ERRATA FOR
MODEL CATEGORIES AND THEIR LOCALIZATIONS

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Proposition 4.2.5

Proposition 4.2.5 is incorrect. It was correct for early drafts of the book, in which the definition of a cellular model category required that the domains of the elements of the set J of generating trivial cofibrations be cofibrant. The current definition of a cellular model category, however, requires only that the domains of the elements of J be small with respect to I (see Definition 12.1.1).

To correct the error:

- In the third line of the statement of Proposition 4.2.5, change "cofibrant domains" to "domains that are small relative to I",
- in the proof of Proposition 4.2.5, in lines 4–5 of page 74, change "with cofibrant domain" to "whose domain is small relative to I", and
- in the proof of Theorem 4.3.1 on page 75, in the first line of that proof, change "Theorem 12.4.3" to "Proposition 11.2.3".

Theorem 7.6.5

Part 3 of Theorem 7.6.5 is incorrect as written. That part claims that if A and B are objects of a model category $\mathcal{M}$, then the category $(A\downarrow\mathcal{M}\downarrow B)$ of objects of $\mathcal{M}$ under A and over B is a model category. The problem with that statement is that $(A\downarrow\mathcal{M}\downarrow B)$ is usually neither complete nor cocomplete. This is because if you have two objects of $(A\downarrow\mathcal{M}\downarrow B)$, $A \xrightarrow{s} X \xrightarrow{t} B$ and $A \xrightarrow{u} Y \xrightarrow{v} B$, there can be no morphisms between them unless $ts = vu$.

The correction is to choose a fixed map $f\colon A \to B$ in $\mathcal{M}$ and to let $(A\downarrow\mathcal{M}\downarrow B)_f$ be the full subcategory of $(A\downarrow\mathcal{M}\downarrow B)$ with objects the diagrams $A \xrightarrow{s} X \xrightarrow{t} B$ such that $ts = f$ (that is, $(A\downarrow\mathcal{M}\downarrow B)_f$ is the category of factorizations of f). To form the limit of a diagram in $(A\downarrow\mathcal{M}\downarrow B)_f$, you form its limit in the category $(\mathcal{M}\downarrow B)$; there is then a natural map from A to that limit, and this forms the limit in $(A\downarrow\mathcal{M}\downarrow B)_f$. Dually, to form the colimit of a diagram in $(A\downarrow\mathcal{M}\downarrow B)_f$, you form its colimit in $(A\downarrow\mathcal{M})$; there is then a natural map from that colimit to B, and this forms the colimit in $(A\downarrow\mathcal{M}\downarrow B)_f$.

The corrected statement of part 3 is then

(3) For every map $f\colon A \to B$ in $\mathcal{M}$ the full subcategory $(A\downarrow\mathcal{M}\downarrow B)_f$ of $(A\downarrow\mathcal{M}\downarrow B)$ with objects equal to the diagrams $A \xrightarrow{s} X \xrightarrow{t} B$ for which $ts = f$ is a model category in which a map is a weak equivalence, fibration, or cofibration if it is one in $\mathcal{M}$. This is the *model category of factorizations of f*.

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There are several theorems and proofs that make reference to part 3 of Theorem 7.6.5, and those references need to be rephrased to take this change into account:

Proof of Proposition 3.3.15. In the fifth line from the bottom of page 62, the sentence that begins "Thus, in the category $(A\downarrow\mathcal{M}\downarrow Z)$ of objects of $\mathcal{M}$ under A and over Z" should instead begin "Thus, in the category $(A\downarrow\mathcal{M}\downarrow Z)_{ui}$ of factorizations of $ui : A \to Z$".

Statement and proof of Proposition 7.6.13.

- In both of the diagrams (one in the statement and one in the proof), the map from B to Y should be labelled "s".
- The last line of the statement of the proposition is "as maps in $(A\downarrow\mathcal{M}\downarrow Y)$, the category of objects of $\mathcal{M}$ under A and over Y.", and that should be changed to "as maps in $(A\downarrow\mathcal{M}\downarrow Y)_{si}$, the category of factorizations of $si\colon A \to Y$.".
- In the part of the proof that follows the diagram displayed in the proof, the sentence that begins on the second line begins "In the category $(A\downarrow\mathcal{M}\downarrow Y)$ of objects of $\mathcal{M}$ under A and over Y", and that should be changed to "In the category $(A\downarrow\mathcal{M}\downarrow Y)_{si}$ of factorizations of $si\colon A \to Y$".
- In the last two lines of that proof there are two appearances of "$(A\downarrow\mathcal{M}\downarrow Y)$", and both of those should be changed to "$(A\downarrow\mathcal{M}\downarrow Y)_{si}$".

