duality give rise to a universal coefficient exact sequence, computing $\underline{\pi}_{-V-*}(I_{\mathbb{Z}}(H\underline{\mathbb{Z}}))$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$. But what is the Anderson dual $I_{\mathbb{Z}}(H\underline{\mathbb{Z}})$?

- **Lemma (The Miracle)**

  If $G = C_{p^n}$, $I_{\mathbb{Z}}(H\underline{\mathbb{Z}}) \simeq H\underline{\mathbb{Z}}^* \simeq S^{2-\lambda_0} \wedge H\underline{\mathbb{Z}}$

  - Where $\underline{\mathbb{Z}}^*(G/H) \cong \mathbb{Z}$ and all transfers are isomorphisms.

Duality, II

- There are two dualities inside.
- The equivariant Anderson duality give rise to a universal coefficient exact sequence, computing $\underline{\pi}_{-V-*}(I_{\mathbb{Z}}(H\underline{\mathbb{Z}}))$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$. But what is the Anderson dual $I_{\mathbb{Z}}(H\underline{\mathbb{Z}})$?

- Lemma (The Miracle)

  If $G = C_{p^n}$, $I_{\mathbb{Z}}(H\underline{\mathbb{Z}}) \simeq H\underline{\mathbb{Z}}^* \simeq S^{2-\lambda_0} \wedge H\underline{\mathbb{Z}}$

  - Where $\underline{\mathbb{Z}}^*(G/H) \cong \mathbb{Z}$ and all transfers are isomorphisms.
  - This lemma plays an important role in equivariant duality of many C_2-spectra, like $Tmf_1(3)$ of Hill and Meier.

Duality, II

- There are two dualities inside.
- The equivariant Anderson duality give rise to a universal coefficient exact sequence, computing $\underline{\pi}_{-V-*}(I_{\mathbb{Z}}(H\underline{\mathbb{Z}}))$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$. But what is the Anderson dual $I_{\mathbb{Z}}(H\underline{\mathbb{Z}})$?

- Lemma (The Miracle)

  If $G = C_{p^n}$, $I_{\mathbb{Z}}(H\underline{\mathbb{Z}}) \simeq H\underline{\mathbb{Z}}^* \simeq S^{2-\lambda_0} \wedge H\underline{\mathbb{Z}}$

  - Where $\underline{\mathbb{Z}}^*(G/H) \cong \mathbb{Z}$ and all transfers are isomorphisms.
  - This lemma plays an important role in equivariant duality of many C_2-spectra, like $Tmf_1(3)$ of Hill and Meier.
  - Combining with Anderson duality, we can use short exact sequences to compute $\underline{\pi}_{\lambda_0-V-2-*}(H\underline{\mathbb{Z}})$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$.

Duality, II

- There are two dualities inside.
- The equivariant Anderson duality give rise to a universal coefficient exact sequence, computing $\underline{\pi}_{-V-*}(I_{\mathbb{Z}}(H\underline{\mathbb{Z}}))$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$. But what is the Anderson dual $I_{\mathbb{Z}}(H\underline{\mathbb{Z}})$?

- Lemma (The Miracle)

  If $G = C_{p^n}$, $I_{\mathbb{Z}}(H\underline{\mathbb{Z}}) \simeq H\underline{\mathbb{Z}}^* \simeq S^{2-\lambda_0} \wedge H\underline{\mathbb{Z}}$

  - Where $\underline{\mathbb{Z}}^*(G/H) \cong \mathbb{Z}$ and all transfers are isomorphisms.
  - This lemma plays an important role in equivariant duality of many C_2-spectra, like $Tmf_1(3)$ of Hill and Meier.
  - Combining with Anderson duality, we can use short exact sequences to compute $\underline{\pi}_{\lambda_0-V-2-*}(H\underline{\mathbb{Z}})$ from $\underline{\pi}_{V+*}(H\underline{\mathbb{Z}})$.
  - It is a duality with centre of symmetry $(-1, \frac{\lambda_0}{2})$.

