CHAPTER 6: The Chicago Stems (π_N^S, $32 \le N \le 45$)

1. Introduction

The stems of dimensions 29 to 45 were first computed using the classical Adams spectral sequenceby Barratt, Mahowald and Tangora [10], [37] as corrected by Bruner [16]. Since all three of these authors have connections to the Chicago area we have taken the liberty to name these stems after that city. We continue our calculations from Chapter 5 to recompute these stems from the Atiyah-Hirzebruch spectral sequence. We compute the first seven of these stems in Section 2 and the remainder in Section 3. In Section 4 we collect all the computer computations of tentative differentials. The results of this chapter are summarized in Appendices 1 to 4.

2. Computation of π_N^S, $32 \le N \le 38$

In the tables of leaders below, all leaders of degree greater than 40 will have an asterisk at the left. They will be omitted from all other tables of leaders in this section except for the last one.

Recall from Figure 5.3.7 that there are five leaders of degree 33 and three leaders of degree 34. Now $A[30]M_1^2$ transgresses to $\nu A[30]$. By Lemma 3.3.14, $\eta^2 A[30]$ is divisible by two. As we shall see, the other elements of π_{32}^S all have order two. Thus, $\eta^2 A[30] = 0$ and $\eta A[30]M_1$ must bound. It follows from the following lemma that $d^8(A[16]M_1^6\overline{M}_2) = A[23]M_1^2\overline{M}_2$. Therefore, there is only one possibility: $d^{18}(\sigma^2 M_1^4 \overline{M}_2^2) = \eta A[30]M_1$.

LEMMA 6.2.1 (a) $\eta^2 A[30] = 0$

(b) $\sigma A[16] = A[23]$ and $\nu A[23] = 0$

PROOF. (b) Since $d^{10}(2\sigma M_1^5) = A[16]$, it follows from Theorem 2.4.2 that $A[16] \in \langle\sigma^2, 2, \eta\rangle$. Therefore, $\sigma\cdot A[16] \in \sigma\langle\sigma^2,2,\eta\rangle \subset \langle\sigma^3,2,\eta\rangle = \langle\nu C[18],2,\eta\rangle$. By Theorem 2.4.2, $A[23] = d^6(2C[18]M_1^3)$ is an element of the last triple product. This triple product has indeterminacy $\nu C[18]\cdot\pi_2^S + \eta\cdot\pi_{22}^S$ $= Z_2(4\nu C[20])$ because $\pi_2^S = Z_2\eta^2$ and $\pi_{22}^S = Z_2\nu A[19] \oplus Z_2\eta^2 C[20]$. Thus, $\sigma A[16]$ $= A[23] + 4k\nu C[20]$. However, $A[23]$ was only defined as $d^6(C[18]M_1^3) \in E^6_{0,23}$ $= \pi_{23}^S/Z_8\nu C[20]$. Thus, we can define $A[23] \in \pi_{23}^S$ so that it equals $\sigma A[16]$. ∎

We are thus left with three nonbounding leaders of degree 33 which define nonzero elements of π_{32}^S: $d^{24}(\eta^2\sigma M_1^5\bar{M}_3) = A[32,1]$, $d^{22}(2\beta_1 M_1^{11}) = A[32,2]$ and $d^{12}(\nu C[18]M_1^3\bar{M}_2) = A[32,3]$. As we shall see, $\eta A[32,2] \neq 0$ and $\nu A[32,3] \neq 0$. From the former relation it follows that $A[32,2]$ is not divisible by 2. Our computer computations show that $A[32,1]M_1^2$, $A[32,2]M_1^2$ is a d^{24}-boundary, d^{22}-boundary, respectively. Therefore $\nu A[32,1]$ and $\nu A[32,2]$ are divisible by η. It follows that $2\nu A[32,1] = 2\nu A[32,2] = 0$. Hence $A[32,3]$ can not be divisible by 2. Thus, π_{32}^S is an elementary abelian group. We have thus proved the following theorem.

THEOREM 6.2.2 $\pi_{32}^S = Z_2A[32,1] \oplus Z_2A[32,2] \oplus Z_2A[32,3] \oplus Z_2\eta\gamma_3$ and $\eta A[31] = 0$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
9	35	$\eta^2\sigma M_1^7\bar{M}_2^2$	*23	63	$\gamma_2 M_1^{20}$
11	35	$\beta_1 M_1^6\bar{M}_2^2$	23	35	$4\nu C[20]M_1^3\bar{M}_2$
15	37	$\gamma_1 M_1^8\bar{M}_2$	*24	60	$\eta A[23]M_1^{15}\bar{M}_2$
*17	51	$\eta^2\gamma_1 M_1^{17}$	28	36	$A[8]C[20]M_1\bar{M}_2$
18	38	$C[18](M_1^4\bar{M}_2^2+2M_1^7\bar{M}_2)$	30	34	$A[30]M_1^2$

*19	55	$\beta_2 M_1^{18}$	31	39	$\eta A[30]M_1\bar{M}_2$
*21	53	$\nu C[18]M_1^6\bar{M}_2\bar{M}_3$	31	35	$A[31]M_1^2$
*22	62	$\nu A[19]M_1^7M_2^2\langle M_3\rangle$	32	34	$A[32,1]M_1, A[32,2]M_1,$
*23	67	$\sigma A[16]M_1^6\bar{M}_2^3\bar{M}_3$			$A[32,3]M_1$

FIGURE 6.2.1: Leaders from Rows 1 to 32 of Degree at Least 34

There are four leaders of degree 34 and four leaders of degree 35. We know that $4\nu C[20]M_1^3\bar{M}_2$ transgresses to $A[14]C[20]$ and $A[31]M_1^2$ transgresses to $\nu A[31]$. Since $A[30] = d^{24}(2\sigma M_1^{12})$, $A[30]M_1^2$ could only bound from below the 7 row, and there are no such leaders of degree 35. Thus, $\nu A[30] = d^4(A[30]M_1^2) \neq 0$. Our computer computations in Section 4 show that $2\beta_1 M_1^6\bar{M}_2^2$ in E^{24} is in Image r_{Δ_1}. Therefore it can not hit $A[32,k]M_1$ for $k = 1,2,3$ and must transgress. If $\eta^2 A[32,k] \neq 0$ then by Theorem 3.3.14 it must be divisible by two. Assume that $\eta^2\sigma M_1^7\bar{M}_2^2$ transgresses. Then there is one leader of degree 36, $A[8]C[20]M_1\bar{M}_2$, which can hit an $\eta A[32,k_3]M_1$ and there are two leaders of degree 35 to transgress to elements of order four: $d^{26}(\eta^2\sigma M_1^7\bar{M}_2^2) = B_1$, $2B_1 = \eta^2 A[32,k_1]$ and $d^{24}(\beta_1 M_1^6\bar{M}_2^2) = B_2$, $2B_2 = \eta^2 A[32,k_2]$. Since $\eta^3 = 4\nu$, $\eta^3 A[32,k_1] = \eta^3 A[32,k_2] = 0$. Thus $\eta^2 A[32,k_1]M_1$ and $\eta^2 A[32,k_2]M_1$ must both bound. However, there is only one leader in degree 37 which can hit such an element: $\gamma_1 M_1^8\bar{M}_2$. This contradiction implies that $\eta^2\sigma M_1^7\bar{M}_2^2$ does not transgress and hits one of the $A[32,k]M_1$. Since $A[32,1]$ bounds from the 9 row it follows that if $A[32,1]M_1$ were to bound it would have to bound from below the 9 row. There are no leaders there. Thus, $A[32,1]M_1$ does not bound and $k \neq 1$. Recall from the proof of Theorem 5.3.9 that $d^{18}(\beta_1 M_1^{10}) = A[8]C[20]M_1$. Apply r_{Δ_1} to see that $2\beta_1 M_1^9$ hits $A[8]C[20]$, and let $R\{2\beta_1 M_1^9\}$ represent $2\beta_1 M_1^9$ such that $\partial R\{2\beta_1 M_1^9\} = A[8]C[20]$. Since $2\beta_1 M_1^{11}$ transgresses there must be an S in filtration degree 20 such that

$(R\{2\beta_1 M_1^9\} \wedge \mu_2) \cup S$ represents $2\beta_1 M_1^{11}$ and

$\partial S = A[32,2] \cup (R\{2\beta_1 M_1^9\} \wedge \nu) \cup (A[8]C[20] \wedge \mu_2)$. Then

$\partial \{[S \wedge \eta] \cup [R\{2\beta_1 M_1^9\} \wedge B_{\eta\nu}] \cup [B_{A[8]C[20]\eta} \wedge \mu_2] \cup [A[8]C[20](\mu_2 \cup_1 \eta)]$
$\cup [R\{2\beta_1 M_1^9\}(\nu \cup_1 \eta)]\} = (A[32,2] \wedge \eta) \cup (A[8]C[20]B_{\eta\nu}) \cup (B_{A[8]C[20]\eta} \wedge \nu)$.
Therefore, $\eta A[32,2] \in \langle A[8]C[20],\eta,\nu\rangle$ and $d^6(A[8]C[20]M_1M_2) = \eta A[32,2]M_1$.
In E^6, $A[8]M_1M_2$ is homologous to $A[8]M_1\bar{M}_2$. Therefore, $d^6(A[8]C[20]M_1\bar{M}_2)$
$= \eta A[32,2]M_1$. Note that $A[8]C[20]M_1\bar{M}_2$ is the only leader of degree 36 which could hit a leader of the form $\eta A[32,h]M_1$. Thus, $d^6(A[8]C[20]M_1\bar{M}_2)$ must be nonzero. Therefore, $k = 3$ and $d^{24}(\eta^2\sigma M_1^7\bar{M}_2^2) = A[32,3]M_1$. We have thus proved the following theorem.

THEOREM 6.2.3 $\pi_{33}^S = Z_2\nu A[30] \oplus Z_2\eta A[32,1] \oplus Z_2\eta A[32,2] \oplus Z_2\alpha_4 \oplus Z_2\eta^2\gamma_3$ and $\eta A[32,3] = 0$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
*9	63	$\eta^2\sigma M_1^{21}\bar{M}_2^2$	31	39	$\eta A[30]M_1\bar{M}_2$
11	35	$\beta_1 M_1^6\bar{M}_2^2$	31	35	$A[31]M_1^2$
15	37	$\gamma_1 M_1^8\bar{M}_2$	32	40	$A[32,1]M_1^4$
18	38	$C[18](M_1^4\bar{M}_2^2+2M_1^7\bar{M}_2)$	32	38	$A[32,2]\bar{M}_2$
23	35	$4\nu C[20]M_1^3\bar{M}_2$	32	36	$A[32,3]M_1^2$
28	36	$A[8]C[20]M_1\bar{M}_2$	33	37	$\nu A[30]M_1^2$
30	38	$A[30]M_1^4$	33	35	$\eta A[32,1]M_1$, $\eta A[32,2]M_1$

FIGURE 6.2.2: Leaders from Rows 1 to 33 of Degree at Least 35

There are five leaders of degree 35 and two leaders of degree 36. Now $A[32,3]M_1^2$ transgresses to $\nu A[32,3]$, and we proved above that $A[8]C[20]M_1\bar{M}_2$ hits $\eta A[32,2]M_1$. Thus, the other four leaders of degree 35 transgress to nonzero elements. One of them B[34] must have order four with 2B[34] = η^2A[32,1]. The only possibility is B[34] = $d^{24}(\beta_1 M_1^6 M_2^2)$. By Lemma 3.3.14,

$$B[34] \in \langle \eta, 2, A[32,1] \rangle \qquad [6.1]$$

We have thus proved the following theorem.

THEOREM 6.2.4 $\pi_{34}^S = Z_4 B[34] \oplus Z_2 A[14]C[20] \oplus Z_2 \nu A[31] \oplus Z_2 \eta\alpha_4$ where 2B[34] = η^2A[32,1] and η^2A[32,2] = 0.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
11	39	$Z_4(2\beta_1 M_1^{11} M_2)$	32	40	$A[32,1]M_1^4$
15	37	$\gamma_1 M_1^8 \bar{M}_2$	32	38	$A[32,2]\bar{M}_2$
18	38	$C[18](M_1^4 \bar{M}_2^2 + 2M_1^7 \bar{M}_2)$	32	36	$A[32,3]M_1^2$
28	38	$A[8]C[20]M_1^2 \bar{M}_2$	33	37	$\nu A[30]M_1^2$
30	38	$A[30]M_1^4$	*33	41	$\eta A[32,2]M_1 \bar{M}_2$
31	39	$\eta A[30]M_1 \bar{M}_2$	34	40	$\nu A[31]M_1^3$, $B[34]M_1^3$
31	37	$A[31]M_2$	34	36	$A[14]C[20]M_1$, $2B[34]M_1$

FIGURE 6.2.3: Leaders from Rows 1 to 34 of Degree at Least 36

There are three leaders of degree 36 and three leaders of degree 37. Clearly $A[31]M_2$ and $\nu A[30]M_1^2$ transgress. $d^2(2B[34]M_1) = 2\eta B[34] = 0$ and thus $2B[34]M_1$ must bound. The only possibility is $d^{20}(\gamma_1 M_1^8 \bar{M}_2) = 2B[34]M_1$. Since $A[32,1]\bar{M}_2$ is a d^{24}-boundary, νA[32,1] can not be ηA[14]C[20] and νA[32,1] must be zero. We have thus proved the following theorem.

THEOREM 6.2.5 $\pi_{35}^S = Z_2 \eta A[14]C[20] \oplus Z_2 \nu A[32,3] \oplus Z_8 \beta_4$ and ηB[34] = νA[32,1] = 0.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
11	39	$Z_4(2\beta_1 M_1^{11} M_2)$	32	38	$A[32,2]\bar{M}_2$, $A[32,3]M_2$
15	39	$Z_4 \gamma_1 M_1^{12}$	33	37	$\nu A[30]M_1^2$
18	38	$C[18](M_1^4\bar{M}_2^2 + 2M_1^7\bar{M}_2)$	34	40	$\nu A[31]M_1^3$, $B[34]M_1^3$
28	38	$A[8]C[20]M_1^2\bar{M}_2$	34	38	$A[14]C[20]M_1^2$
30	38	$A[30]M_1^4$	*34	42	$2B[34]M_1\bar{M}_2$
31	39	$\eta A[30]M_1\bar{M}_2$	*35	41	$\nu A[32,3]M_1^3$
31	37	$A[31]M_2$	35	37	$\eta A[14]C[20]M_1$
32	40	$A[32,1]M_1^4$			

FIGURE 6.2.4: Leaders from Rows 1 to 35 of Degree at Least 37

We will use the following lemma to see that $\nu A[30]M_1^2$ bounds.

LEMMA 6.2.6 (a) $\nu A[30] \in \langle\sigma, C[18], \sigma\rangle = \langle C[18], \sigma, 2\sigma\rangle$.

(b) $\nu^2 A[30] = 0$.

PROOF. (a) $d^{12}(2\sigma M_1^{12}) = A[30]$ so we can represent $2\sigma M_1^{12}$ by $\mathcal{M} = 2\sigma\langle M^4\rangle^3 \cup (B_{2\sigma 3\sigma} \wedge \langle M_1^4\rangle^2)$ union an element of filtration degree 14.

Then $\nu A[30] = \partial\ [(\nu \wedge \mathcal{M}) \cup \partial\ (B_{\nu 2\sigma} \wedge \langle M_1^4\rangle^3)]$

$= \partial\ [(\nu \wedge B_{2\sigma 3\sigma} \wedge \langle M_1^4\rangle^2) \cup (B_{\nu 2\sigma} \wedge 3\sigma\langle M_1^4\rangle^2) \cup F]$ where F has filtration degree 14;

$= \partial\ [3C[18]\langle M_1^4\rangle^2 \cup F]$ since $C[18] = \langle\nu, 2\sigma, \sigma\rangle$. By Theorems 2.4.2 and 2.3.7(a),

$$\nu A[30] \in \langle\sigma, C[18], \sigma\rangle = \langle C[18], \sigma, 2\sigma\rangle. \qquad [6.2]$$

(b) $\nu^2 A[30] \in \nu\langle\sigma, C[18], \sigma\rangle = \langle\nu, \sigma, C[18]\rangle\sigma \in \sigma \cdot \pi_{29}^S = 0.$ ∎

From Figure 6.2.4, we see that there are three leaders of degree 37 and six leaders of degree 38. Clearly $A[30]M_1^4$, $A[32,3]M_2$ and $A[14]C[20]M_1^2$ transgress. Since $d^{18}(\beta_1 M_1^{10}) = A[8]C[20]M_1$, we can apply r_{Δ_1} to see that $2\beta_1 M_1^9$ hits

A[8]C[20]. Using $r_{2\Delta_1+\Delta_2}$, we see that $d^{18}(2\beta_1 M_1^{11}M_2) = A[8]C[20]M_1^2\bar{M}_2$. Since $d^8(A[8]M_1^2\bar{M}_2) = \eta A[14]M_1$, $d^8(A[8]C[20]M_1^2\bar{M}_2) = \eta A[14]C[20]M_1$. Therefore, $\eta A[14]C[20]M_1$ must be zero in E^8. The only possibility is: $d^4(A[32,2]\bar{M}_2) = \eta A[14]C[20]M_1$. It follows that

$$\nu A[32,2] = \eta A[14]C[20]. \qquad [6.3]$$

By Lemma 6.2.6(b), $\nu A[30]M_1^2$ must bound. The only remaining possibility is that $d^{16}(C[18](M_1^4\bar{M}_2^2+2M_1^7\bar{M}_2)) = \nu A[30]M_1^2$. Now there is no possibility for $A[31]M_2$ to bound. Thus, $A[36] = d^6(A[31]M_2)$ defines a nonzero element of π_{36}^S. We have thus proved the following theorem.