Statement of Proposition 7.6.14. There are two appearances of "$(A\downarrow\mathcal{M}\downarrow Y)$", and they should both be changed to "$(A\downarrow\mathcal{M}\downarrow Y)_{qi}$".

Proof of Proposition 13.2.1. In both of the diagrams in that proof (on page 243), the map $B \to Y$ should be labelled as "s". The third line from the bottom of page 243 begins "The category of objects under A and over Y", and that should be changed to "The category $(A\downarrow\mathcal{M}\downarrow Y)_{sg}$ of factorizations of $sg\colon A \to Y$".

Theorem 14.5.4

The proof of Theorem 14.5.4 has two errors.

(1) The object $(\mathrm{F}\boldsymbol{X}, p_{\boldsymbol{X}})$ may not be a terminal object of $\widetilde{\mathcal{D}}$ because there may be distinct objects $(\widetilde{\boldsymbol{X}}, i)$ and $(\widetilde{\boldsymbol{X}}', i')$ of $\mathcal{D}$ such that $(\mathrm{F}\widetilde{\boldsymbol{X}}, i\circ p_{\widetilde{\boldsymbol{X}}})$ and $(\mathrm{F}\widetilde{\boldsymbol{X}}', i'\circ p_{\widetilde{\boldsymbol{X}}'})$ are the same object of $\widetilde{\mathcal{D}}$.

(2) The definition of $\widetilde{\mathrm{F}}$ may not work because $\widetilde{\mathrm{F}}$ is defined to be the identity on $\widetilde{\mathcal{D}}$ but is not the identity on any objects of $\mathcal{D}$. This is a problem because those two subcategories of $\mathcal{D}'$ might have objects in common.

The proof we present here uses a different definition of $\mathcal{D}'$ and does not use the subcategory $\widetilde{\mathcal{D}}$. We are indebted to Fabian Lenhardt and Alberto Vezzani for help in identifying the problems and correcting the proof.

Proof of Theorem 14.5.4. Let $\mathcal{D}$ be a small category of functors over $\boldsymbol{X}$ relative to $\mathcal{W}$. Let $\mathcal{D}'$ be the smallest subcategory of functors over $\boldsymbol{X}$ relative to $\mathcal{W}$ containing

(1) $\mathcal{D}$,
(2) the object $(\mathrm{F}\boldsymbol{X}, p_{\boldsymbol{X}})$,
(3) for each object $(\widetilde{\boldsymbol{X}}, i)$ in $\mathcal{D}'$
    - the object $(\mathrm{F}\widetilde{\boldsymbol{X}}, i\circ p_{\widetilde{\boldsymbol{X}}})$,

- the map $p_{\widetilde{\boldsymbol{X}}}\colon (\mathrm{F}\widetilde{\boldsymbol{X}}, i \circ p_{\widetilde{\boldsymbol{X}}}) \to (\widetilde{\boldsymbol{X}}, i)$, and
- the map $\mathrm{F}(i)\colon (\mathrm{F}\widetilde{\boldsymbol{X}}, i \circ p_{\widetilde{\boldsymbol{X}}}) \to (\mathrm{F}\boldsymbol{X}, p_{\boldsymbol{X}})$,

and

(4) for each map $g\colon (\widetilde{\boldsymbol{X}}, i) \to (\widetilde{\boldsymbol{X}'}, i')$ in $\mathcal{D}'$ the map $\mathrm{F}(g)\colon (\mathrm{F}\widetilde{\boldsymbol{X}}, i \circ p_{\widetilde{\boldsymbol{X}}}) \to (\mathrm{F}\widetilde{\boldsymbol{X}'}, i' \circ p_{\widetilde{\boldsymbol{X}}'})$.

The category $\mathcal{D}'$ is small because it can be constructed as the union of an increasing sequence of small categories, where each category in the sequence is obtained by "applying F to everything in the preceding category" (i.e., adding the objects and maps described in the last two items above).

We will show that $\mathrm{B}\mathcal{D}'$ is contractible by constructing a functor $\mathrm{G}\colon \mathcal{D}' \to \mathcal{D}'$ together with

- a natural transformation ϕ from G to the identity functor $1_{\mathcal{D}'}$ and
- a natural transformation ψ from G to a constant functor.

Proposition 14.3.10 will then imply that the identity map of $\mathrm{B}\mathcal{D}'$ is homotopic to a constant map.