Generalized Miracle

Generalized Miracle

- Theorem (Z.)

  For $G = C_{p^n}$*, and* $\underline{M}$ *be a Mackey functor with* $\underline{M}(G/H) \cong \mathbb{Z}$ *for all* $H < G$*, then*
  $$H\underline{M} \simeq S^V \wedge H\underline{\mathbb{Z}}$$
  for a $V \in RO(G)$*.*
  Furthermore, it gives a one-to-one correspondence between all such $\underline{M}$*(Forms of* $\underline{\mathbb{Z}}$*) and all* $V \in RO(G)$ *with* $S^V \wedge H\underline{\mathbb{Z}}$ *Eilenberg-Mac Lane.*

Duality, III

Duality, III

- Another duality is the honest universal coefficient spectral sequence. The actual theorem is very general, but we only pick the case we need.

Duality, III

- Another duality is the honest universal coefficient spectral sequence. The actual theorem is very general, but we only pick the case we need.

- Theorem (Lewis, Mandell)

  There is a spectral sequence of Mackey functors with E_2-page

$$\underline{Ext}^{i,j}_{\underline{\mathbb{Z}}}(\underline{\pi}_{V+*}(H\underline{\mathbb{Z}}), \underline{\mathbb{Z}})$$

  converging to $\underline{\pi}_{-V+*}(H\underline{\mathbb{Z}})$.

Duality, III

- Another duality is the honest universal coefficient spectral sequence. The actual theorem is very general, but we only pick the case we need.
- Theorem (Lewis, Mandell)

  There is a spectral sequence of Mackey functors with E_2-page

$$\underline{Ext}^{i,j}_{\underline{\mathbb{Z}}}(\underline{\pi}_{V+*}(H\underline{\mathbb{Z}}), \underline{\mathbb{Z}})$$

  converging to $\underline{\pi}_{-V+*}(H\underline{\mathbb{Z}})$.
  - It is a duality with centre of symmetry $(0, 0)$.

Homological Algebra of $\mathbb{Z}$-Modules

Homological Algebra of $\underline{\mathbb{Z}}$-Modules

- Theorem (Bouc, Stancu and Webb, Arnold)

  If G is finite and cyclic, the global projective dimension of $\underline{\mathbb{Z}}$-module is 3. That means,

Homological Algebra of $\underline{\mathbb{Z}}$-Modules

- Theorem (Bouc, Stancu and Webb, Arnold)

  If G is finite and cyclic, the global projective dimension of $\underline{\mathbb{Z}}$-module is 3. That means,
  Each $\underline{\mathbb{Z}}$-module has a projective resolution of length 3.

Homological Algebra of $\underline{\mathbb{Z}}$-Modules

- Theorem (Bouc, Stancu and Webb, Arnold)

  If G is finite and cyclic, the global projective dimension of $\underline{\mathbb{Z}}$-module is 3. That means,
  Each $\underline{\mathbb{Z}}$-module has a projective resolution of length 3.

- Lemma (Z.)

  If $G = C_{p^n}$, and $\underline{M}$ be a $\underline{\mathbb{Z}}$-module that $\underline{M}(G/e) \cong 0$, then $\underline{Ext}^i_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated at $i = 3$, and

$$\underline{Ext}^3_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}}) \cong \underline{M}^E$$

Homological Algebra of $\underline{\mathbb{Z}}$-Modules

- Theorem (Bouc, Stancu and Webb, Arnold)

  If G is finite and cyclic, the global projective dimension of $\underline{\mathbb{Z}}$-module is 3. That means,
  Each $\underline{\mathbb{Z}}$-module has a projective resolution of length 3.

- Lemma (Z.)