THEOREM 6.2.7 $\pi_{36}^S = Z_2A[36]$

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
11	39	$Z_4(2\beta_1 M_1^{11}M_2)$	32	38	$A[32,3]M_2$
15	39	$Z_4\gamma_1 M_1^{12}$	*33	45	$\nu A[30]M_1^6$
18	40	$2C[18](M_1^4\bar{M}_3+3M_1^8\bar{M}_2)$	*33	41	$\eta A[32,2]M_1\bar{M}_2$
28	38	$A[8]C[20]M_1^2\bar{M}_2$	34	40	$\nu A[31]M_1^3$, $B[34]M_1^3$
30	38	$A[30]M_1^4$	34	38	$A[14]C[20]M_1^2$
31	39	$\eta A[30]M_1\bar{M}_2$	*35	43	$\eta A[14]C[20]M_1\bar{M}_2$
32	40	$A[32,1]M_1^4$	36	38	$A[36]M_1$

FIGURE 6.2.5: Leaders from Rows 1 to 36 of Degree at Least 38

The following relations were discovered by Bruner [16].

LEMMA 6.2.8 $\eta A[36] = \nu^2 A[31] = 0$

PROOF. Since $A[36] = d^6(A[31]M_2)$, Theorem 2.4.4(b) implies that

$$A[36] = \langle \nu, \eta, A[31] \rangle. \qquad [6.4]$$

Therefore, $\eta A[36] = \eta\langle\nu,\eta,A[31]\rangle = \langle\eta,\nu,\eta\rangle A[31] = \nu^2 A[31]$. Now $A[31] = d^{10}(\nu A[19]M_1^2\bar{M}_2)$, so $\nu^2 A[31] = \eta A[36]$ is hit by $\nu^3 A[19]M_1^2\bar{M}_2$ $= \eta A[8]A[19]M_1^2\bar{M}_2$. Thus, $A[36]$ is hit by $A[8]A[19]M_1^2\bar{M}_2$, and $d^{10}(A[8]A[19]M_1^3\bar{M}_2) = A[36]M_1$. Therefore, $\eta A[36] = 0$. ∎

There are five leaders of degree 38 and three leaders of degree 39. We have already observed in the proof of Theorem 6.2.7 that $d^{18}(2\beta_1 M_1^{11}M_2)$ $= A[8]C[20]M_1^2\bar{M}_2$. Recall that $A[8]\bar{M}_2 = d^6(2\nu M_1^3\bar{M}_2)$ and $A[14] = d^{12}(4\nu M_1^3\bar{M}_2)$. Note that $A[8]\bar{M}_2$ is represented by $(A[8] \wedge \bar{\mu}_2) \cup (B_{A[8]\nu} \wedge \mu_1) \cup B_{\langle A[8],\nu,\eta\rangle}$. Therefore, $2A[8]\bar{M}_2 \cup \partial\ [(B_{2A[8]} \wedge \bar{\mu}_{01}) \cup (B_{\langle 2,A[8],\nu\rangle} \wedge \mu_1)]$ $= (B_{2A[8]} \wedge B_{\nu\eta}) \cup 2B_{\langle A[8],\nu,\eta\rangle} \cup (B_{\langle 2,A[8],\nu\rangle} \wedge \eta)$ represeents $A[14]$. One conseqeuce is that

$$A[14] \in \langle 2,A[8],\nu,\eta\rangle. \qquad [6.5]$$

A second consequence is that $4\beta_1 M_1^9 M_2$ has a representative with boundary $A[14]C[20]$. Since $4\beta_1 M_1^{11}M_2$ survives to E^{24}, $d^{24}(4\beta_1 M_1^{11}M_2) = A[14]C[20]M_1^2$. Observe next that $A[30]M_1^4$ could only be hit from below the 7 row and there are no such leaders of degree 39. Therefore $\sigma A[30] = d^8(A[30]M_1^4) \neq 0$. Since $r_{2\Delta_1}(A[32,3]M_2) = A[32,3]M_1$ which bounds from the 9 row, it follows that $A[32,3]M_2$ can only bound from below the 9 row, and there are no such leaders of degree 39. Thus, $A[37] = d^6(A[32,3]M_2)$ is a nonzero element of π_{37}^S. By the proof of the preceding lemma, $d^{10}(A[8]A[19]M_1^3\bar{M}_2) = A[36]M_1$. Since $A[8]A[19] \in \mathrm{CokJ}_{27} = 0$, $A[36]M_1$ must bound from above the 27 row. There is only one possibility: $d^6(\eta A[30]M_1\bar{M}_2) = A[36]M_1$. (It follows that $(\eta A[30] + A[31])M_1^3$ is a d^{18}-boundary and the image of the leading differential $d^{18}(\sigma^2 M_1^4 M_2^2)$ should be denoted by $(\eta A[30] + A[31])M_1$ even though $A[31]M_1$ is zero in E^{18}.) We have thus proved the following theorem.

THEOREM 6.2.9 $\pi_{37}^S = Z_2 A[37] \oplus Z_2 \sigma A[30]$ and $\nu B[34] = \nu A[14]C[20] = 0$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
*11	41	$2\beta_1 M_1^6 \overline{M}_2^3$	*32	42	$A[32,3]M_1^5$
15	39	$Z_4(\gamma_1 M_1^{12})$	34	40	$\nu A[31]M_1^3$, $A[14]C[20]\overline{M}_2$, $B[34]M_1^3$
18	40	$2C[18](M_1^4\overline{M}_3+3M_1^8\overline{M}_2)$	*34	42	$2B[34]M_1\overline{M}_2$
*30	60	$A[30]\langle M_4\rangle$	*35	41	$\nu A[32,3]M_1^3$
31	41	$\eta A[30]M_1^5$	36	40	$A[36]M_1^2$
32	40	$A[32,1]M_1^4$	37	39	$A[37]M_1$, $\sigma A[30]M_1$

FIGURE 6.2.6: Leaders from Rows 1 to 37 of Degree at Least 39

The proof of the following lemma requires knowledge of the computation of π_N^S for $N \le 50$. Thus, the reader will understand the proof better after reading the rest of this chapter and Section 7.2.

LEMMA 6.2.10 $\eta\sigma A[30] = \eta A[37] = \nu^2 A[32,3]$ and $\sigma A[37] = 0$.

PROOF. Since $A[37] = d^6(A[32,3]M_2)$, Theorem 2.4.4(b) implies that

$$A[37] = \langle \nu, \eta, A[32,3]\rangle. \qquad [6.6]$$

Thus, $\sigma A[37] \in \sigma\langle \nu,\eta,A[32,3]\rangle = \langle\sigma,\nu,\eta\rangle A[32,3] = 0$. Also, $\eta A[37] = \eta\langle\nu,\eta,A[32,3]\rangle = \langle\eta,\nu,\eta\rangle A[32,3] = \nu^2 A[32,3]$. We shall see that there is an element $C[44]$ of order eight in π_{44}^S such that $2C[44] = d^{12}(\nu A[30]M_1^6)$ and $4C[44] = \sigma^2 A[30]$. By Theorem 2.4.2,

$$2C[44] \in \langle\nu,\nu A[30],\sigma\rangle = \langle\nu A[30],\nu,\sigma\rangle. \qquad [6.7]$$

Then $\sigma^2 A[30] = 4C[44] \in 2\langle\nu A[30],\nu,\sigma\rangle = \langle 2,\nu A[30],\nu\rangle\sigma$ and $\sigma A[30] + kA[37] \in \langle 2,\nu A[30],\nu\rangle$. Therefore, $\eta\sigma A[30] + k\eta A[37] \in \eta\langle 2,\nu A[30],\nu\rangle$ $= \langle\eta,2,\nu A[30]\rangle\nu = \langle\nu,\eta,2\rangle A[30] = 0$. Thus, $\eta\sigma A[30] = k\eta A[37]$. We shall see that it is only possible for one of $\eta\sigma A[30]$, $\eta A[37]$ to be zero. Thus, $\eta A[37] \neq 0$. Assume that $\eta\sigma A[30] = 0$. If one were to continue the computation through degree 50 one would have the table of leaders given in Appendix 4 with the following changes: delete $4C[44]M_1^3$, $A[40,1]M_1^2\bar{M}_2$ and add $\eta A[39,3]M_1\bar{M}_2$, $\sigma A[30]M_1^3\bar{M}_2$, $\eta A[37]M_1^3\bar{M}_2$. Then $\eta A[39,3]M_1\bar{M}_2 = \nu A[37]M_1\bar{M}_2$ would transgress to a nonzero element $\xi \in \langle\nu,\eta,\nu A[37],\eta\rangle = \langle\nu,\eta A[37],\nu,\eta\rangle$. Thus, $d^{10}(\eta A[37]M_1^3\bar{M}_2) = \xi M_1$. Now there is no way for $\eta^2 D[45]M_1$ to bound. Thus $0 \neq \eta^3 D[45] = 4\nu D[45]$. However $4\nu D[45]M_1 = d^4(4D[45]\bar{M}_2)$ and $4D[45]\bar{M}_2 = d^8(B[38]\langle M_3\rangle)$, a contradiction. Therefore, $\eta\sigma A[30] \neq 0$, $k = 1$ and $\eta\sigma A[30] = \eta A[37] = \nu^2 A[32,3]$. ∎

In Figure 6.2.6, there are three leaders of degree 39 and six leaders of degree 40. Clearly $A[36]M_1^2$ transgresses to $\nu A[36]$. Since $A[14]C[20]M_1^2$ bounds, $\nu A[14]C[20]$ is divisible by η and is zero by Theorem 6.2.9. Thus, $A[14]C[20]\bar{M}_2$ transgresses. By Lemma 6.2.8, $\nu A[31]M_1^3$ transgresses. By Lemma 6.2.10(a), $B[34]M_1^3$ transgresses. $A[32,1]M_1^4$ transgresses to $\sigma A[32,1]$. By Lemma 6.2.10(b), $(A[37]+\sigma A[30])M_1$ must bound. There is only one possibility: $d^{20}(2C[18](M_1^4\bar{M}_3+3M_1^8\bar{M}_2)) = (A[37]+\sigma A[30])M_1$. All the other leaders of degree 40 transgress. Thus, we have shown that π_{38}^S has a composition series of Z_2 and Z_4.

THEOREM 6.2.11 $\pi_{38}^S = Z_4 B[38] \oplus Z_2\eta\sigma A[30]$ where $\eta\sigma A[30] = \eta A[37]$ and $\sigma A[31]=0$.
PROOF. Let $B[38] = d^{12}(\gamma_1 M_1^{12})$. It remains to show that $4B[38] = 0$, not $\eta\sigma A[30]$. If $4B[38] = \eta\sigma A[30]$ then $4\gamma_1(M_1^{11}\bar{M}_3+2M_1^{15}\bar{M}_2+2M_1^{12}\bar{M}_2^2)$ hits $\eta\sigma A[30]M_1^3\bar{M}_2$ instead of $C[42]\langle M_1^4\rangle$. Now $\eta\sigma A[30]M_1^3\bar{M}_2$ can not hit $\eta^2 D[45]M_1$, $C[42]\langle M_1^4\rangle$ transgresses to $\sigma C[42]$ and there is no leader than can hit $\eta^2 D[45]M_1$. Therefore, $4\nu D[45] = \eta^3 D[45] \neq 0$ which leads to a contradiction as in the proof of Lemma 6.2.10(b). Thus, $4B[38] = 0$. Note that $A[31] = d^{10}(\nu A[19]M_1^2\bar{M}_2)$ is only defined modulo $Z_2\eta A[30]$. Therefore, we can define $A[31]$ so that $\sigma A[31] = 0$. ∎
The computations of Section 4 show that we have the following table of leaders. The leaders of all degrees have been included and those leaders of degree greater than 47 have an asterisk at the left.

Row	Degree	Leader	Row	Degree	Leader
*9	63	$\eta^2\sigma M_1^{21}M_2^2$	32	40	$A[32,1]M_1^4$
11	41	$2\beta_1 M_1^6\overline{M}_2^3$	32	42	$A[32,3]M_1^5$
15	41	$4\gamma_1 M_1^{10}\overline{M}_2$	33	45	$\nu A[30]M_1^6$
*17	51	$\eta^2\gamma_1 M_1^{17}$	33	41	$\eta A[32,2]M_1\overline{M}_2$
18	46	$4C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2)$	34	40	$\nu A[31]M_1^3$, $A[14]C[20]\overline{M}_2$,
*19	55	$\beta_2 M_1^{18}$			$B[34]M_1^3$
*21	53	$\nu C[18]M_1^6\overline{M}_2\overline{M}_3$	34	42	$2B[34]M_1\overline{M}_2$
*22	62	$\nu A[19]M_1^7M_2^2\langle M_3\rangle$	35	41	$\nu A[32,3]M_1^3$
*23	67	$\sigma A[16]M_1^6\overline{M}_2^3\overline{M}_3$	35	43	$\eta A[14]C[20]M_1\overline{M}_2$
*23	63	$\gamma_2 M_1^{20}$	36	40	$A[36]M_1^2$
*24	60	$\eta A[23]M_1^{15}\overline{M}_2$	37	41	$A[37]M_1^2$
*30	60	$A[30]\langle M_4\rangle$	37	45	$\sigma A[30]M_1^4$
31	41	$\eta A[30]M_1^5$	38	40	$\eta\sigma A[30]M_1$, $B[38]M_1$

FIGURE 6.2.7: Leaders from Rows 1 to 38 of Degree at Least 40

3. Computation of π_N^S, $39 \le N \le 45$.

We continue the computations of the preceding section. In the tables of leaders below, all leaders of degree greater than 47 will have an asterisk at the left. They will be omitted from all other tables of leaders except for the last one at the end of this section.

From the table of leaders in Figure 6.2.7, we see that there are seven leaders of degree 40 and six leaders of degree 41. By Lemma 6.2.10(b), $d^4(\nu A[32,3]M_1^3) = \eta A[37]M_1$ and clearly $A[37]M_1^2$ transgresses. Clearly $\eta A[32,2]M_1\overline{M}_2$ survives to E^6 and $d^6(\eta A[32,2]M_1\overline{M}_2)$ can not equal $B[38]M_1$ because $B[38]M_1$ can only bound from below the 15 row. Thus, $\eta A[32,2]M_1\overline{M}_2$ transgresses. Since $d^{24}(2\gamma_1 M_1^{13}) = 2B[38]M_1$ and $4B[38] = 0$, it follows that $4\gamma_1 M_1^{13}$ must transgress. Since

$d^{24}(\beta_1 M_1^6 \overline{M}_2^3) \neq 0$ in the 34 row, it follows that a nonzero differential on $2\beta_1 M_1^6 \overline{M}_2^3$ must land above the 34 row. There are three possibilities for a differential on $2\beta_1 M_1^6 \overline{M}_2^3$: $A[36]M_1^2$, $B[38]M_1$ or an element of π_{40}^S. Assume that $d^{26}(2\beta_1 M_1^6 \overline{M}_2^3) = B[38]M_1$. Then the new B[38]-leader is $B[38]M_1^3$ which is the only leader of degree 44 that can hit an element we will call $\eta A[40,1]M_1$ and which we will show must bound. It follows that $\nu B[38] = \eta A[40,1] \neq 0$. However, the computer calculations in Section 4 show that $B[38]M_1^2$ and $B[38]\overline{M}_2$ are d^{24}-boundaries which implies that $\nu B[38] = 0$, a contradiction. Thus $2\beta_1 M_1^6 \overline{M}_2^3$ must hit $A[36]M_1^2$ or transgress. If it transgresses to ξ then the computer calculation in Section 4 shows that ξM_1 does not bound and thus $\eta\xi \neq 0$. There is no possibility to bound $\eta\xi M_1$ and thus $\eta^2\xi \neq 0$. By Theorem 3.3.14, $\eta^2\xi$ is divisible by two. However, $\eta^2 d^{26}(4\gamma_1 M_1^{13})$ will also be divisible by two. There will be only one element which can be half of one of these elements. Therefore, $2\beta_1 M_1^6 \overline{M}_2^3$ can not transgress and must hit $A[36]M_1^2$. We have thus proved that π_{39}^S has a composition series of Z_2, $3Z_2$, Z_2 and Z_{16} from $\mathrm{Im}J_{39}$.

THEOREM 6.3.1 $\pi_{39}^S = Z_2\sigma A[32,1] \oplus Z_2\eta B[38] \oplus Z_2 A[39,1] \oplus Z_2 A[39,2] \oplus Z_2 A[39,3] \oplus Z_{16}\gamma_4$

and $\eta^2\sigma A[30] = 0$.