If $(\widetilde{\boldsymbol{X}}, i)$ is an object in $\mathcal{D}'$ then we let $\mathrm{G}(\widetilde{\boldsymbol{X}}, i) = (\mathrm{F}\widetilde{\boldsymbol{X}}, i \circ p_{\widetilde{\boldsymbol{X}}})$, and if $g\colon (\widetilde{\boldsymbol{X}}, i) \to (\widetilde{\boldsymbol{X}'}, i')$ is a map in $\mathcal{D}'$ then we let $\mathrm{G}(g) = \mathrm{F}(g)$.

We define a natural transformation ϕ from G to the identity functor of $\mathcal{D}'$ by letting $\phi(\widetilde{\boldsymbol{X}}, i) = p_{\widetilde{\boldsymbol{X}}}\colon (\mathrm{F}\widetilde{\boldsymbol{X}}, i \circ p_{\widetilde{\boldsymbol{X}}}) \to (\widetilde{\boldsymbol{X}}, i)$.

We define a natural transformation ψ from G to the constant functor that takes every object of $\mathcal{D}'$ to $(\mathrm{F}\boldsymbol{X}, p_{\boldsymbol{X}})$ by letting $\psi(\widetilde{\boldsymbol{X}}, i) = \mathrm{F}(i)\colon (\mathrm{F}\widetilde{\boldsymbol{X}}, i \circ p_{\widetilde{\boldsymbol{X}}}) \to (\mathrm{F}\boldsymbol{X}, p_{\boldsymbol{X}})$. □

Chapter 3

Page 51, line 35:	"$\mathcal{C}$-local objects and $\mathcal{C}$-local" should be "$\mathcal{C}$-colocal objects and $\mathcal{C}$-colocal".
Page 59, line 5:	"Proposition 9.1.9" should be "Proposition 8.1.23".
Page 59, line 11:	"Proposition 8.3.26" should be "Theorem 7.5.10".
Page 59, line 14:	"Proposition 8.3.20" should be "Proposition 7.3.4".
Page 59, line 14:	"Proposition 8.4.4" should be "Proposition 7.6.8".
Page 59, line 16:	"Definition 9.6.2" should be "Definition 8.3.2".
Page 59, line 16:	"Lemma 9.6.3" should be "Lemma 8.3.4".
Page 59, line 17:	"Theorem 9.6.9" should be "Theorem 8.3.10".
Page 63, line 15:	"Theorem 4.1.10" should be "Theorem 3.2.13".
Page 67, line 3:	"Corollary 8.5.4" should be "Corollary 7.7.4".
Page 67, line 7:	"Proposition 8.3.7" should be "Proposition 7.3.10".
Page 67, line 23:	"is an trivial cofibration" should be "is a trivial cofibration".
Page 68, line 5:	"Definition 11.2.12" should be "Definition 13.3.12".

Chapter 4

Page 72, line 6:	"$\mathrm{L}_{\mathcal{C}}\mathcal{M}$" should be "$\mathrm{L}_{S}\mathcal{M}$".
Page 72, line 6:	"$\mathcal{C}$-local" should be "S-local".
Page 72, line 8:	"$\mathrm{L}_{\mathcal{C}}\mathcal{M}$" should be "$\mathrm{L}_{S}\mathcal{M}$".
Page 72, line 10:	"$\mathrm{L}_{\mathcal{C}}\mathcal{M}$" should be "$\mathrm{L}_{S}\mathcal{M}$".
Page 72, line 12:	"$\mathcal{C}$-local" should be "S-local".

Chapter 5

Page 83, line 11: "K-local equivalences" should be "K-colocal equivalences".
Page 83, line 15: "$\mathcal{C}$-colocal" should be "K-colocal".
Page 83, line 20: "$\mathcal{C}$-colocal" should be "K-colocal".
Page 83, line 21: "$\mathcal{C}$-local" should be "K-colocal".

Chapter 9

Page 163, line 4: "$X \times K^+$" should be "$X \wedge K^+$".

Summary of Part 2

Page 103, line 4 from bottom: "$\mathbf{R}U\colon \mathcal{N} \to \mathcal{M}$" should be "$\mathbf{R}U\colon \operatorname{Ho}\mathcal{N} \to \operatorname{Ho}\mathcal{M}$"

Chapter 15

Page 303, Proposition 15.6.19: The three appearances of "$\partial(\overrightarrow{\mathcal{C}}\downarrow\alpha)^{\mathrm{op}}$" should all be "$\partial(\alpha\downarrow\overleftarrow{\mathcal{C}})^{\mathrm{op}}$" and, on the last line, "be the element" should be "by the element".

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