  If $G = C_{p^n}$, and $\underline{M}$ be a $\underline{\mathbb{Z}}$-module that $\underline{M}(G/e) \cong 0$, then $\underline{Ext}^i_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated at $i = 3$, and

$$\underline{Ext}^3_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}}) \cong \underline{M}^E$$

  - Where $\underline{M}^E$ is the level-wise $Ext^1(-, \mathbb{Z})$ of abelian groups on $\underline{M}$.

Homological Algebra of $\underline{\mathbb{Z}}$-Modules

- Theorem (Bouc, Stancu and Webb, Arnold)

  If G is finite and cyclic, the global projective dimension of $\underline{\mathbb{Z}}$-module is 3. That means,
  Each $\underline{\mathbb{Z}}$-module has a projective resolution of length 3.

- Lemma (Z.)

  If $G = C_{p^n}$, and $\underline{M}$ be a $\underline{\mathbb{Z}}$-module that $\underline{M}(G/e) \cong 0$, then $\underline{Ext}^i_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated at $i = 3$, and

  $$\underline{Ext}^3_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}}) \cong \underline{M}^E$$

  - Where $\underline{M}^E$ is the level-wise $Ext^1(-, \mathbb{Z})$ of abelian groups on $\underline{M}$.
  - So in our case we can understand the universal coefficient spectral sequence!(At least E_2 page)

Homological Algebra of $\underline{\mathbb{Z}}$-Modules

- Theorem (Bouc, Stancu and Webb, Arnold)

  If G is finite and cyclic, the global projective dimension of $\underline{\mathbb{Z}}$-module is 3. That means,
  Each $\underline{\mathbb{Z}}$-module has a projective resolution of length 3.

- Lemma (Z.)

  If $G = C_{p^n}$, and $\underline{M}$ be a $\underline{\mathbb{Z}}$-module that $\underline{M}(G/e) \cong 0$, then $\underline{Ext}^i_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated at $i = 3$, and

  $$\underline{Ext}^3_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}}) \cong \underline{M}^E$$

  - Where $\underline{M}^E$ is the level-wise $Ext^1(-, \mathbb{Z})$ of abelian groups on $\underline{M}$.
  - So in our case we can understand the universal coefficient spectral sequence!(At least E_2 page)
  - This lemma also gives an algebraic interpretation of the "gap" in the upper-left part.

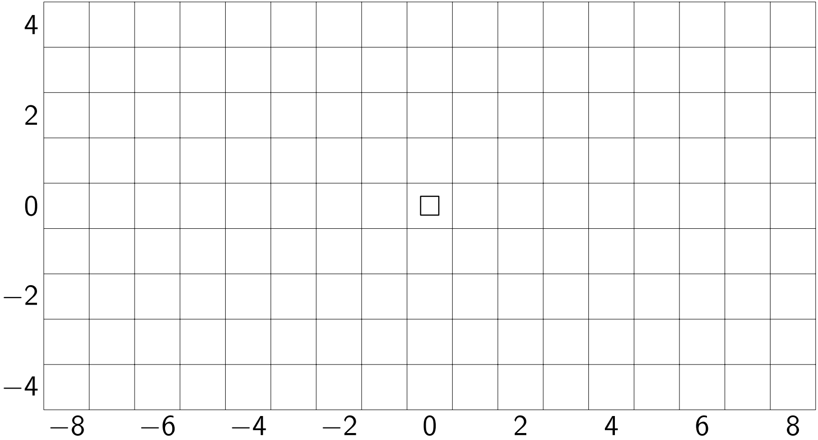
Duality, :‖

Duality, :

If you start with $\underline{\pi}_*(H\underline{\mathbb{Z}})$, the integer grading. By using these two dualites, you can actually recover the whole figure.