PROOF. Let $A[39,1] = d^6(\nu A[31]M_1^3)$, $A[39,2] = d^6(A[14]C[20]\overline{M}_2)$ and $A[39,3] = d^6(B[34]M_1^3) = d^6(B[34]\overline{M}_2)$. The only possibility for a nontrivial extension is that $2A[39,k]$ could equal $\eta B[38]$ instead of 0 for some $1 \leq k \leq 3$. By Theorem 2.4.2,

$$A[39,1] \in \langle \eta, \nu, \nu A[31] \rangle. \qquad [6.8]$$

Thus, $2A[39,1] \in 2\langle \eta, \nu, \nu A[31] \rangle = \langle 2, \eta, \nu \rangle \nu A[31] = 0$. By Theorem 2.4.4(c),

$$A[39,2] \in \langle \eta, \nu, A[14]C[20] \rangle. \qquad [6.9]$$

Thus $2A[39,2] \in 2\langle \eta, \nu, A[14]C[20] \rangle = \langle 2, \eta, \nu \rangle A[14]C[20] = 0$. By Theorem 2.4.4(c)

$$A[39,3] \in \langle \eta, \nu, B[34] \rangle. \qquad [6.10]$$

Thus, $2A[39,3] \in 2\langle \eta, \nu, B[34] \rangle = \langle 2, \eta, \nu \rangle B[34] = 0$. Therefore, CokJ_{39} is an elementary abelain group.∎

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
11	43	$2\beta_1 M_1^7 \overline{M}_2^3$	34	42	$2B[34]M_1\overline{M}_2$
15	41	$4\gamma_1 M_1^{13}$	35	43	$\eta A[14]C[20]M_1\overline{M}_2$
18	46	$4C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2+M_1^4\overline{M}_2\overline{M}_3)$	36	42	$A[36]M_1^3$
31	45	$\eta A[30]M_1^5$	37	41	$A[37]M_1^2$
*32	50	$A[32,1]M_1^2\overline{M}_3$	37	45	$\sigma A[30]M_1^4$
32	42	$A[32,3]M_1^5$	*38	52	$\eta\sigma A[30]M_1^3\overline{M}_2$
33	45	$\nu A[30]M_1^6$	*38	50	$B[38]M_1^6$
33	41	$\eta A[32,2]M_1\overline{M}_2$	39	45	$\sigma A[32,1]M_1^3$
34	46	$B[34]M_1^6$	39	41	$\eta B[38]M_1, A[39,1]M_1,$ $A[39,2]M_1, A[39,3]M_1$

FIGURE 6.3.1: Leaders from Rows 1 to 39 of Degree at Least 41

LEMMA 6.3.2 (a) $\eta B[38] = \nu A[36]$.

(b) $\nu A[37] = A[8]A[32,1] = \eta A[39,3] + \eta\sigma A[32,1]$.

PROOF. (a) $\gamma_1 M_1^{10}\overline{M}_2$ hits $2B[34]M_2$ and if it had survived to E^{24} it would have hit $B[38]M_1$. Now $\beta_1 M_1^6\overline{M}_2^3$ hits $B[34]M_2$. Therefore, if $2\beta_1 M_1^6\overline{M}_2^3$ had survived to E^{28} it would have hit $B[38]M_1$. However, $d^{26}(2\beta_1 M_1^6\overline{M}_2^3) = A[36]M_1^2$. Let $\mathscr{C}$ denote the mapping cone of $A[36]$. Let $'E^r$ denote the Atiyah-Hirzebruch spectral sequence with $'E^2_{n,t} = H_n BP \otimes \pi_t \mathscr{C} \Longrightarrow \mathscr{C}_{n+t}BP$. Then the canonical map $\phi: S \longrightarrow \mathscr{C}$ induces a map of Atiyah-Hirzebruch spectral sequences $\phi_r: E^r \longrightarrow {}'E^r$. Clearly $\phi_2(2\beta_1 M_1^6\overline{M}_2^3)$ survives to $'E^{28}$ and $d^{28}\phi_2(2\beta_1 M_1^6\overline{M}_2^3) = \phi_{28}(B[38]M_1)$. Thus, in π_{39}^S

$\eta B[38] \in A[36]\cdot\pi_3^S$, i.e. $\eta B[38] = k\nu A[36]$. Since $\eta B[38] \neq 0$ and $2\nu A[36] = 0$, k must equal 1.

(b) If $A[37]M_1^2$ is not a boundary then $\nu A[37] \neq 0$ and is not divisible by η. The $\nu A[37]$-leader will be $\nu A[37]M_1^3$. As we shall see, there is no leader of degree 47 below the 40 row. Therefore, $\xi = d^6(\nu A[37]M_1^3)$ is a nonzero element of π_{45}^S. ξM_1 is not a d^6-boundary and as we shall see the one leader of degree 48 below the 40 row bounds a different element. Therefore, $\eta\xi \neq 0$. By Theorem 2.4.2, $\xi \in \langle\eta, A[37], \nu^2\rangle$. Thus, $\eta\xi \in \eta\langle\eta, A[37], \nu^2\rangle \subset \langle\eta^2, A[37], \nu^2\rangle$ $= \langle\langle 2,\eta,2\rangle, A[37], \nu^2\rangle = \langle 2, \langle\eta, 2, A[37]\rangle, \nu^2\rangle + \langle 2, \eta, \langle 2, A[37], \nu^2\rangle\rangle$. Under our present assumptions ($A[37]M_1^2$ does not bound and $\sigma A[32,1]M_1 = d^8(A[32,1]M_1^2\bar{M}_2)$), $\pi_{44}^S = Z_8 C[44] \oplus Z_2 A[44]$ where $A[44] = d^6(\sigma A[32,1]M_1^3)$ and $\eta C[44] \neq 0$. Then
$\eta\xi \in \langle 2, \langle A[37], 2, \eta\rangle, \nu^2\rangle + \langle 2, \eta, hA[44]+2kC[44]\rangle$
$= \langle 2, \langle A[37], 2, \eta\rangle, \nu^2\rangle + h\langle 2, \eta, A[44]\rangle + kC[44]\langle 2, \eta, 2\rangle$.
By Theorem 2.4.2, $A[44] \in \langle\nu, A[32,1], \eta\sigma\rangle$. Thus, $\eta\xi \in$
$\langle\langle 2, A[37], 2\rangle, \eta, \nu^2\rangle + \langle 2, A[37], \langle 2, \eta, \nu^2\rangle\rangle + h\langle 2, \eta, \langle\nu, A[32,1], \eta\sigma\rangle\rangle + 2k\eta^2 C[44]$
$\supset \langle\eta A[37], \eta, \nu^2\rangle + \langle 2, A[37], 0\rangle + h\langle 2, \eta, \nu, A[32,1]\rangle\eta\sigma$ by Theorem 2.3.7(b). (Note that multiplication by η is a monomorphism on π_8^S and $\eta\langle 2, \eta, \nu^2\rangle = \langle\eta, 2, \eta\rangle\nu^2$ $= 2\nu^3 = 0$.) The four-fold Toda bracket above is defined by Theorem 2.2.7(a) because $\langle 2, \eta, \nu\rangle = 0$ and $\langle\eta, \nu, A[32,1]\rangle$ contains $d^6(A[32,1]\bar{M}_2) = 0$. Then
$\eta\xi \in \langle 0, \eta, \nu^2\rangle + (2, \eta\sigma, \eta^2) = (\nu^2, 2, \eta\sigma, \eta^2)$. Thus, either 2 divides $\eta\xi$, ν divides $\eta\xi$ or σ divides ξ. We will see that $2\cdot\pi_{46}^S = \nu\cdot\pi_{43}^S = 0$ and thus neither 2 nor ν can divide $\eta\xi$. Therefore, $\xi = \sigma B[38]$. Hence ξM_1 $= d^8(B[38]M_1^2\bar{M}_2)$. However, $B[38]M_1^2\bar{M}_2$ bounds from the 15 row, a contradiction. Therefore, $A[37]M_1^2$ must be a boundary. It can only bound from the 32 row or below. The only such leader is $A[32,3]M_1^5$ which maps to zero under $r_{2\Delta_1}$, and therefore does not hit $A[37]M_1^2$. Thus, $A[37]M_1^2$ must bound by a hidden differential. That is, there is a differential which we thought originated on the 32 row or below and landed in the 39 or 41 row which really hits $A[37]M_1^2$.

There is only one possibility: $d^6(A[32,1]M_1^2\overline{M}_2) = A[37]M_1^2$. Now $\sigma A[32,1]M_1$ becomes a nonbounding leader defining the nonzero element $\eta\sigma A[32,1]$ in π_{40}^S. By Theorem 2.4.4(c),

$$A[37] \in <\eta, \nu, A[32,1]>. \qquad [6.10]$$

Thus, $\nu A[37] \in \nu<\eta,\nu,A[32,1]> = <\nu,\eta,\nu>A[32,1] = A[8]A[32,1] \in <\eta,\nu,2\nu>A[32,1]$ $= \eta<\nu,2\nu,A[32,1]> = \eta<\nu^2,2,A[32,1]>= \eta<<\eta,\nu,\eta>,2,A[32,1]>$ $= \eta<\eta,<\nu,\eta,2>,A[32,1]> + \eta<\eta,\nu,<\eta,2,A[32,1]>>$ $= \eta<\eta,0,A[32,1]> + \eta<\eta,\nu,<\eta,2,A[32,1]>> = \eta<\eta,\nu,B[34]> + k\eta\sigma A[32,1]$ since $2<\eta,2,A[32,1]> = <2,\eta,2>A[32,1] = \eta^2A[32,1] = 2B[34]$. ($B[34] = d^{24}(\beta_1M_1^6\overline{M}_2^2)$ is only defined modulo $Z_2\nu A[31] \oplus Z_2A[14]C[20] \oplus Z_2(2B[34])$, so we can define $B[34]$ modulo $Z_2(2B[34])$ by insisting that $B[34] \in <\eta,2,A[32,1]>$.) By Theorem 2.4.4(c), $A[39,3] = d^6(B[34]\overline{M}_2) \in <\eta,\nu,B[34]>$. Thus, $\nu A[37] = \eta A[39,3] + k\eta\sigma A[32,1]$. We shall see that $d^{12}(B[34]<M_2^2>) = 2D[45]$ and $d^6(\eta A[39,3]M_1^3) = 4D[45]$. Then $4D[45] \in <2B[34],\sigma,\nu> = <\eta^2A[32,1],\sigma,\nu>$ $= <\eta,\eta\sigma A[32,1],\nu> = d^6(\eta A[39,3]M_1^3)$ in E^6. Thus, $\eta A[39,3]M_1^3 = \eta\sigma A[32,1]M_1^3$ in E^6. This can only occur if $d^4(A[37]M_1^2\overline{M}_2) = \eta A[39,3]M_1^3 + \eta\sigma A[32,1]M_1^3$. Therefore, $k = 1$. ∎

In addition to the hidden differential $d^6(A[32,1]M_1^2\overline{M}_2) = A[37]M_1^2$ which we uncovered in the proof of Lemma 6.3.2(b), there are eight leaders of degree 41 and three leaders of degree 42. Since $d^{24}(\beta_1M_1^7\overline{M}_2^3) = B[34]M_1\overline{M}_2$, $d^{24}(2\beta_1M_1^7\overline{M}_2^3) = 2B[34]M_1\overline{M}_2$. By Lemma 6.3.2(a), $d^4(A[36]M_1^3) = \eta B[38]M_1$. Since $2\cdot\pi_{41}^S = 0$, $\eta^2A[39,1] = 0$ by Lemma 3.3.14 . If $\eta A[39,1] \neq 0$ then $\eta A[39,1]M_1$ must bound and the only possibility is $d^{26}(2\gamma_1(M_1^{11}\overline{M}_2+10M_1^{14})) = \eta A[39,1]M_1$. However, this can not occur because $2\gamma_1(M_1^{11}\overline{M}_2+10M_1^{14})) \in \text{Image } r_{\Delta_1}$ and $\eta A[39,1]M_1 \notin \text{Image } r_{\Delta_1}$. It follows that $\eta A[39,1] = 0$. Thus $A[39,1]M_1$ must bound. The only possibility is $d^8(A[32,3]M_1^5) = A[39,1]M_1$ from which it follows that

$$\sigma A[32,3] = A[39,1]. \qquad [6.12]$$

Thus, π^S_{40} has a composition series of $3Z_2$, Z_2, Z_2, Z_2 and a Z_2 from ImJ_{40}.

THEOREM 6.3.3 $\pi^S_{40} = Z_4B[40] \oplus Z_2\eta A[39,3] \oplus Z_2\eta\sigma A[32,1] \oplus Z_2A[40,1] \oplus Z_2A[40,2] \oplus Z_2\eta\gamma_4$.

where $B[40] = C[20]^2$, $2B[40] = \eta A[39,2]$ and $\eta A[39,1] = \eta^2B[38] = 0$.

PROOF. Let $B[40] = d^8(\eta A[32,2]M_1\bar{M}_2)$. In the following argument which identifies $B[40]$ as $C[20]^2$ and $2B[40]$ as $\eta A[39,2]$ we compute modulo ImJ. By Theorem 2.4.5(a), $B[40] \in \langle\eta,\eta A[32,2],\eta,\nu\rangle$. Thus, $2B[40] \in 2\langle\eta,\eta A[32,2],\eta,\nu\rangle \subset \langle\langle 2,\eta,\eta A[32,2]\rangle,\eta,\nu\rangle$. Since $d^6(A[8]C[20]M_1\bar{M}_2) = \eta A[32,2]M_1$, $\eta A[32,2] \in \langle\nu,\eta,A[8]C[20]\rangle$ and $\langle 2,\eta,\eta A[32,2]\rangle \subset \langle 2,\eta,\langle\nu,\eta,A[8]C[20]\rangle\rangle$
$= \langle 2,\langle\eta,\nu,\eta\rangle,A[8]C[20]\rangle + \langle\langle 2,\eta,\nu\rangle,\eta,A[8]C[20]\rangle$
$= \langle 2,\nu^2,A[8]C[20]\rangle + \langle 0,\eta,A[8]C[20]\rangle = \langle 2,\nu^2,A[8]\rangle C[20]$ since $\sigma A[8] = 0$ and $2CokJ_{35} = 0$. Now $\langle 2,\nu^2,A[8]\rangle = \langle 2,\nu^2,\langle\eta,2,\nu^2\rangle\rangle$
$= \langle 2,\langle\nu^2,\eta,2\rangle,\nu^2\rangle + \langle\langle 2,\nu^2,\eta\rangle,2,\nu^2\rangle = \langle 2,0,\nu^2\rangle + \langle A[8],2,\nu^2\rangle = \langle A[8],2,\nu^2\rangle$
$= \langle A[8],2,\langle\eta,\nu,\eta\rangle\rangle = \langle A[8],2,\eta,\nu\rangle\eta$. This four-fold Toda bracket is defined by Theorem 2.2.7(b) since $0 \in \langle A[8],2,\eta\rangle$ and $0 = \langle 2,\eta,\nu\rangle$. Note that $A[8]^2 \in A[8]\langle\nu,\eta,\nu\rangle = \langle A[8],\nu,\eta\rangle\nu = 0$. Now $A[8]\langle A[8],2,\eta,\nu\rangle$
$= \langle\langle A[8],A[8],2\rangle,\eta,\nu\rangle$. However, $\langle A[8],A[8],2\rangle = \langle\langle\nu,\eta,\nu\rangle,A[8],2\rangle$
$= \nu\langle\eta,\nu,A[8],2\rangle = \nu A[14]$. This four-fold Toda bracket is defined by Theorem 2.2.7(a) since $\langle\eta,\nu,A[8]\rangle = 0$ and $\langle\nu,A[8],2\rangle = 0$. Thus, $A[8]\langle A[8],2,\eta,\nu\rangle = \langle\nu A[14],\eta,\nu\rangle \supset A[14]\langle\nu,\eta,\nu\rangle = A[14]A[8]$. Therefore, $A[14] + k\sigma^2 \in \langle A[8],2,\eta,\nu\rangle$ and $\eta A[14] \in \langle 2,\nu^2,A[8]\rangle$. Thus, $2B[40] \in \langle\eta A[14]C[20],\eta,\nu\rangle \supset C[20]\langle\eta A[14],\eta,\nu\rangle = 2C[20]^2$ by 5.6. We shall see that $\eta^2B[40]$ is nonzero and not divisible by two. By Lemma 3.3.14, $2C[20]^2 = 2B[40] \neq 0$. We showed above that $2B[40] \in \langle\langle 2,\eta,\eta A[32,2]\rangle,\eta,\nu\rangle$
$= \langle\langle\eta A[32,2],\eta,2\rangle,\eta,\nu\rangle \subset \langle\langle\eta,\eta A[32,2],2\rangle,\eta,\nu\rangle$

$= \eta\langle\eta A[32,2],2,\eta,\nu\rangle$ modulo $Z_2(\nu A[37])$ $\subset \langle\eta A[32,2],2,\langle\eta,\nu,\eta\rangle\rangle = \langle\eta A[32,2],2,\nu^2\rangle = \eta\langle A[32,2],2,\nu^2\rangle$. Now $\nu\langle A[32,2],2,\nu^2\rangle \subset \langle\nu A[32,2],2,\nu^2\rangle = \langle\eta A[14]C[20],2,\nu^2\rangle = A[14]C[20]\langle\eta,2,\nu^2\rangle$ $= A[14]C[20]A[8] = \eta^2C[20]^2$ which as we remarked above must be nonzero. Note that the four-fold Toda bracket above is defined by Theorem 2.2.7(b) because $0 \in \langle\eta A[32,2],2,\eta\rangle$ and $0 = \langle 2,\eta,\nu\rangle$. Since $A[39,2] = d^6(A[14]C[20]\bar{M}_2)$, $A[39,2] \in \langle\eta,\nu,A[14]C[20]\rangle$ and $\nu A[39,2] \in \nu\langle\eta,\nu,A[14]C[20]\rangle$ $= \langle\nu,\eta,\nu\rangle A[14]C[20] = A[8]A[14]C[20] = \eta^2C[20]$. Thus,

$$\nu A[39,2] = \eta^2C[20]^2 \qquad [6.13]$$

Now $\sigma\langle\eta A[32,2],2,\eta,\nu\rangle \subset \langle\eta A[32,2],2,\langle\eta,\nu,\sigma\rangle\rangle = \langle\eta A[32,2],2,0\rangle \in \eta A[32,2]\cdot\pi^S_{13}$ $= 0$. As we shall see, the only element $\xi \in \pi^S_{39}$ such that $\eta\xi \neq 0$, $\nu\xi = \eta^2C[20]$ and $\sigma\xi = 0$ is $A[39,2]$. Thus $2C[20]^2 = \eta A[39,2]$ modulo $Z_2(\nu A[37])$. Since $\nu A[37] = \eta A[39,3] + \eta\sigma A[32,1]$, $4C[20]^2 = 0$. Write $2C[20]^2 = \eta A[39,2] + h\eta A[39,3] + k\eta\sigma A[32,1]$. Redefine $A[39,2]$ as $A[39,2] + hA[39,3] + k\sigma A[32,1]$ so that η times the new $A[39,2]$ equals $2C[20]^2$. Note that $\nu A[39,2]$ and $\sigma A[39,2]$ remain unchanged. Let $A[40,1] = d^6(\eta A[30]M^5_1)$. By Theorem 2.4.2,

$$A[40,1] \in \langle\eta,(\eta A[30],\nu A[32,3]),(\sigma,\nu)^T\rangle. \qquad [6.14]$$

Then $2A[40,1] \in 2\langle\eta,(\eta A[30],\nu A[32,3]),(\sigma,\nu)^T\rangle = \langle 2,\eta,(\eta A[30],\nu A[32,3])\rangle(\sigma,\nu)^T$ $= \langle 2,\eta,\eta A[30]\rangle\sigma + \langle 2,\eta,\nu A[32,3]\rangle\nu = \langle 2,\eta,\eta A[30]\rangle\sigma + \langle 2,\eta,\nu\rangle\nu A[32,3]$

since $2\nu\cdot\pi^S_{37} = 0$

$= \langle 2,\eta,\eta A[30]\rangle\sigma \in \sigma\cdot\pi^S_{33} = \eta\sigma\cdot\pi^S_{32}$. Since $\eta\langle 2,\eta,\eta A[30]\rangle = \langle\eta,2,\eta\rangle\eta A[30]$ $= 2\nu\eta A[30] = 0$, $2A[40,1] \in \{0,\eta\sigma A[32,2]\} = \{0\}$. Let $A[40,2] = d^{26}(4\gamma_1 M^{13}_1)$. Since $d^{24}(2\gamma_1 M^{12}_1) = 2B[38]$, it follows from Theorem 2.4.2 that

$$A[40,2] \in \langle\eta,2,2B[38]\rangle. \qquad [6.15]$$

Thus, $2A[40,2] \in 2\langle\eta,2,2B[38]\rangle = \langle 2,\eta,2\rangle 2B[38] = 2\eta^2B[38] = 0$. ∎

The computations of Section 5 show that we have the following leaders.

Row	Degree	Leader
11	43	$2\beta_1 M_1^7\bar{M}_2^3$
15	43	$Z_8(2\gamma_1(M_1^{11}\bar{M}_2+10M_1^{14}))$
18	46	$4C[18](M_1^7\bar{M}_3+M_1^{11}\bar{M}_2+M_1^4\bar{M}_2\bar{M}_3)$
31	45	$\eta A[30]M_1M_2^2$
*32	50	$A[32,1]M_1^2\bar{M}_3$
33	45	$\nu A[30]M_1^6$
34	46	$B[34]M_1^6$
34	42	$2B[34]M_1\bar{M}_2$
35	43	$\eta A[14]C[20]M_1\bar{M}_2$
36	44	$A[36]M_1M_2$
37	43	$A[37]\bar{M}_2$
37	45	$\sigma A[30]M_1^4$
*39	53	$\sigma A[32,1]M_1^4\bar{M}_2$
*39	51	$\eta B[38]M_1^3\bar{M}_2$
*39	49	$A[39,1]M_1^2M_2$, $A[39,1]M_1^5$, $A[39,3]M_1^2\bar{M}_2$
39	45	$A[39,2]\bar{M}_2$
40	42	$\eta A[39,3]M_1$, $A[40,1]M_1$, $A[40,2]M_1$
40	42	$C[20]^2M_1$, $2C[20]^2M_1$, $\eta\sigma A[32,1]M_1$

FIGURE 6.3.2: Leaders from Rows 1 to 40 of Degree at Least 42

Observe that there are seven leaders of degree 42 and four leaders of degree 43. In the derivation of Theorem 6.3.3, we showed that $d^{24}(2\beta_1M_1^7\bar{M}_2^3) = 2B[34]M_1\bar{M}_2$. By Lemma 6.3.2(b), $d^4(A[37]\bar{M}_2) = \eta A[39,3]M_1 + \eta\sigma A[32,1]M_1$. In addition, $d^6(\eta A[14]C[20]M_1\bar{M}_2) = 2C[20]^2M_1$ because $d^6(\eta A[14]M_1^3) = 2C[20]$. Since $2\pi^S_{41} = 0$, $\eta^2\sigma A[32,1] = 0$ by Lemma 3.3.14. Thus, $\eta\sigma A[32,1]M_1$ must bound. There is only one possibility: $d^{26}(2\gamma_1(M_1^{11}\bar{M}_2+10M_1^{14})) = \eta\sigma A[32,1]M_1$. Now the remaining three leaders of degree 42 must transgress to define nonzero elements of π^S_{41}. All of these elements have order two because they are divisible by η. Note that $\nu B[38] = 0$ because $B[38]\bar{M}_2$ is a d^{24}-boundary. We have thus proved the following theorem.

THEOREM 6.3.4 $\pi^S_{41} = Z_2\eta C[20]^2 \oplus Z_2\eta A[40,1] \oplus Z_2\eta A[40,2] \oplus Z_2\alpha_5 \oplus Z_2\eta^2\gamma_4$ and $\eta^2\sigma A[32,1] = \eta^2A[39,2] = \eta^2A[39,3] = \nu B[38] = 0$.

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
*11	57	$4\beta_1(M_1^7\bar{M}_2^3\bar{M}_3+M_1^{10}\bar{M}_2^2\bar{M}_3+M_1^{14}\bar{M}_2^3)$	37	45	$\sigma A[30]M_1^4$
15	43	$Z_4(4\gamma_1)(M_1^{11}\bar{M}_2+2M_1^{14})$	39	45	$A[39,2]\bar{M}_2$
18	46	$4C[18](M_1^7\bar{M}_3+M_1^{11}\bar{M}_2+M_1^4\bar{M}_2\bar{M}_3)$	*40	52	$\eta A[39,3]M_1^3\bar{M}_2$, $A[40,1]M_1^6$
31	45	$\eta A[30]M_1M_2^2$	*40	66	$A[40,2]M_1^6\bar{M}_3$
33	45	$\nu A[30]M_1^6$	40	46	$C[20]^2\bar{M}_2$, $\eta\sigma A[32,1]M_1^3$
34	46	$B[34]M_1^6$, $2B[34]M_1^3\bar{M}_2$	*40	48	$2C[20]^2M_1\bar{M}_2$
36	44	$A[36]M_1M_2$	41	43	$\eta C[20]^2M_1$, $\eta A[40,1]M_1$, $\eta A[40,2]M_1$
*37	51	$(A[37]+\sigma A[30])M_1^4M_2$			

FIGURE 6.3.3: Leaders from Rows 1 to 41 of Degree at Least 43

There are four leaders of degree 43 and one leader of degree 44. Since

$A[40,1] \in \langle\eta,(\eta A[30],\nu A[32,3]),(\sigma,\nu)^T\rangle$,

$\eta^2A[40,1] \in \eta^2\langle\eta,(\eta A[30],\nu A[32,3]),(\sigma,\nu)^T\rangle \subset \langle\eta^3,(\eta A[30],\nu A[32,3]),(\sigma,\nu)^T\rangle$

$= \langle 4\nu,(\eta A[30],\nu A[32,3]),(\sigma,\nu)^T\rangle \subset \langle\nu,(4\eta A[30],4\nu A[32,3]),(\sigma,\nu)^T\rangle$

$= \langle\nu,0,(\sigma,\nu)^T\rangle = \sigma\cdot\pi_{35}^S + \nu\cdot\pi_{39}^S = \nu\cdot\pi_{39}^S$. Thus, if $\eta^2A[40,1] \neq 0$ then there is $\xi \in \pi_{39}^S$ such that either ξM_1^3 or $\xi\bar{M}_2$ is a leader and $\nu\xi = \eta^2A[40,1]$. There is only one possibility: $\xi = A[39,2]$. In the proof of Theorem 6.3.3 we showed that $\nu A[39,2] = \eta^2C[20]$. Thus $\eta^2A[40,1] = 0$ and $\eta A[40,1]M_1$ must bound. There is only one possibility: $d^6(A[36]M_1M_2) = \eta A[40,1]M_1$. By Theorem 2.4.4(b),

$$\eta A[40,1] \in \langle\nu,\eta,A[36]\rangle \qquad [6.16]$$

In addition, we have shown that

$$\nu A[39,1] = \nu A[39,3] = 0 \qquad [6.17]$$

Now the other three leaders of degree 43 must transgress to nonzero elements of π_{42}^S. Thus, π_{42}^S has a composition series of $2Z_2$, Z_4 and a Z_2 from ImJ_{42}.

THEOREM 6.3.5 $\pi_{42}^S = Z_8C[42] \oplus Z_2\eta^2C[20]^2 \oplus Z_2\eta\alpha_5$, $4C[42] = \eta^2A[40,2]$ and $\eta^2A[40,1] = 0$.

PROOF. By Lemma 3.3.14, $\eta^2 A[40,2]$ must be divisible by two. There is only one possibility: $4d^{28}(4\gamma_1 M_1^{11} M_2) = \eta^2 A[40,2]$. Thus, $C[42] = d^{28}(4\gamma_1(M_1^{11}\overline{M}_2+2M_1^{14}))$ has order eight and $4C[42] = \eta^2 A[40,2]$. ∎

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
15	45	$16\gamma_1 M_1^8 \langle M_3 \rangle$	39	45	$A[39,2]\overline{M}_2$
18	46	$4C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2+M_1^4\overline{M}_2\overline{M}_3)$	40	46	$\eta\sigma A[32,1]M_1^3, C[20]^2\overline{M}_2$
31	45	$\eta A[30]M_1 M_2^2$	41	47	$\eta A[40,1]M_1^3$
33	45	$\nu A[30]M_1^6$	42	46	$C[42]M_1^2$
34	46	$B[34]M_1^6$, $2B[34]M_1^3\overline{M}_2$	*42	48	$2C[42]M_1^3$
*36	54	$A[36]M_1^2\langle M_3 \rangle$	42	44	$4C[42]M_1$, $\eta^2 C[20]^2 M_1$
37	45	$\sigma A[30]M_1^4$			

FIGURE 6.3.4: Leaders from Rows 1 to 42 of Degree at Least 44

There are two leaders of degree 44 and five leaders of degree 45. In the proof of Theorem 6.3.3 we showed that $\nu A[39,2] = \eta^2 C[20]$. Thus, $d^4(A[39,2]\overline{M}_2) = \eta^2 C[20]^2 M_1$. Since $r_{\Delta_1}(16\gamma_1 M_1^8\langle M_3\rangle)$ is homologous to $16\gamma_1 M_1^{11} M_2$ in E^{16}, it follows that $d^{28}(16\gamma_1 M_1^8 \overline{M}_3) = 4C[42]M_1$. Thus all the leaders of degree 44 bound and we have proved the following theorem.

THEOREM 6.3.6 $\pi_{43}^S = Z_8 \beta_5$

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
*15	67	$7\gamma_1(M_1^{26}+2e)$	37	45	$\sigma A[30]M_1^4$
18	46	$4C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2+M_1^4\overline{M}_2\overline{M}_3)$	40	46	$\eta\sigma A[32,1]M_1^3, C[20]^2\overline{M}_2$

31	45	$\eta A[30]M_1M_2^2$	41	47	$\eta A[40,1]M_1^3$
33	45	$\nu A[30]M_1^6$	42	46	$C[42]M_1^2$
34	46	$B[34]M_1^6$, $2B[34]M_1^3\overline{M}_2$	*42	50	$\eta^2C[20]^2M_1\overline{M}_2$

FIGURE 6.3.5: Leaders from Rows 1 to 43 of Degree at Least 45

There are three leaders of degree 45 and six leaders of degree 46. If $\eta A[30]M_1M_2^2$, $\nu A[30]M_1^6$ or $\sigma A[30]M_1^4$ were to bound it would have to be hit by a leader of degree 46 which is below the 30 row. The only such leader is $4C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2+M_1^4\overline{M}_2\overline{M}_3)$. However, $d^{20}(2C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2+M_1^4\overline{M}_2\overline{M}_3))$ $= (\sigma A[30]+A[37])M_1M_2$. Therefore, $4C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2+M_1^4\overline{M}_2\overline{M}_3)$ survives to E^{22} and can not hit $\eta A[30]M_1M_2^2$, $\nu A[30]M_1^6$ or $\sigma A[30]M_1^4$. Thus, we have shown that π_{44}^S has a composition series of Z_2, Z_2, Z_2.

THEOREM 6.3.7 $\pi_{44}^S = Z_8C[44]$ where $4C[44] = \sigma^2A[30]$.

PROOF. Let $C[44] = d^{14}(\eta A[30]M_1M_2^2)$. Note that $\mu_1 \wedge \mu_1$ has boundary $(\eta \wedge \mu_1) \cup (\mu_1 \wedge \eta)$, and $d^4(M_1^2) = \nu$. Therefore, $2d^{14}(\eta A[30]M_1M_2^2)$ is represented by $d^{12}(\nu A[30]M_2^2)$. Since $\nu A[30]M_2^2$ is homologous in E^4 to $\nu A[30]M_1^6$, $2C[44] = d^{12}(\nu A[30]M_2^2)$. By Theorem 2.4.2,

$$2C[44] \in \langle\nu,\nu A[30],\sigma\rangle = \langle\nu A[30],\nu,\sigma\rangle. \qquad [6.18]$$

Note that $\mu_2 \wedge \mu_2$ has boundary $(\nu \wedge \mu_2) \cup (\mu_2 \wedge \nu)$, and $d^8\langle M_1^4\rangle = \sigma$. Therefore, $2d^{12}(\nu A[30]M_1^6)$ is represented by $d^8(\sigma A[30]M_1^4) = \sigma^2A[30]$. Thus, $4C[44] = \sigma^2A[30]$, and $C[44]$ has order eight.∎

The computations of Section 4 show that we have the following leaders.

Row	Degree	Leader	Row	Degree	Leader
18	46	$4C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2+M_1^4\overline{M}_2\overline{M}_3)$	42	46	$C[42]M_1^2$
34	46	$B[34]M_1^6$, $2B[34]M_1^3\overline{M}_2$	44	46	$C[44]M_1$

40	46	$\eta\sigma A[32,1]M_1^3$, $C[20]^2\overline{M}_2$	*44	56	$2C[44]M_1^6$
41	47	$\eta A[40,1]M_1^3$	*44	58	$4C[44]M_1^7$

FIGURE 6.3.6: Leaders from Rows 1 to 44 of Degree at Least 46

There are seven leaders of degree 46 and one leader of degree 47. Clearly $\eta A[40,1]M_1^3$ transgresses. Thus π_{45}^S has a composition series of Z_2, Z_2, $2Z_2$, $2Z_2$, Z_2.

THEOREM 6.3.8 $\pi_{45}^S = Z_{16}D[45] \oplus Z_2A[45,1] \oplus Z_2A[45,2] \oplus Z_2\eta C[44]$ where $8D[45] = \nu C[42]$.