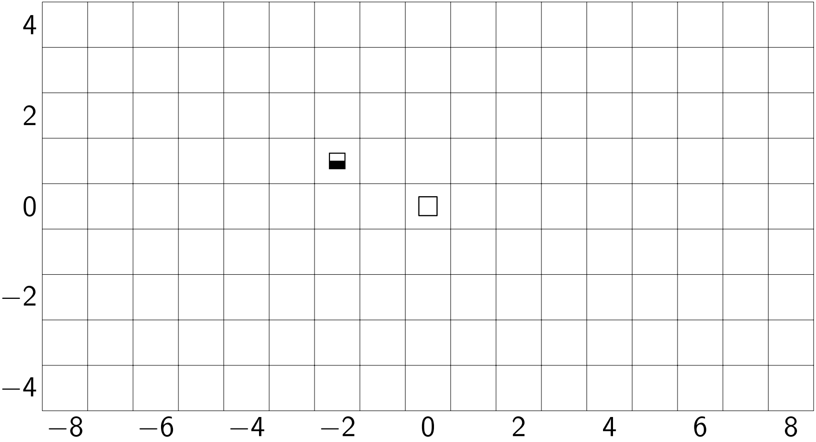
Duality, :‖

You start with $\mathbb{Z}$ in $(0, 0)$.



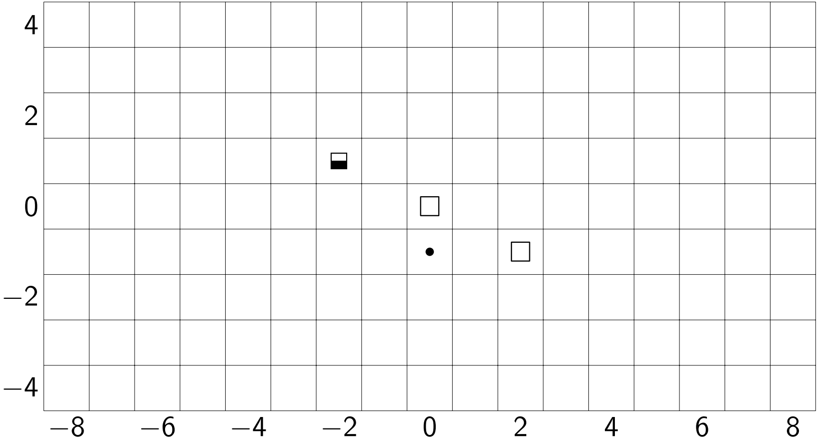
Duality,

Apply Anderson duality.



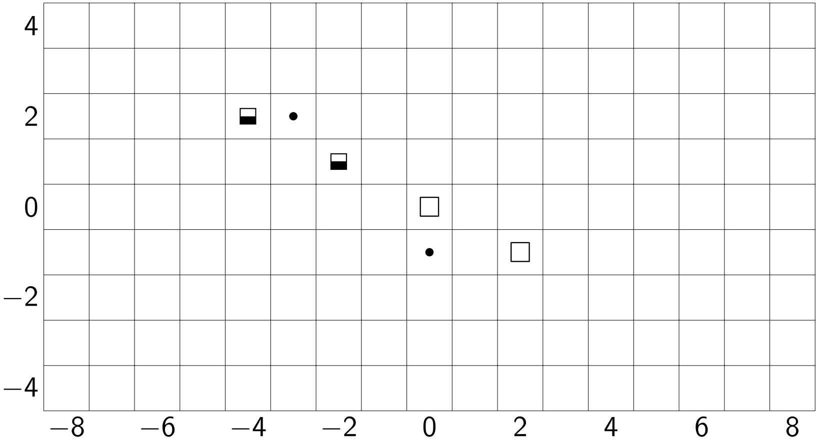
Duality, :‖

Compute the universal coefficient spectral sequence.



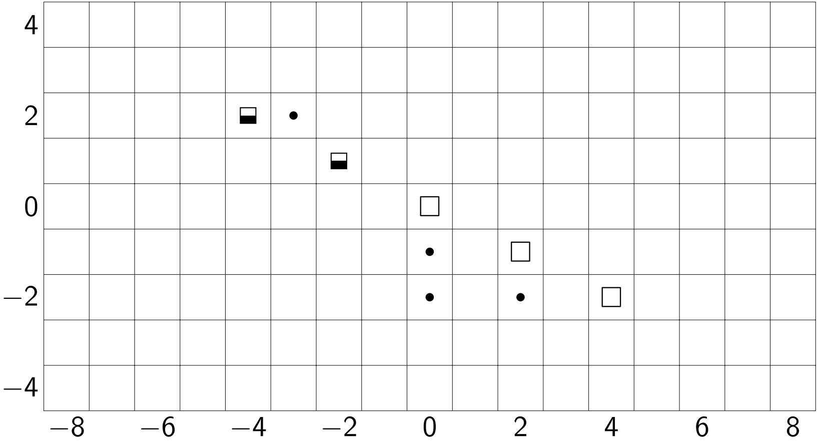
Duality, $\because\!\!\|$

Apply Anderson duality again.



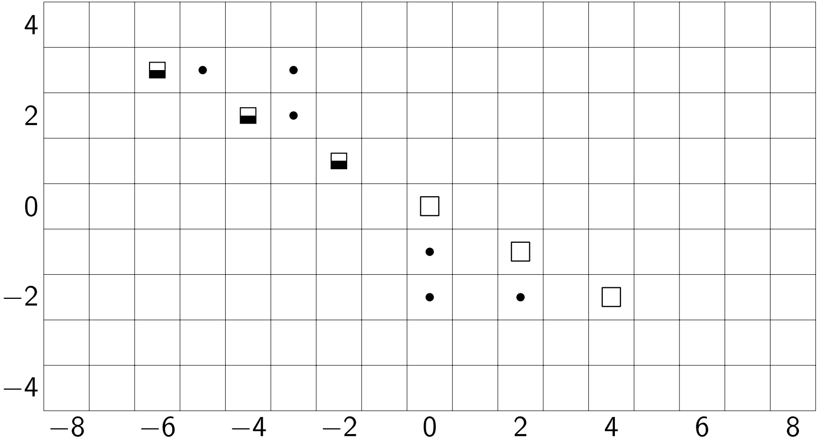
Duality, :‖

‖:Universal coefficient spectral sequence. Anderson Duality :‖



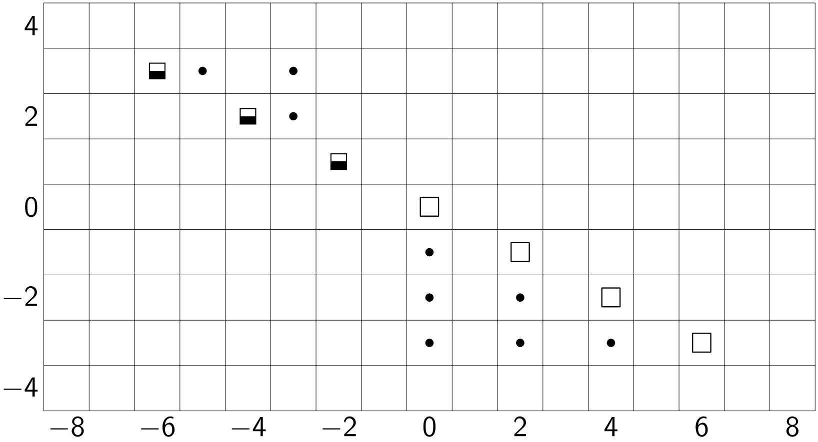
Duality, :‖

‖:Universal coefficient spectral sequence. Anderson Duality :‖



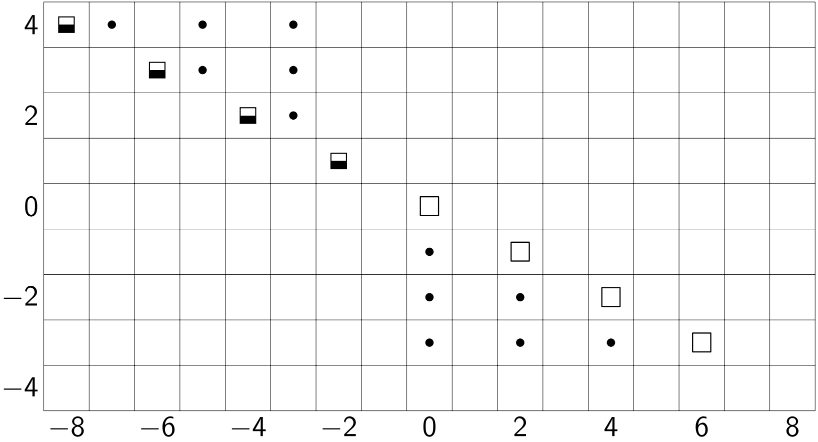
Duality, :‖

‖:Universal coefficient spectral sequence. Anderson Duality :‖



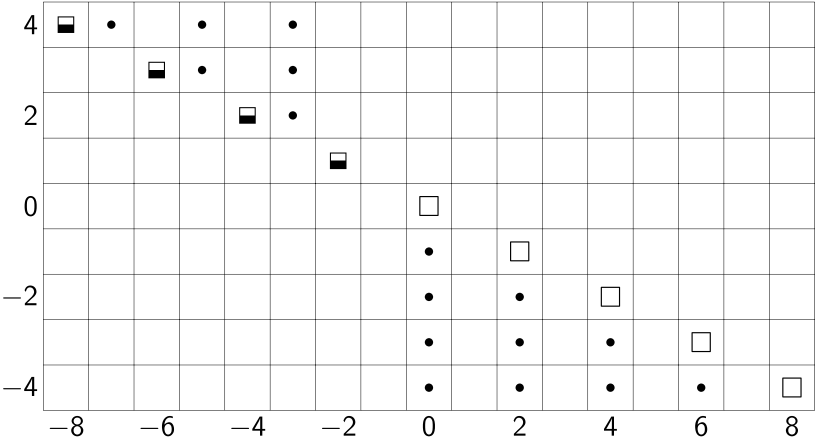
Duality,

Universal coefficient spectral sequence. Anderson Duality



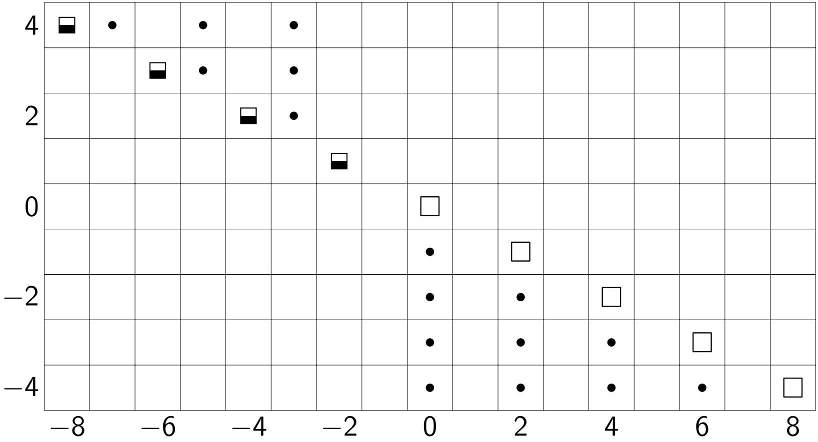
Duality, :‖

‖:Universal coefficient spectral sequence. Anderson Duality :‖



Duality, :‖

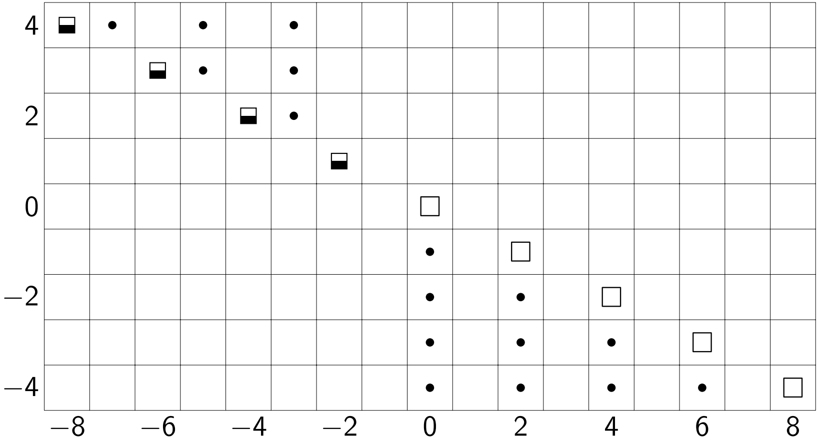
‖:Universal coefficient spectral sequence. Anderson Duality :‖



- The way these dualities spinning holds for all G that the miracle holds, especially for C_{p^n}.

Duality, :‖

‖:Universal coefficient spectral sequence. Anderson Duality :‖



- The way these dualities spinning holds for all G that the miracle holds, especially for C_{p^n}.
- However as a n-dimensional grid.

Application in Slice Spectral Sequence

Application in Slice Spectral Sequence