PROOF. Note that $2\gamma_1M_1^{11}M_2$ hits $\eta\sigma A[32,1]M_1$ and $4\gamma_1M_1^{11}M_2$ hits $C[42]$. It follows that

$$C[42] \in \langle 2, \eta, \eta\sigma A[32,1]\rangle. \qquad [6.19]$$

Thus, $\nu C[42] \in \nu\langle 2,\eta,\eta\sigma A[32,1]\rangle = 2\langle\eta,\eta\sigma A[32,1],\nu\rangle$. By Theorem 2.4.4(a), $\langle\eta,\eta\sigma A[32,1],\nu\rangle$ contains $d^6(\eta\sigma A[32,1]M_1^3)$. Therefore, $\nu C[42] = d^4(C[42]M_1^2)$ is twice $d^6(\eta\sigma A[32,1]M_1^3)$. Since $\eta^2A[40,2] = 4C[42]$, $4C[42] \in 2\langle\eta,2,A[40,2]\rangle$ and

$$2C[42] \in \langle\eta,2,A[40,2]\rangle. \qquad [6.20]$$

Thus, $2\nu C[42] \in \nu\langle\eta,2,A[40,2]\rangle = \langle\nu,\eta,2\rangle A[40,2] = 0$. Let $\xi = d^{12}(B[34]M_1^6)$. We shall see in Chapter 7 that $\sigma B[34] = \eta A[40,1]$. Therefore by Theorem 2.4.2, $\xi \in \langle\nu,(B[34],A[40,1]),(\sigma,\eta)^T\rangle$. Thus, $2\xi \in 2\langle\nu,(B[34],A[40,1]),(\sigma,\eta)^T\rangle$
$\subset \langle\nu,(2B[34],2A[40,1]),(\sigma,\eta)^T\rangle = \langle\nu,(\eta^2A[32,1],0),(\sigma,\eta)^T\rangle$
$= \langle\nu,\eta^2A[32,1],\sigma\rangle$ modulo $\eta\cdot\pi_{44}^S$
$\supset \langle\nu,\eta,\eta\sigma A[32,1]\rangle$ modulo $\eta\cdot\pi_{44}^S$ and $\langle\nu,\eta,\eta\sigma A[32,1]\rangle$contains $d^6(\eta\sigma A[32,1]M_2)$.
Thus, 2ξ projects in E^6 to $d^6(\eta\sigma A[32,1]M_1^3)$. Let $A[45,1] = d^{12}(2B[34]M_1^3\overline{M}_2)$. Since $A[45,1]M_1$ can not bound, $\eta A[45,1] \neq 0$ and $A[45,1]$ is not divisible by two. Let $A[45,2] = d^6(C[20]^2\overline{M}_2)$. By Theorem 2.4.4(c),

$$A[45,2] \in \langle\eta,\nu,C[20]^2\rangle. \qquad [6.21]$$

Thus, $2A[45,2] \in 2\langle\eta,\nu,C[20]^2\rangle = \langle 2,\eta,\nu\rangle C[20]^2 = 0$, and $2A[45,2] = 0$. Since

$A[45,2]M_1$ can not bound, $\eta A[45,2] \neq 0$ and $A[45,2]$ can not be divisible by two. Now there is no possibility for $2A[45,1]$ to be nonzero. Let $D[45] = d^{28}(4C[18](M_1^7\bar{M}_3+M_1^{11}\bar{M}_2+M_1^4\bar{M}_2\bar{M}_3))$. We shall see in Chapter 7 that $\eta^2 D[45]$ is nonzero and not divisible by two. By Lemma 3.3.14, $2D[45] \neq 0$. The only possibility is that $2D[45] = \xi$ which we already know has order eight. In addition, we showed above that

$$2D[45] \in \langle \nu, (B[34], A[40,1]), (\sigma, \eta)^T \rangle \qquad [6.22]$$

Since $4D[45] = d^6(\eta\sigma A[32,1]M_1^3)$, it follows from that Theorem 2.4.2 that

$$4D[45] \in \langle \eta, \eta\sigma A[32,1], \nu \rangle \qquad [6.23]$$ ∎

The computations of Section 4 show that we have the following table of leaders. This table contains the leaders of all degrees.

Row	Degree	Leader	Row	Degree	Leader
9	63	$\eta^2\sigma M_1^{21}M_2^2$	39	51	$\eta B[38]M_1^3\bar{M}_2$
11	57	$4\beta_1(M_1^7\bar{M}_2^3\bar{M}_3+M_1^{10}\bar{M}_2^2\bar{M}_3+M_1^{14}\bar{M}_2^3)$	39	49	$A[39,1]M_1^2M_2$, $A[39,1]M_1^5$,
15	67	$\gamma_1(M_1^{26}+2e)$			$A[39,3]M_1^2\bar{M}_2$
17	51	$\alpha_2 M_1^{14}\bar{M}_2$	40	52	$A[40,1]M_1^6$,
18	64	$4C[18]M_1^7\bar{M}_2M_2^2\bar{M}_3$			$(\eta A[39,3]+\eta\sigma A[32,1])M_1^3\bar{M}_2$
19	55	$\beta_2 M_1^{18}$	40	66	$A[40,2]M_1^6\bar{M}_3$
21	53	$\nu C[18]M_1^6\bar{M}_2\bar{M}_3$	40	48	$2C[20]^2M_1\bar{M}_2$
22	62	$\nu A[19]M_1^7M_2^2\langle M_3\rangle$	41	47	$\eta A[40,1]M_1^3$
23	67	$\sigma A[16]M_1^6\bar{M}_2^3\bar{M}_3$	42	48	$2C[42]\bar{M}_2$
23	63	$\gamma_2 M_1^{20}$	42	50	$\eta^2C[20]^2M_1\bar{M}_2$
24	60	$\eta A[23]M_1^{15}\bar{M}_2$	44	48	$C[44]M_1^2$
30	60	$A[30]\langle M_4\rangle$	44	56	$2C[44]M_1^6$
32	50	$A[32,1]M_1^2\bar{M}_3$	44	58	$4C[44]M_1^7$
34	48	$B[34]M_1^4M_2$	45	47	$D[45]M_1$
34	64	$2B[34]M_1^5\bar{M}_2\bar{M}_3$	45	51	$2D[45]M_1^3$, $8D[45]M_1^3$

36	54	$A[36]M_1^2\langle M_3\rangle$	45	49	$4D[45]M_1^2$
38	50	$\eta\sigma A[30]M_1^3\bar{M}_2$, $B[38]M_1^6$	45	47	$\eta C[44]M_1$, $A[45,1]M_1$,
39	53	$\sigma A[32,1]M_1^4\bar{M}_2$			$A[45,2]M_1$, $D[45]M_1$

FIGURE 6.3.7: Leaders from Rows 1 to 45 of Degree at Least 47

4. Tentative Differentials

In this section we give the computer calculations of the tenative differentials which are determined by the leading differentials on leaders of degrees 33 through 46 which we established in Sections 2 and 3. We use the same notation and conventions as in Sections 4.4 and 5.4. These tables are ordered by the row numbers of the leaders.

<u>DEGREE 9:</u> $\eta^2\sigma$ and α_1

The leading differential $d^{24}(\eta^2\sigma M_1^5\bar{M}_3) = A[32,1]$ determines tentative differentials by making the following assignments to monomials of $[Z_2\eta^2\sigma M_1 \otimes B\langle 2\rangle] \oplus [Z_2\alpha_1 \otimes B\langle 2\rangle]$ of degree 33: $\eta^2\sigma M_1^3\bar{M}_2^3$ and $\eta^2\sigma M_1^5\bar{M}_3$ are assigned 1 and all other monomials are assigned 0. The kernel of these tentaive differentials in degrees less than 70 is given by the table below, and the new $\eta^2\sigma$-leader is $\eta^2\sigma M_1^7\bar{M}_2^2$.

DEGREE	BASIS				DEGREE	BASIS				DEGREE	BASIS			
(26,9)	7	2	0	0	(42,9)	7	0	2	0	(50,9)	5	2	2	0
											7	6	0	0
											11	0	2	0
(54,9)	7	2	2	0		21	2	0	0	(56,9)	3	1	1	1
	21	2	0	0		27	0	0	0		13	5	0	0
											21	0	1	0
											6	0	1	1
(58,9)	23	2	0	0										

The leading differential $d^{24}(\eta^2\sigma M_1^{13}) = A[32,3]M_1$ determines tentative differentials by making the following assignments to monomials of $[Z_2\eta^2\sigma M_1 \otimes B\langle 2\rangle] \oplus [Z_2\alpha_1 \otimes B\langle 2\rangle]$ of degree 35: $\eta^2\sigma M_1^3\bar{M}_2\bar{M}_3$, $\eta^2\sigma M_1^7\bar{M}_2^2$ are assigned 1 and all other monomials are assigned 0. The only remaining element in degrees less than 70 is $\eta^2\sigma(M_1^{21}\bar{M}_2^2+M_1^{27})$.

DEGREE 11: β_1

The leading differential $d^{18}(2\beta_1M_1^{14}) = A[8]C[20]M_1^2\bar{M}_2$ determines tentative differentials by making the following assignments to monomials of $Z_8\beta_1 \otimes H_*BP$ of degree 39: $\beta_1M_1^{11}M_2$ is assigned 1; $\beta_1M_1^5M_2^3$, $\beta_1M_1^8M_2^2$ are assigned 2; $\beta_1M_1^{14}$ is assigned 6; $\beta_1M_1^7M_3$, $\beta_1M_1^4M_2M_3$ are assigned 4 and all the other monomials are assigned 0. The kernel of these tentative differentials in degrees less than 70 is given by the table below, and the β_1-leader remains $2\beta_1M_1^8\bar{M}_2$.

DEGREE	GROUP	BASIS	DEGREE	GROUP	BASIS	DEGREE	GROUP	BASIS
(22,11)	Z_2	2/ 8 1 0 0	(24,11)	Z_2	6 2 0 0	(26,11)	Z_2	1/ 6 0 1 0 2/ 10 1 0 0
	Z_2	2/ 10 1 0 0	(28,11)	Z_2	2/ 11 1 0 0 1/ 4 1 1 0 3/ 14 0 0 0		Z_2	4/ 11 1 0 0
(30,11)	Z_2	2/ 12 1 0 0	(32,11)	Z_2	3/ 7 3 0 0 4/ 13 1 0 0 1/ 6 1 1 0 7/ 10 2 0 0		Z_4	1/ 7 3 0 0 7/ 10 2 0 0
	Z_4	6 3 0 0						
(34,11)	Z_2	1/ 7 1 1 0 7/ 10 0 1 0 3/ 14 1 0 0		Z_2	2/ 8 3 0 0	(36,11)	Z_2	2/ 9 3 0 0 6/ 15 1 0 0 1/ 2 3 1 0 3/ 8 1 1 0 7/ 12 2 0 0
				Z_2	2/ 14 1 0 0			
	Z_2	2/ 9 3 0 0 2/ 8 1 1 0		Z_2	4/ 15 1 0 0	(38,11)	Z_2	2/ 6 2 1 0 2/ 10 3 0 0
	Z_2	2/ 10 3 0 0 6/ 12 0 1 0	(40,11)	Z_2	3/ 7 2 1 0 6/ 11 3 0 0 6/ 13 0 1 0 1/ 4 3 1 0 3/ 10 1 1 0 6/ 14 2 0 0		Z_2	6/ 7 2 1 0 4/ 13 0 1 0 1/ 6 0 2 0 6/ 10 1 1 0 6/ 14 2 0 0
	Z_4	6 2 1 0						

Z_2 2/ 7 2 1 0
4/ 11 3 0 0
4/ 13 0 1 0
2/ 10 1 1 0

Z_2 4/ 11 3 0 0

Z_4 1/ 7 2 1 0
2/ 10 1 1 0
1/ 14 2 0 0

(42,11) Z_2 6/ 5 3 1 0
6/ 11 1 1 0
1/ 6 0 0 1
6/ 14 0 1 0

Z_2 2/ 5 3 1 0
2/ 11 1 1 0
6/ 12 3 0 0

Z_2 2/ 12 3 0 0

Z_2 2/ 14 0 1 0

Z_4 1/ 5 3 1 0
3/ 11 1 1 0
1/ 12 3 0 0
1/ 14 0 1 0

(44,11) Z_2 6/ 13 3 0 0
1/ 4 1 0 1
6/ 6 3 1 0
7/ 10 4 0 0
6/ 12 1 1 0

Z_2 2/ 6 3 1 0
2/ 12 1 1 0

Z_2 4/ 13 3 0 0

Z_4 1/ 4 6 0 0
1/ 6 3 1 0
7/ 10 4 0 0
2/ 12 1 1 0

(46,11) Z_2 2/ 6 1 2 0
6/ 10 2 1 0

Z_2 2/ 14 3 0 0

Z_2 2/ 8 5 0 0
2/ 10 2 1 0
2/ 14 3 0 0

Z_4 6 1 2 0

Z_8 1/ 7 3 1 0
4/ 13 1 1 0
7/ 10 2 1 0
1/ 14 3 0 0

(48,11) Z_2 1/ 4 2 2 0
6/ 6 1 0 1
3/ 6 6 0 0
6/ 8 3 1 0
1/ 10 0 2 0

Z_2 5/ 7 1 2 0
6/ 9 5 0 0
4/ 11 2 1 0
2/ 15 3 0 0
1/ 6 1 0 1
7/ 10 0 2 0
2/ 14 1 1 0

Z_2 2/ 7 1 2 0
4/ 9 5 0 0
6/ 11 2 1 0
2/ 15 3 0 0
2/ 6 6 0 0
2/ 8 3 1 0
6/ 10 0 2 0
6/ 14 1 1 0

Z_2 4/ 9 5 0 0
6/ 11 2 1 0
6/ 15 3 0 0
2/ 6 6 0 0
2/ 8 3 1 0
2/ 14 1 1 0

Z_2 2/ 11 2 1 0
2/ 15 3 0 0
6/ 14 1 1 0

Z_4 1/ 7 1 2 0
7/ 10 0 2 0
2/ 14 1 1 0

Z_2 2/ 8 1 2 0

Z_2 2/ 12 2 1 0

Z_2 4/ 15 1 1 0

Z_2 4/ 15 1 1 0
2/ 10 5 0 0
2/ 12 2 1 0

(50,11) Z_2 3/ 7 1 0 1
4/ 15 1 1 0
1/ 4 2 0 1
6/ 4 7 0 0
6/ 6 4 1 0
2/ 10 0 0 1
3/ 10 5 0 0

Z_2 3/ 5 7 0 0
6/ 7 4 1 0
4/ 9 1 2 0
7/ 11 5 0 0
3/ 13 2 1 0
1/ 2 3 0 1
1/ 4 5 1 0
7/ 6 2 2 0
7/ 8 1 0 1
5/ 14 4 0 0
6/ 26 0 0 0

Z_2 1/ 7 1 0 1
2/ 4 7 0 1
2/ 6 4 1 0
7/ 10 0 0 1
5/ 10 5 0 0

Z_2 2/ 6 4 1 0

Z_2 7/ 5 7 0 0
2/ 9 1 2 0
3/ 11 5 0 0
5/ 13 2 1 0
1/ 4 0 1 1
3/ 4 5 1 0
6/ 6 2 2 0
6/ 8 1 0 1
7/ 14 4 0 0
5/ 26 0 0 0

	Z_2	7/	5	7 0 0
		6/	9	1 2 0
		1/	11	5 0 0
		3/	13	2 1 0
		1/	4	5 1 0
		6/	6	2 2 0
		2/	8	1 0 1
		2/	14	4 0 0
		3/	26	0 0 0
	Z_2	2/	10	3 1 0

	Z_2	2/	5	7 0 0
		2/	7	4 1 0
		4/	9	1 2 0
		2/	11	5 0 0
		4/	13	2 1 0
		1/	6	2 2 0
		1/	26	0 0 0
	Z_2	4/	11	5 0 0
	Z_2	4/	13	2 1 0

	Z_2	6/	5	7 0 0
		6/	11	5 0 0
		2/	13	2 1 0
		2/	8	1 0 1
		2/	14	4 0 0
	Z_2	2/	5	7 0 0
		2/	7	4 1 0
		4/	9	1 2 0
		6/	11	5 0 0
		6/	14	4 0 0
		2/	26	0 0 0

(54,11)	Z_2	2/	5	5 1 0
		2/	9	1 0 1
		1/	2	6 1 0
		1/	4	3 2 0
		2/	6	0 3 0
		1/	6	2 0 1
		7/	6	7 0 0
		7/	10	1 2 0
		5/	12	5 0 0
		5/	14	2 1 0
		1/	20	0 1 0
		1/	24	1 0 0
	Z_4	6/	9	1 0 1
		1/	4	3 2 0
		1/	6	0 3 0
		2/	6	2 0 1
		3/	6	7 0 0
		1/	10	1 2 0
		2/	24	1 0 0

	Z_2	4/	11	3 1 0
		2/	4	3 2 0
		2/	6	2 0 1
		2/	6	7 0 0
		2/	10	1 2 0
	Z_2	2/	6	0 3 0
		2/	6	7 0 0
	Z_2	2/	10	1 2 0
		2/	12	5 0 0
		2/	24	1 0 0
	Z_4		6	0 3 0

	Z_2	2/	5	5 1 0
		2/	9	1 0 1
		6/	14	2 1 0
		6/	20	0 1 0
	Z_2	2/	10	1 2 0
		2/	12	5 0 0
		3/	20	0 1 0
		2/	24	1 0 0
	Z_2	2/	14	2 1 0
		2/	24	1 0 0
	Z_4	6/	9	1 0 1
		1/	6	2 0 1
		2/	6	7 0 0

(56,11)	Z_2	1/	5	3 2 0
		2/	7	0 3 0
		6/	7	7 0 0
		3/	11	1 2 0
		6/	13	5 0 0
		1/	15	2 1 0
		2/	25	1 0 0
		1/	0	7 1 0
		2/	4	1 3 0
		1/	4	3 0 1
		5/	6	0 1 1
		5/	10	1 0 1
		7/	10	6 0 0
		6/	12	3 1 0
		7/	14	0 2 0

	Z_2	3/	5	3 2 0
		6/	7	0 3 0
		3/	7	7 0 0
		7/	11	1 2 0
		2/	13	5 0 0
		1/	4	1 3 0
		7/	4	3 0 1
		2/	6	0 1 1
		5/	6	5 1 0
		7/	10	1 0 1
		6/	10	6 0 0
		2/	12	3 1 0
		2/	14	0 2 0
		1/	22	2 0 0

	Z_2	5/	5	3 2 0
		5/	7	0 3 0
		7/	7	7 0 0
		7/	11	1 2 0
		6/	13	5 0 0
		1/	4	3 0 1
		7/	6	0 1 1
		1/	10	1 0 1
		6/	14	0 2 0
		6/	22	2 0 0
	Z_2	2/	13	5 0 0
		4/	15	2 1 0
		6/	25	1 0 0
		3/	22	2 0 0

Z_2 5/ 5 3 2 0
2/ 7 0 3 0
7/ 7 7 0 0
7/ 11 1 2 0
6/ 13 5 0 0
1/ 6 0 1 1
2/ 6 5 1 0
6/ 10 1 0 1
5/ 10 6 0 0
5/ 14 0 2 0
1/ 22 2 0 0

Z_2 4/ 15 2 1 0
4/ 25 1 0 0
2/ 6 5 1 0
2/ 10 1 0 1
2/ 12 3 1 0
1/ 22 2 0 0

Z_2 2/ 5 3 2 0
4/ 7 0 3 0
2/ 7 7 0 0
6/ 11 1 2 0
4/ 13 5 0 0
2/ 12 3 1 0
6/ 14 0 2 0

Z_2 4/ 11 1 2 0

Z_4 1/ 5 3 2 0
1/ 7 0 3 0
7/ 7 7 0 0
3/ 11 1 2 0
6/ 15 2 1 0
2/ 25 1 0 0
7/ 10 6 0 0
2/ 12 3 1 0
6/ 14 0 2 0
5/ 22 2 0 0

Z_2 2/ 7 0 3 0
4/ 15 2 1 0
4/ 25 1 0 0
2/ 14 0 2 0
2/ 22 2 0 0

Z_2 4/ 7 7 0 0
2/ 10 1 0 1
6/ 10 6 0 0
2/ 12 3 1 0
6/ 22 2 0 0

Z_4 1/ 7 0 3 0
2/ 25 1 0 0
3/ 14 0 2 0
6/ 22 2 0 0

(58,11) Z_2 4/ 7 0 1 1
2/ 7 5 1 0
4/ 11 1 0 1
2/ 13 3 1 0
2/ 4 1 1 1
6/ 12 1 2 0
6/ 14 0 0 1
2/ 20 3 0 0
3/ 22 0 1 0
6/ 26 1 0 0

Z_2 2/ 7 5 1 0
6/ 13 3 1 0
6/ 14 5 0 0
2/ 22 0 1 0
6/ 26 1 0 0

Z_2 2/ 12 1 2 0
6/ 26 1 0 0

Z_4 6 3 2 0
26 1 0 0

Z_2 2/ 5 3 0 1
6/ 11 1 0 1
4/ 13 3 1 0
2/ 8 7 0 0
6/ 14 0 0 1
2/ 14 5 0 0

Z_2 2/ 4 6 1 0
2/ 6 3 2 0
2/ 14 5 0 0
2/ 26 1 0 0

Z_2 4/ 11 1 0 1

Z_2 2/ 8 7 0 0
2/ 12 1 2 0

Z_4 1/ 5 3 0 1
1/ 11 1 0 1
2/ 13 3 1 0
1/ 4 1 1 1
1/ 6 3 2 0
1/ 8 7 0 0
7/ 10 4 1 0
4/ 14 0 0 1
1/ 20 3 0 0
6/ 22 0 1 0

Z_2 1/ 7 0 1 1
2/ 7 5 1 0
4/ 11 1 0 1
2/ 13 3 1 0
6/ 4 6 1 0
5/ 10 4 1 0
7/ 14 0 0 1
7/ 20 3 0 0
3/ 26 1 0 0

Z_2 2/ 6 3 2 0

Z_2 2/ 14 5 0 0
3/ 20 3 0 0
6/ 22 0 1 0
5/ 26 1 0 0

Z_4 1/ 5 3 0 1
3/ 11 1 0 1
1/ 8 7 0 0
7/ 14 0 0 1
1/ 14 5 0 0
7/ 20 3 0 0
2/ 22 0 1 0
3/ 26 1 0 0

The leading differential $d^{22}(2\beta_1 M_1^8 \overline{M}_2) = A[32,2]$ determines tentative differentials by making the following assignments to monomials of $Z_8\beta_1 \otimes H_*BP$ of degree 33: $\beta_1 M_1^5 M_2^2$ is assigned 2, $\beta_1 M_1^{11}$ is assigned 1 and all the other monomials are assigned 0. The following table gives the kernel of these tentative differentials in degrees less than 70, and the new β_1-leader is $\beta_1 M_1^6 M_2^2$.

DEGREE	GROUP	BASIS	DEGREE	GROUP	BASIS	DEGREE	GROUP	BASIS
(24,11)	Z_2	6 2 0 0	(26,11)	Z_2	1/ 6 0 1 0 2/ 10 1 0 0	(28,11)	Z_2	4/ 11 1 0 0
	Z_2	2/ 11 1 0 0 1/ 4 1 1 0 3/ 14 0 0 0	(30,11)	Z_4	6 3 0 0	(32,11)	Z_2	3/ 7 3 0 0 4/ 13 1 0 0 1/ 6 1 1 0 7/ 10 2 0 0
	Z_4	1/ 7 3 0 0 7/ 10 2 0 0	(34,11)	Z_2	1/ 7 1 1 0 7/ 10 0 1 0 1/ 14 1 0 0	(36,11)	Z_2	6/ 15 1 0 0 1/ 2 3 1 0 1/ 8 1 1 0 3/ 12 2 0 0
	Z_2	4/ 15 1 0 0	(38,11)	Z_4	1/ 6 2 1 0 2/ 10 3 0 0	(40,11)	Z_2	4/ 11 3 0 0
	Z_2	3/ 7 2 1 0 6/ 11 3 0 0 6/ 13 0 1 0 1/ 4 3 1 0 3/ 10 1 1 0 6/ 14 2 0 0		Z_2	4/ 7 2 1 0 4/ 11 3 0 0 1/ 6 0 2 0 6/ 14 2 0 0		Z_4	1/ 7 2 1 0 2/ 10 1 1 0 1/ 14 2 0 0
(42,11)	Z_2	1/ 6 0 0 1 6/ 14 0 1 0		Z_2	2/ 5 3 1 0 2/ 11 1 1 0 6/ 12 3 0 0 6/ 14 0 1 0		Z_4	1/ 5 3 1 0 3/ 11 1 1 0 7/ 12 3 0 0 1/ 14 0 1 0
(44,11)	Z_2	6/ 13 3 0 0 1/ 4 1 0 1 4/ 6 3 1 0 3/ 10 4 0 0		Z_2	4/ 13 3 0 0	(46,11)	Z_4	6 1 2 0
				Z_4	1/ 4 6 0 0 7/ 6 3 1 0 3/ 10 4 0 0		Z_8	1/ 7 3 1 0 4/ 13 1 1 0 1/ 10 2 1 0 1/ 14 3 0 0
(48,11)	Z_2	5/ 7 1 2 0 6/ 9 5 0 0 2/ 11 2 1 0 1/ 4 2 2 0 5/ 6 1 0 1 5/ 6 6 0 0		Z_2	5/ 7 1 2 0 6/ 9 5 0 0 2/ 11 2 1 0 1/ 6 1 0 1 7/ 10 0 2 0		Z_2	2/ 7 1 2 0 4/ 15 3 0 0 6/ 10 0 2 0
							Z_4	1/ 7 1 2 0 7/ 10 0 2 0 2/ 14 1 1 0
(50,11)	Z_2	3/ 7 1 0 1 4/ 15 1 1 0 1/ 4 2 0 1 6/ 4 7 0 0 2/ 10 0 0 1 3/ 10 5 0 0		Z_2	1/ 7 1 0 1 2/ 4 7 0 0 7/ 10 0 0 1 5/ 10 5 0 0 6/ 12 2 1 0		Z_2	4/ 15 1 1 0

(52,11)

Z_2

6/	5	7	0	0
2/	9	1	2	0
4/	13	2	1	0
1/	2	3	0	1
2/	6	2	2	0
5/	8	1	0	1
7/	14	4	0	0

Z_2

7/	5	7	0	0
2/	9	1	2	0
3/	11	5	0	0
5/	13	2	1	0
1/	4	0	1	1
3/	4	5	1	0
6/	6	2	2	0
6/	8	1	0	1
7/	14	4	0	0
5/	26	0	0	0

Z_2

1/	5	7	0	0
6/	9	1	2	0
3/	11	5	0	0
1/	13	2	1	0
1/	4	5	1	0
6/	6	2	2	0
3/	26	0	0	0

Z_2

4/	11	5	0	0
4/	13	2	1	0
1/	6	2	2	0
6/	14	4	0	0
7/	26	0	0	0

Z_2

4/	11	5	0	0

Z_2

4/	13	2	1	0

(54,11)

Z_2

4/	5	5	1	0
4/	9	1	0	1
1/	2	6	1	0
1/	4	3	2	0
4/	6	0	3	0
1/	6	2	0	1
5/	6	7	0	0
1/	10	1	2	0
3/	12	5	0	0
3/	14	2	1	0
3/	24	1	0	0

Z_2

4/	11	3	1	0
2/	4	3	2	0
2/	6	2	0	1
2/	6	7	0	0
2/	10	1	2	0

Z_2

5/	20	0	1	0

Z_4

2/	5	5	1	0
1/	4	3	2	0
3/	6	0	3	0
2/	6	2	0	1
1/	6	7	0	0
3/	10	1	2	0
2/	12	5	0	0
6/	14	2	1	0
7/	20	0	1	0

Z_4

6/	5	5	1	0
6/	9	1	0	1
1/	6	0	3	0
6/	6	7	0	0
6/	10	1	2	0
2/	12	5	0	0
2/	14	2	1	0
2/	20	0	1	0
6/	24	1	0	0

Z_4

4/	5	5	1	0
2/	9	1	0	1
1/	6	2	0	1
2/	14	2	1	0
7/	20	0	1	0
2/	24	1	0	0

(56,11)

Z_2

4/	5	3	2	0
1/	7	7	0	0
2/	11	1	2	0
5/	15	2	1	0
6/	25	1	0	0
1/	0	7	1	0
3/	4	1	3	0
7/	6	0	1	1
3/	6	5	1	0
2/	10	1	0	1
1/	10	6	0	0
6/	12	3	1	0
5/	14	0	2	0

Z_2

1/	5	3	2	0
2/	7	0	3	0
1/	7	7	0	0
1/	11	1	2	0
6/	13	5	0	0
1/	4	1	3	0
7/	4	3	0	1
2/	6	0	1	1
1/	6	5	1	0
3/	10	1	0	1
2/	10	6	0	0
5/	22	2	0	0

Z_2

4/	11	1	2	0

Z_2

5/	5	3	2	0
5/	7	0	3	0
7/	7	7	0	0
7/	11	1	2	0
6/	13	5	0	0
4/	15	2	1	0
4/	25	1	0	0
1/	4	3	0	1
7/	6	0	1	1
6/	6	5	1	0
7/	10	1	0	1
6/	12	3	1	0
2/	14	0	2	0
5/	22	2	0	0

Z_2	5/ 5	3 2 0
	2/ 7	0 3 0
	7/ 7	7 0 0
	7/ 11	1 2 0
	6/ 13	5 0 0
	1/ 6	0 1 1
	2/ 6	5 1 0
	6/ 10	1 0 1
	5/ 10	6 0 0
	5/ 14	0 2 0
	1/ 22	2 0 0
Z_2	2/ 7	0 3 0
	4/ 15	2 1 0
	4/ 25	1 0 0
	2/ 14	0 2 0
	2/ 22	2 0 0
Z_2	2/ 13	5 0 0
	4/ 15	2 1 0
	6/ 25	1 0 0
	3/ 22	2 0 0
Z_4	1/ 5	3 2 0
	7/ 7	0 3 0
	7/ 7	7 0 0
	7/ 11	1 2 0
	6/ 15	2 1 0
	6/ 25	1 0 0
	2/ 6	5 1 0
	7/ 10	6 0 0
	6/ 22	2 0 0
Z_2	1/ 7	0 3 0
	2/ 25	1 0 0
	3/ 14	0 2 0
	6/ 22	2 0 0

(58,11)	Z	4/ 7	0 1 1
		2/ 7	5 1 0
		4/ 11	1 0 1
		2/ 13	3 1 0
		2/ 4	1 1 1
		2/ 4	6 1 0
		2/ 6	3 2 0
		6/ 14	0 0 1
		2/ 14	5 0 0
		2/ 20	3 0 0
		7/ 22	0 1 0
		2/ 26	1 0 0
	Z_4	1/ 5	3 0 1
		1/ 11	1 0 1
		2/ 13	3 1 0
		1/ 4	1 1 1
		1/ 6	3 2 0
		7/ 8	7 0 0
		7/ 10	4 1 0
		1/ 20	3 0 0
		6/ 22	0 1 0
		2/ 26	1 0 0
	Z_2	2/ 5	3 0 1
		6/ 11	1 0 1
		4/ 13	3 1 0
		2/ 8	7 0 0
		6/ 14	0 0 1
		2/ 14	5 0 0
	Z_2	2/ 7	5 1 0
		6/ 13	3 1 0
		2/ 6	3 2 0
		1/ 20	3 0 0
		5/ 26	1 0 0
	Z_2	4/ 11	1 0 1
	Z_4	1/ 5	3 0 1
		3/ 11	1 0 1
		7/ 8	7 0 0
		7/ 14	0 0 1
		1/ 14	5 0 0
		7/ 20	3 0 0
		2/ 22	0 1 0
		5/ 26	1 0 0
	Z_2	1/ 7	0 1 1
		4/ 11	1 0 1
		4/ 13	3 1 0
		6/ 4	6 1 0
		5/ 10	4 1 0
		3/ 14	0 0 1
		6/ 14	5 0 0
		7/ 20	3 0 0
		6/ 22	0 1 0
		5/ 26	1 0 0
	Z_4	6	3 2 0
		26	1 0 0

The leading differential $d^{24}(\beta_1 M_1^{12}) = B[34]$ determines tentative differntials by making the following assignments to monomials of $Z_8\beta_1 \otimes H_*BP$ of degree 35: $\beta_1 M_1^6 M_2^2$ is assigned 1 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the new β_1-leader is $4\beta_1 M_1^{11}\overline{M}_2$.

DEGREE	GROUP	BASIS	
(28,11)	Z_2	4/ 11	1 0 0
(30,11)	Z_2	2/ 6	3 0 0
(32,11)	Z_2	2/ 7	3 0 0
		6/ 10	2 0 0

(36,11) Z_2 4/ 15 1 0 0

(38,11) Z_2 2/ 6 2 1 0

(40,11) Z_2 2/ 7 2 1 0
2/ 14 2 0 0

Z_2 4/ 11 3 0 0

(42,11) Z_2 2/ 5 3 1 0
6/ 11 1 1 1
6/ 12 3 0 0
2/ 14 0 1 0

Z_2 4/ 11 1 1 0

(44,11) Z_2 4/ 13 3 0 0
2/ 4 1 0 1
6/ 4 6 0 0
2/ 6 3 1 0

Z_2 4/ 13 3 0 0

(46,11) Z_2 2/ 6 1 2 0

Z_4 2/ 7 3 1 0
2/ 10 2 1 0
2/ 14 3 0 0

(48,11) Z_2 2/ 7 1 2 0
6/ 10 0 2 0

Z_2 4/ 15 3 0 0

(50,11) Z_2 4/ 15 1 1 0

(52,11) Z_2 5/ 5 7 0 0
4/ 9 1 2 0
5/ 11 5 0 0
3/ 13 2 1 0
1/ 2 3 0 1
7/ 4 5 1 0
5/ 8 1 0 1
7/ 14 4 0 0
1/ 26 0 0 0

Z_2 4/ 11 5 0 0

Z_2 4/ 13 2 1 0

(54,11) Z_2 4/ 5 5 1 0
4/ 9 1 0 1
2/ 2 6 1 0
2/ 4 3 2 0
6/ 6 0 3 0
2/ 6 2 0 1
6/ 6 7 0 0
6/ 10 1 2 0
2/ 12 5 0 0
2/ 14 2 1 0
2/ 24 1 0 0

Z_2 4/ 5 5 1 0
2/ 4 3 2 0
6/ 6 0 3 0
2/ 6 7 0 0
6/ 10 1 2 0
6/ 20 0 1 0

Z_2 4/ 5 5 1 0
4/ 11 3 1 0
2/ 6 0 3 0
6/ 6 2 0 1
2/ 20 0 1 0

Z_2 4/ 9 1 0 1
2/ 6 2 0 1
6/ 20 0 1 0

Z_2 5/ 20 0 1 0

(56,11) Z_2 6/ 5 3 2 0
6/ 7 0 3 0
2/ 7 7 0 0
6/ 11 1 2 0
4/ 13 5 0 0
2/ 4 1 3 0
2/ 4 3 0 1
6/ 6 0 1 1
2/ 6 5 1 0
6/ 10 1 0 1
2/ 14 0 2 0
2/ 22 2 0 0

Z_2 2/ 7 0 3 0
4/ 25 1 0 0
6/ 14 0 2 0

Z_2 4/ 15 2 1 0
6/ 22 2 0 0

Z_2 4/ 11 1 2 0

(58,11) Z_2 2/ 5 3 0 1
2/ 11 1 0 1
4/ 13 3 1 0
2/ 4 1 1 1
2/ 6 3 2 0
6/ 8 7 0 0
6/ 10 4 1 0
2/ 20 3 0 0

Z_2 2/ 5 3 0 1
6/ 11 1 0 1
6/ 8 7 0 0
6/ 14 0 0 1
2/ 14 5 0 0
6/ 20 3 0 0
2/ 26 1 0 0

Z_2 4/ 7 0 1 1
6/ 4 6 1 0
2/ 6 3 2 0
6/ 10 4 1 0
3/ 20 3 0 0
5/ 22 0 1 0
5/ 26 1 0 0

Z_2	2/ 6 3 2 0 2/ 26 1 0 0	Z_2	4/ 11 1 0 1	Z_2	4/ 13 3 1 0 2/ 20 3 0 0 6/ 26 1 0 0

The leading differential $d^{24}(2\beta_1 M_1^7\overline{M}_2^3) = 2B[34]M_1\overline{M}_2$ determines tentative differentials by making the following assignments to monomials of $Z_8\beta_1 \otimes H_*BP$ of degree 43: $\beta_1 M_1^7 M_2^3$ is assigned 1; $\beta_1 M_1^{13}M_2$, $\beta_1 M_1^6 M_2 M_3$, $\beta_1 M_1^{10}M_2^2$ are assigned 2; $\beta_1 M_1 M_4$, $\beta_1 M_1^3 M_2^2 M_3$, $\beta_1 M_1^4 M_2^4$ are assigned 4 and all other monomials are assigned 0. The kernel of these tentative differntials is given by the table below, and the β_1-leader remains $4\beta_1 M_1^{11}\overline{M}_2$.