- Understanding the Mackey functor structure of $\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$ allows us to understand the whole $RO(G)$-grading of many slice spectral sequences.

Application in Slice Spectral Sequence

- Understanding the Mackey functor structure of $\underline{\pi}_\bigstar(H\underline{\mathbb{Z}})$ allows us to understand the whole $RO(G)$-grading of many slice spectral sequences.
- The multiplicative structure can help in deducing differentials in these gradings from those known slice differentials.

Loosing Ends

Loosing Ends

- We see that $\underline{Ext}^*_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated in degree 3, and it is $\underline{M}^E$.

Loosing Ends

- We see that $\underline{Ext}^*_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated in degree 3, and it is $\underline{M}^E$. We really like to say $\underline{\mathbb{Z}}$ is a Gorenstein Green functor of shift 3.

Loosing Ends

- We see that $\underline{Ext}^*_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated in degree 3, and it is $\underline{M}^E$. We really like to say $\underline{\mathbb{Z}}$ is a Gorenstein Green functor of shift 3. But in this way, the level-wise *Ext* must play a role.

Loosing Ends

- We see that $\underline{Ext}^*_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated in degree 3, and it is $\underline{M}^E$. We really like to say $\underline{\mathbb{Z}}$ is a Gorenstein Green functor of shift 3. But in this way, the level-wise *Ext* must play a role. So, what should be a good definition of Gorenstein in Green functors?

Loosing Ends

- We see that $\underline{Ext}^*_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated in degree 3, and it is $\underline{M}^E$. We really like to say $\underline{\mathbb{Z}}$ is a Gorenstein Green functor of shift 3. But in this way, the level-wise *Ext* must play a role. So, what should be a good definition of Gorenstein in Green functors?
- What do these dualities look like for other groups? Especially for those the miracle fails.

Loosing Ends

- We see that $\underline{Ext}^*_{\underline{\mathbb{Z}}}(\underline{M}, \underline{\mathbb{Z}})$ is concentrated in degree 3, and it is $\underline{M}^E$. We really like to say $\underline{\mathbb{Z}}$ is a Gorenstein Green functor of shift 3. But in this way, the level-wise *Ext* must play a role. So, what should be a good definition of Gorenstein in Green functors?
- What do these dualities look like for other groups? Especially for those the miracle fails.
- Tate diagram can also effectively compute $H\underline{\mathbb{F}}_p$ in all cyclic p-groups. Then what does A_* look like? Say, C_4?

The End

The End

Thank you!