DEGREE	GROUP	BASIS	DEGREE	GROUP	BASIS	DEGREE	GROUP	BASIS
(28,11)	Z_2	4/ 11 1 0 0	(30,11)	Z_2	2/ 6 3 0 0	(36,11)	Z_2	4/ 15 1 0 0
(38,11)	Z_2	2/ 6 2 1 0	(40,11)	Z_2	4/ 11 3 0 0	(42,11)	Z_2	4/ 11 1 1 0
	Z_2	2/ 5 3 1 0 6/ 11 1 1 0 6/ 12 3 0 0 2/ 14 0 1 0	(44,11)	Z_2	4/ 13 3 0 0	(46,11)	Z_2 Z_2	2/ 6 1 2 0 4/ 7 3 1 0
(48,11)	Z_2	4/ 15 3 0 0	(50,11)	Z_2	4/ 15 1 1 0	(52,11)	Z_2 Z_2	4/ 11 5 0 0 4/ 13 2 1 0
(54,11)	Z_2	4/ 5 5 1 0 4/ 9 1 0 1 2/ 2 6 1 0 2/ 4 3 2 0 6/ 6 0 3 0 2/ 6 2 0 1 6/ 6 7 0 0 6/ 10 1 2 0 2/ 12 5 0 0 2/ 14 2 1 0 2/ 24 1 0 0		Z_2 Z_2	4/ 5 5 1 0 2/ 4 3 2 0 6/ 6 0 3 0 2/ 6 7 0 0 6/ 10 1 2 0 6/ 20 0 1 0 5/ 20 0 1 0		Z_2 Z_2	4/ 5 5 1 0 4/ 11 3 1 0 2/ 6 0 3 0 6/ 6 2 0 1 2/ 20 0 1 0 4/ 9 1 0 1 2/ 6 2 0 1 6/ 20 0 1 0
(56,11)	Z_2	4/ 15 2 1 0 6/ 22 2 0 0		Z_2	4/ 11 1 2 0			
(58,11)	Z_2	2/ 5 3 0 1 2/ 11 1 0 1 4/ 13 3 1 0 2/ 4 1 1 1 2/ 6 3 2 0 6/ 8 7 0 0 6/ 10 4 1 0 2/ 20 3 0 0		Z_2	2/ 5 3 0 1 6/ 11 1 0 1 6/ 8 7 0 0 6/ 14 0 0 1 2/ 14 5 0 0 6/ 20 3 0 0 2/ 26 1 0 0		Z_2	4/ 7 0 1 1 6/ 4 6 1 0 2/ 6 3 2 0 6/ 10 4 1 0 3/ 20 3 0 0 5/ 22 0 1 0 5/ 26 1 0 0

Z_2	2/ 6	3 2 0
	2/ 26	1 0 0
Z_2	4/ 11	1 0 1
Z_2	4/ 13	3 1 0
	2/ 20	3 0 0
	6/ 26	1 0 0

The tentative differential $d^{24}(4\beta_1 M_1^{11}\overline{M}_2) = A[14]C[20]M_1^2$ determines tentative differentials by making the following assignments to monomials of $Z_8\beta_1 \otimes H_*BP$ of degree 39: $\beta_1 M_1^{11}M_2$ is assigned 1; $\beta_1 M_1^5 M_2^3$, $\beta_1 M_1^8 M_2^2$ are assigned 2; $\beta_1 M_1^{14}$ is assigned 6; $\beta_1 M_1^7 M_3$, $\beta_1 M_1^4 M_2 M_3$ are assigned 4 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the new β_1-leader is $2\beta_1 M_1^6\overline{M}_2^3$.

DEGREE	GROUP	BASIS	
(30,11)	Z_2	2/ 6	3 0 0
(38,11)	Z_2	2/ 6	2 1 0
(42,11)	Z_2	2/ 5	3 1 0
		6/ 11	1 1 0
		6/ 12	3 0 0
		2/ 14	0 1 0
(46,11)	Z_2	2/ 6	1 2 0
	Z_2	4/ 7	3 1 0
(54,11)	Z_2	4/ 5	5 1 0
		4/ 9	1 0 1
		2/ 2	6 1 0
		2/ 4	3 2 0
		6/ 6	0 3 0
		2/ 6	2 0 1
		6/ 6	7 0 0
		6/ 10	1 2 0
		2/ 12	5 0 0
		2/ 14	2 1 0
		2/ 24	1 0 0
	Z_2	4/ 5	5 1 0
		2/ 4	3 2 0
		6/ 6	0 3 0
		2/ 6	7 0 0
		6/ 10	1 2 0
		6/ 20	0 1 0
	Z_2	4/ 9	1 0 1
		2/ 6	2 0 1
		6/ 20	0 1 0
	Z_2	5/ 20	0 1 0
(58,11)	Z_2	2/ 5	3 0 1
		2/ 11	1 0 1
		4/ 13	3 1 0
		2/ 4	1 1 1
		2/ 6	3 2 0
		6/ 8	7 0 0
		6/ 10	4 1 0
		2/ 20	3 0 0
	Z_2	2/ 6	3 2 0
		2/ 26	1 0 0
	Z_2	2/ 5	3 0 1
		6/ 11	1 0 1
		6/ 8	7 0 0
		6/ 14	0 0 1
		2/ 14	5 0 0
		6/ 20	3 0 0
		2/ 26	1 0 0
	Z_2	4/ 7	0 1 1
		4/ 13	3 1 0
		2/ 4	6 1 0
		6/ 6	3 2 0
		2/ 10	4 1 0
		7/ 20	3 0 0
		3/ 22	0 1 0
		1/ 26	1 0 0

The leading differential $d^{26}(2\beta_1 M_1^6 M_2^3\ ) = A[36]M_1^2$ determines tentative differentials by making the following assignments to monomials of $Z_8\beta_1 \otimes H_*BP$ of degree 41: $\beta_1 M_1^6 M_2^3$ is assigned 1; $\beta_1 M_1^8 M_3$ is assigned 2; $\beta_1 M_1^5 M_2 M_3$ $\beta_1 M_1^9 M_2^2$,

$\beta_1 M_1^{12} M_2$ are assigned 6; $\beta_1 M_1 M_3^2$, $\beta_1 M_1^3 M_2^4$, $\beta_1 M_4$ are assigned 4 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the new β_1-leader is $4\beta_1 M_1^7 \overline{M}_2^3 \overline{M}_3$.

DEGREE	GROUP	BASIS
(46,11)	Z_2	4/ 7 3 1 0
(54,11)	Z_2	4/ 5 5 1 0
		4/ 9 1 0 1
		2/ 4 3 2 0
		6/ 6 0 3 0
		2/ 6 2 0 1
		2/ 6 7 0 0
		6/ 10 1 2 0
	Z_2	5/ 20 0 1 0
(58,11)	Z_2	4/ 5 3 0 1
		4/ 7 0 1 1
		2/ 4 1 1 1
		6/ 4 6 1 0
		6/ 14 0 0 1
		2/ 14 5 0 0
		5/ 20 3 0 0
		1/ 22 0 1 0
		1/ 26 1 0 0
	Z_2	4/ 7 0 1 1
		4/ 13 3 1 0
		6/ 4 6 1 0
		6/ 10 4 1 0
		1/ 20 3 0 0
		5/ 22 0 1 0
		1/ 26 1 0 0

DEGREE 14: σ^2

The leading differential $d^{18}(\sigma^2 M_1^4 M_2^2) = \eta A[30]M_1 + A[31]M_1$ determines tentative differentials which are a monomorphism on $E^{18}_{*,14}$ in degrees less than 69. There are no remaining elements.

DEGREE 15: γ_1

The leading differential $d^{20}(\gamma_1 M_1^8 \overline{M}_2) = \eta^2 A[32,1]M_1$ determines tentative differentials by making the following assignments to monomials of $Z_{32}\gamma_1 \otimes H_*BP$ of degree 37: $\gamma_1 M_1^{11}$ is assigned 1 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the γ_1-leader is $\gamma_1 M_1^{12}$.

DEGREE	GROUP	GENERATOR
(24,15)	Z_4	12 0 0 0
(26,15)	Z_4	2/ 10 1 0 0
(28,15)	Z_2	2/ 11 1 0 0
		22/ 14 0 0 0

	Z_{16}	14	0 0 0
(30,15)	Z_4	26/ 15	0 0 0
		16/ 8	0 1 0
		30/ 12	1 0 0
	Z_{32}	14/ 15	0 0 0
		1/ 8	0 1 0
		7/ 12	1 0 0
(32,15)	Z_4	6/ 13	1 0 0
		1/ 10	2 0 0
	Z_8	2/ 13	1 0 0
(34,15)	Z_4	2/ 10	0 1 0
	Z_{32}	10	0 1 0
		14	1 0 0
(36,15)	Z_2	20/ 15	1 0 0
		2/ 8	1 1 0
		6/ 12	2 0 0
	Z_2	2/ 11	0 1 0
		28/ 15	1 0 0
		22/ 12	2 0 0
	Z_{16}	14/ 15	1 0 0
		1/ 12	2 0 0
	Z_{16}	2/ 15	1 0 0
(38,15)	Z_4	20/ 13	2 0 0
		2/ 10	3 0 0
		22/ 12	0 1 0
	Z_4	2/ 13	2 0 0
		2/ 6	2 1 0
		8/ 10	3 0 0
		20/ 12	0 1 0
	Z_{32}	6	2 1 0
(40,15)	Z_2	10/ 11	3 0 0
		18/ 13	0 1 0
		4/ 8	4 0 0
		30/ 10	1 1 0
		10/ 14	2 0 0
	Z_4	26/ 11	3 0 0
		14/ 13	0 1 0
		1/ 8	4 0 0
		2/ 10	1 1 0
		24/ 14	2 0 0
	Z_8	2/ 10	1 1 0
		2/ 14	2 0 0
	Z_8	2/ 11	3 0 0
	Z_{16}	14	2 0 0
(42,15)	Z_4	18/ 11	1 1 0
		6/ 15	2 0 0
		2/ 8	2 1 0
		6/ 12	3 0 0
		24/ 14	0 1 0
	Z_4	2/ 6	5 0 0
		4/ 12	3 0 0
	Z_8	2/ 11	1 1 0
		28/ 12	3 0 0
		28/ 14	0 1 0
	Z_{32}	1/ 6	5 0 0
		2/ 12	3 0 0
		1/ 14	0 1 0
	Z_{32}	10/ 15	2 0 0
		1/ 8	2 1 0
		23/ 12	3 0 0
		27/ 14	0 1 0
(44,15)	Z_2	24/ 13	3 0 0
		4/ 15	0 1 0
		2/ 6	3 1 0
		22/ 10	4 0 0
		26/ 12	1 1 0
	Z_2	2/ 9	2 1 0
		2/ 15	0 1 0
		10/ 10	4 0 0
		12/ 12	1 1 0
	Z_8	2/ 13	3 0 0
		30/ 15	0 1 0
		4/ 10	4 0 0
		14/ 12	1 1 0
	Z_{16}	10	4 0 0
	Z_{16}	18/ 13	3 0 0
		20/ 15	0 1 0
		4/ 12	1 1 0
	Z_{32}	6	3 1 0
(46,15)	Z_4	16/ 11	4 0 0
		2/ 4	4 1 0
		2/ 8	0 0 1
		22/ 8	5 0 0
		14/ 10	2 1 0
		4/ 14	3 0 0
	Z_4	2/ 11	4 0 0
		12/ 13	1 1 0
		2/ 8	5 0 0
		24/ 10	2 1 0
		28/ 14	3 0 0
	Z_8	26/ 11	4 0 0
		26/ 13	1 1 0
		4/ 8	0 0 1
		4/ 10	2 1 0
	Z_{32}	30/ 11	4 0 0
		4/ 13	1 1 0
		1/ 4	4 1 0
		1/ 8	0 0 1
		1/ 8	5 0 0
		15/ 10	2 1 0
		7/ 14	3 0 0
	Z_{32}	12/ 11	4 0 0
		2/ 13	1 1 0
		1/ 8	0 0 1
		1/ 8	5 0 0
		10/ 10	2 1 0
		22/ 14	3 0 0

(48, 15)	Z_2 2/ 9 5 0 0 10/ 11 2 1 0 4/ 15 3 0 0 2/ 6 6 0 0 6/ 8 3 1 0 30/ 10 0 2 0 30/ 12 4 0 0 10/ 14 1 1 0	Z_4 6/ 11 2 1 0 1/ 6 6 0 0 2/ 8 3 1 0 7/ 12 4 0 0 6/ 14 1 1 0	Z_4 8/ 9 5 0 0 18/ 11 2 1 0 12/ 15 3 0 0 16/ 8 3 1 0 23/ 10 0 2 0 5/ 12 4 0 0 20/ 14 1 1 1
	Z_8 2/ 9 5 0 0 16/ 15 3 0 0 1/ 10 0 2 0 2/ 12 4 0 0 28/ 14 1 1 0	Z_8 2/ 8 3 1 0 2/ 14 1 1 0 Z_8 2/ 11 2 1 0	Z_{16} 16/ 9 5 0 0 6/ 15 3 0 0 16/ 12 4 0 0 12/ 14 1 1 0 Z_{16} 2/ 14 1 1 0
(50, 15)	Z_4 22/ 9 3 1 0 14/ 13 4 0 0 26/ 15 1 1 0 4/ 4 7 0 0 12/ 6 4 1 0 4/ 8 1 2 0 18/ 10 0 0 1 22/ 10 5 0 0 22/ 12 2 1 0	Z_4 2/ 6 4 1 0 12/ 12 2 1 0 Z_4 8/ 9 3 1 0 12/ 13 4 0 0 4/ 8 1 2 0 10/ 10 0 0 1 28/ 10 5 0 0 16/ 12 2 1 0	Z_4 14/ 9 3 1 0 8/ 13 4 0 0 2/ 15 1 1 0 2/ 10 0 0 1 18/ 10 5 0 0 18/ 12 2 1 0
	Z_{16} 8/ 9 3 1 0 30/ 13 4 0 0 10/ 15 1 1 0 2/ 8 1 2 0 18/ 10 5 0 0 20/ 12 2 1 0	Z_{32} 26/ 9 3 1 0 30/ 13 4 0 0 30/ 15 1 1 0 1/ 4 7 0 0 17/ 10 5 0 0 7/ 12 2 1 0	Z_{32} 26/ 9 3 1 0 22/ 13 4 0 0 16/ 15 1 1 0 1/ 10 0 0 1 25/ 10 5 0 0 20/ 12 2 1 0
(52, 15)	Z_2 28/ 11 5 0 0 12/ 13 2 1 0 2/ 4 5 1 0 10/ 8 1 0 1 26/ 8 6 0 0 20/ 10 3 1 0 2/ 12 0 2 0 4/ 14 4 0 0 16/ 26 0 0 0	Z_2 8/ 11 5 0 0 28/ 13 2 1 0 2/ 8 1 0 1 28/ 8 6 0 0 8/ 10 3 1 0 18/ 12 0 2 0 26/ 14 4 0 0 20/ 26 0 0 0	Z_2 2/ 7 4 1 0 4/ 11 0 0 1 28/ 11 5 0 0 12/ 13 2 1 0 14/ 8 6 0 0 30/ 14 4 0 0 17/ 26 0 0 0
	Z_2 2/ 11 0 0 1 8/ 11 5 0 0 12/ 13 2 1 0 12/ 8 6 0 0 16/ 10 3 1 0 14/ 12 0 2 0 10/ 14 4 0 0 21/ 26 0 0 0	Z_2 24/ 11 5 0 0 16/ 13 2 1 0 16/ 8 6 0 0 8/ 10 3 1 0 12/ 12 0 2 0 24/ 14 4 0 0 31/ 26 0 0 0	Z_8 28/ 13 2 1 0 1/ 8 6 0 0 10/ 10 3 1 0 27/ 12 0 2 0 29/ 14 4 0 0 27/ 26 0 0 0
	Z_8 30/ 11 5 0 0 18/ 13 2 1 0 2/ 10 3 1 0 8/ 12 0 2 0 18/ 14 4 0 0 23/ 26 0 0 0	Z_{16} 30/ 11 5 0 0 28/ 13 2 1 0 1/ 12 0 2 0 18/ 14 4 0 0 4/ 26 0 0 0	Z_{16} 2/ 11 5 0 0 30/ 13 2 1 0 14/ 14 4 0 0 1/ 26 0 0 0

Z_{16}	2/ 13	2 1 0
	3/ 14	4 0 0
	12/ 26	0 0 0

Z_{16}	1/ 14	4 0 0
	22/ 26	0 0 0

(54, 15)

Z_{2}	16/ 9	6 0 0
	16/ 11	3 1 0
	20/ 15	4 0 0
	4/ 6	2 0 1
	28/ 6	7 0 0
	16/ 8	4 1 0
	20/ 10	1 2 0
	28/ 12	0 0 1
	24/ 12	5 0 0
	8/ 14	2 1 0
	10/ 24	1 0 0

Z_{4}	8/ 9	6 0 0
	8/ 11	3 1 0
	20/ 13	0 2 0
	8/ 15	4 0 0
	2/ 6	2 0 1
	18/ 6	7 0 0
	24/ 8	4 1 0
	6/ 10	1 2 0
	30/ 12	0 0 1
	18/ 12	5 0 0

Z_{4}	20/ 11	3 1 0
	2/ 6	7 0 0
	30/ 8	4 1 0
	2/ 12	5 0 0
	20/ 14	2 1 0

Z_{4}	24/ 9	6 0 0
	12/ 11	3 1 0
	20/ 13	0 2 0
	18/ 15	4 0 0
	2/ 8	4 1 0
	26/ 10	1 2 0
	12/ 12	0 0 1
	18/ 12	5 0 0
	8/ 14	2 1 0
	6/ 24	1 0 0

Z_{4}	26/ 9	6 0 0
	20/ 11	3 1 0
	8/ 13	0 2 0
	4/ 15	4 0 0
	2/ 10	1 2 0
	24/ 12	0 0 1
	22/ 12	5 0 0
	28/ 14	2 1 0
	12/ 24	1 0 0

Z_{4}	26/ 9	6 0 0
	16/ 11	3 1 0
	14/ 13	0 2 0
	22/ 15	4 0 0
	4/ 12	0 0 1
	24/ 12	5 0 0
	4/ 24	1 0 0

Z_{8}	8/ 9	6 0 0
	24/ 11	3 1 0
	16/ 13	0 2 0
	20/ 15	4 0 0
	1/ 6	2 0 1
	1/ 8	4 1 0
	19/ 12	5 0 0
	14/ 14	2 1 0
	18/ 24	1 0 0

Z_{8}	2/ 9	6 0 0
	30/ 11	3 1 0
	28/ 13	0 2 0
	16/ 15	4 0 0
	10/ 12	5 0 0
	4/ 14	2 1 0
	12/ 24	1 0 0

Z_{32}	30/ 11	3 1 0
	30/ 13	0 2 0
	20/ 15	4 0 0
	1/ 6	7 0 0
	1/ 8	4 1 0
	5/ 12	5 0 0
	7/ 14	2 1 0

Z_{32}	30/ 11	3 1 0
	20/ 15	4 0 0
	1/ 8	4 1 0
	11/ 12	5 0 0
	30/ 14	2 1 0
	2/ 24	1 0 0

Z_{32}	12/ 11	3 1 0
	24/ 13	0 2 0
	14/ 15	4 0 0
	1/ 12	0 0 1
	1/ 12	5 0 0
	20/ 14	2 1 0
	12/ 24	1 0 0

The leading differential $d^{24}(\gamma_1 M_1^{12}) = B[38]$ determines tentative differentials by making the following assignments to monomials of $Z_{32}\gamma_1 \otimes H_*BP$ of degree 39: $\gamma_1 M_1^{12}$ is assigned 1 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the new γ_1-leader is $4\gamma_1 M_1^{10}\overline{M}_2$.

DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR
(26,15)	Z_2	4/ 10 1 0 0	(28,15)	Z_8	2/ 11 1 0 0	(30,15)	Z_2	20/ 15 0 0 0
					20/ 14 0 0 0			32/ 8 0 1 0
								28/ 12 1 0 0
	Z_8	16/ 15 0 0 0	(32,15)	Z_4	20/ 13 1 0 0	(34,15)	Z_2	4/ 10 0 1 0
		4/ 8 0 1 0			4/ 10 2 0 0			
		4/ 12 1 0 0						
	Z_8	4/ 14 1 0 0	(36,15)	Z_8	2/ 11 0 1 0		Z_8	4/ 15 1 0 0
					20/ 12 2 0 0			
(38,15)	Z_2	28/ 13 2 0 0		Z_2	8/ 13 2 0 0		Z_8	28/ 13 2 0 0
		4/ 6 2 1 0			4/ 10 3 0 0			8/ 12 0 1 0
		12/ 10 3 0 0			12/ 12 0 1 0			
		28/ 12 0 1 0						
(40,15)	Z_4	8/ 11 3 0 0		Z_4	4/ 11 3 0 0		Z_8	30/ 11 3 0 0
		24/ 13 0 1 0			4/ 10 1 1 0			2/ 13 0 1 0
		4/ 8 4 0 0						2/ 8 4 0 0
		12/ 10 1 1 0						2/ 10 1 1 0
								8/ 14 2 0 0
(42,15)	Z_2	4/ 6 5 0 0		Z_2	4/ 11 1 1 0		Z_8	2/ 11 1 1 0
		8/ 12 3 0 0			12/ 15 2 0 0			2/ 6 5 0 0
					4/ 8 2 1 0			28/ 14 0 1 0
					12/ 12 3 0 0			
					16/ 14 0 1 0			
	Z_8	4/ 11 1 1 0		Z_8	4/ 14 0 1 0	(44,15)	Z_4	12/ 13 3 0 0
		4/ 15 2 0 0						12/ 15 0 1 0
		16/ 12 3 0 0						4/ 6 3 1 0
		4/ 14 0 1 0						8/ 10 4 0 0
								24/ 12 1 1 0
	Z_8	2/ 9 2 1 0		Z_8	16/ 13 3 0 0		Z_8	16/ 13 3 0 0
		20/ 13 3 0 0			8/ 15 0 1 0			20/ 15 0 1 0
		10/ 15 0 1 0			4/ 10 4 0 0			16/ 12 1 1 0
		2/ 6 3 1 0			4/ 12 1 1 0			
		30/ 10 4 0 0						
		10/ 12 1 1 0						
(46,15)	Z_2	4/ 4 4 1 0		Z_2	4/ 11 4 0 0		Z_4	20/ 11 4 0 0
		4/ 8 0 0 1			24/ 13 1 1 0			20/ 13 1 1 0
		12/ 8 5 0 0			4/ 8 5 0 0			8/ 8 0 0 1
		28/ 10 2 1 0			16/ 10 2 1 0			8/ 10 2 1 0
		8/ 14 3 0 0			24/ 14 3 0 0			
	Z_8	12/ 11 4 0 0		Z_8	4/ 14 3 0 0	(48,15)	Z_4	4/ 9 5 0 0
		16/ 13 1 1 0						16/ 11 2 1 0
		4/ 8 0 0 1						24/ 15 3 0 0
		24/ 10 2 1 0						4/ 6 6 0 0
								8/ 8 3 1 0
								8/ 12 4 0 0

	Z_4	12/ 9	5 0 0
		4/ 11	2 1 0
		8/ 15	3 0 0
		8/ 10	0 2 0
		28/ 12	4 0 0
		24/ 14	1 1 0
	Z_8	14/ 9	5 0 0
		14/ 11	2 1 0
		8/ 15	3 0 0
		1/ 6	6 0 0
		12/ 8	3 1 0
		13/ 10	0 2 0
		27/ 12	4 0 0
		12/ 14	1 1 0
	Z_8	32/ 9	5 0 0
		12/ 15	3 0 0
		24/ 14	1 1 0
	Z_8	4/ 14	1 1 0
(50,15)	Z_2	16/ 9	3 1 0
		12/ 13	4 0 0
		16/ 15	1 1 0
		8/ 4	7 0 0
		24/ 6	4 1 0
		8/ 8	1 2 0
		8/ 10	5 0 0
		8/ 12	2 1 0
	Z_2	16/ 9	3 1 0
		24/ 13	4 0 0
		8/ 8	1 2 0
		20/ 10	0 0 1
		24/ 10	5 0 0
	Z_2	28/ 9	3 1 0
		16/ 13	4 0 0
		4/ 15	1 1 0
		4/ 10	0 0 1
		4/ 10	5 0 0
		4/ 12	2 1 0
	Z_4	4/ 9	3 1 0
		26/ 13	4 0 0
		20/ 15	1 1 0
		4/ 4	7 0 0
		14/ 6	4 1 0
		2/ 10	0 0 1
		4/ 10	5 0 0
		28/ 12	2 1 0
	Z_8	4/ 9	3 1 0
		14/ 13	4 0 0
		4/ 15	1 1 0
		2/ 6	4 1 0
		14/ 10	0 0 1
	Z_8	28/ 13	4 0 0
		4/ 15	1 1 0
		4/ 8	1 2 0
		20/ 10	5 0 0
		24/ 12	2 1 0
	Z_8	4/ 9	3 1 0
		24/ 13	4 0 0
		20/ 15	1 1 0
		4/ 12	2 1 0
(52,15)	Z_2	24/ 11	5 0 0
		16/ 13	2 1 0
		16/ 8	6 0 0
		8/ 10	3 1 0
		12/ 12	0 2 0
		24/ 14	4 0 0
		31/ 26	0 0 0
	Z_4	28/ 11	5 0 0
		20/ 13	2 1 0
		2/ 4	5 1 0
		10/ 8	1 0 1
		24/ 8	6 0 0
		12/ 12	0 2 0
		10/ 14	4 0 0
		26/ 26	0 0 0
	Z_4	28/ 7	4 1 0
		24/ 11	0 0 1
		4/ 11	5 0 0
		4/ 13	2 1 0
		4/ 8	1 0 1
		16/ 8	6 0 0
		12/ 10	3 1 0
		4/ 12	0 2 0
		28/ 14	4 0 0
		25/ 26	0 0 0
	Z_8	2/ 7	4 1 0
		2/ 11	0 0 1
		20/ 11	5 0 0
		12/ 13	2 1 0
		28/ 10	3 1 0
		28/ 12	0 2 0
		30/ 26	0 0 0
	Z_8	2/ 11	0 0 1
		12/ 11	5 0 0
		16/ 13	2 1 0
		20/ 8	6 0 0
		4/ 12	0 2 0
		8/ 14	4 0 0
		29/ 26	0 0 0
	Z_8	16/ 11	5 0 0
		8/ 13	2 1 0
		4/ 8	6 0 0
		28/ 10	3 1 0
		16/ 12	0 2 0
		4/ 14	4 0 0
		24/ 26	0 0 0
	Z_8	4/ 11	5 0 0
		14/ 26	0 0 0
(54,15)	Z_2	4/ 9	6 0 0
		4/ 13	0 2 0
		12/ 15	4 0 0
		8/ 12	5 0 0
		20/ 24	1 0 0
	Z_2	16/ 15	4 0 0
		4/ 6	2 0 1
		4/ 8	4 1 0
		12/ 12	5 0 0
		24/ 14	2 1 0
		8/ 24	1 0 0

Z_2	12/ 9	6 0 0
	16/ 13	0 2 0
	24/ 15	4 0 0
	4/ 6	7 0 0
	28/ 8	4 1 0
	28/ 10	1 2 0
	16/ 12	0 0 1
	24/ 12	5 0 0
	16/ 14	2 1 0
	8/ 24	1 0 0

Z_2	16/ 9	6 0 0
	24/ 11	3 1 0
	8/ 13	0 2 0
	4/ 15	4 0 0
	4/ 8	4 1 0
	20/ 10	1 2 0
	24/ 12	0 0 1
	4/ 12	5 0 0
	16/ 14	2 1 0
	12/ 24	1 0 0

Z_2	20/ 9	6 0 0
	8/ 11	3 1 0
	16/ 13	0 2 0
	8/ 15	4 0 0
	4/ 10	1 2 0
	16/ 12	0 0 1
	12/ 12	5 0 0
	24/ 14	2 1 0
	24/ 24	1 0 0

Z_2	12/ 9	6 0 0
	16/ 11	3 1 0
	28/ 15	4 0 0
	4/ 12	0 0 1
	28/ 12	5 0 0
	16/ 14	2 1 0
	14/ 24	1 0 0

Z_4	2/ 9	6 0 0
	16/ 11	3 1 0
	14/ 13	0 2 0
	2/ 15	4 0 0
	2/ 6	2 0 1
	2/ 6	7 0 0
	2/ 10	1 2 0
	2/ 12	0 0 1
	22/ 12	5 0 0
	24/ 14	2 1 0
	16/ 24	1 0 0

Z_4	4/ 11	3 1 0
	12/ 13	0 2 0
	12/ 15	4 0 0
	20/ 12	5 0 0
	24/ 14	2 1 0
	28/ 24	1 0 0

Z_8	4/ 13	0 2 0
	16/ 15	4 0 0
	28/ 12	5 0 0
	16/ 14	2 1 0
	12/ 24	1 0 0

Z_8	16/ 15	4 0 0
	4/ 14	2 1 0
	16/ 24	1 0 0

Z_8	16/ 15	4 0 0
	4/ 12	5 0 0
	16/ 14	2 1 0
	14/ 24	1 0 0

The leading differential $d^{26}(4\gamma_1 M_1^{10}\bar{M}_2) = A[40,2]$ determines tentative differentials by making the following assignments to monomials of $Z_{32}\gamma_1 \otimes H_*BP$ of degree 41: $\gamma_1 M_1^{13}$ is assigned 1; $\gamma_1 M_1^{10} M_2$ is assigned 2; $\gamma_1 M_1^{7} M_2^{2}$ is assigned 4 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the new γ_1-leader is $2\gamma_1(M_1^{11}\bar{M}_2+10M_1^{14})$.

DEGREE	GROUP	GENERATOR	
(28,15)	Z_8	2/ 11	1 0 0
		20/ 14	0 0 0
(30,15)	Z_8	16/ 15	0 0 0
		4/ 8	0 1 0
		4/ 12	1 0 0
(32,15)	Z_2	8/ 13	1 0 0
		8/ 10	2 0 0
(34,15)	Z_8	4/ 10	0 1 0
		28/ 14	1 0 0
(36,15)	Z_4	4/ 11	0 1 0
		8/ 12	2 0 0
	Z_8	2/ 11	0 1 0
		4/ 15	1 0 0
		20/ 12	2 0 0
(38,15)	Z_8	8/ 13	2 0 0
		4/ 16	2 1 0
		16/ 10	3 0 0
(40,15)	Z_2	16/ 11	3 0 0
		16/ 13	0 1 0
		8/ 8	4 0 0
		24/ 10	1 1 0
	Z_2	8/ 11	3 0 0
		8/ 10	1 1 0

Z_8 10/ 11 3 0 0
30/ 13 0 1 0
2/ 8 4 0 0
2/ 10 1 1 0
24/ 14 2 0 0

(42, 15) Z_8 2/ 11 1 1 0
12/ 15 2 0 0
2/ 6 5 0 0
4/ 8 2 1 0
20/ 12 3 0 0
20/ 14 0 1 0

Z_8 8/ 15 2 0 0
4/ 8 2 1 0
28/ 12 3 0 0
12/ 14 0 1 0

Z_8 4/ 11 1 1 0
8/ 15 2 0 0
8/ 12 3 0 0
20/ 14 0 1 0

(44, 15) Z_2 24/ 13 3 0 0
24/ 15 0 1 0
8/ 6 3 1 0
16/ 10 4 0 0
16/ 12 1 1 0

Z_4 8/ 15 0 1 0

Z_8 22/ 9 2 1 0
18/ 15 0 1 0
2/ 6 3 1 0
2/ 10 4 0 0
22/ 12 1 1 0

Z_8 16/ 13 3 0 0
8/ 15 0 1 0
4/ 10 4 0 0
4/ 12 1 1 0

(46, 15) Z_2 8/ 11 4 0 0
8/ 13 1 1 0
16/ 8 0 0 1
16/ 10 2 1 0

Z_8 8/ 11 4 0 0
8/ 13 1 1 0
4/ 4 4 1 0
20/ 8 0 0 1
12/ 8 5 0 0
12/ 10 2 1 0
4/ 14 3 0 0

Z_8 8/ 11 4 0 0
24/ 13 1 1 0
4/ 8 0 0 1
28/ 8 5 0 0
8/ 10 2 1 0
8/ 14 3 0 0

(48, 15) Z_2 8/ 9 5 0 0
16/ 15 3 0 0
8/ 6 6 0 0
16/ 8 3 1 0
16/ 12 4 0 0

Z_2 24/ 9 5 0 0
8/ 11 2 1 0
16/ 15 3 0 0
16/ 10 0 2 0
24/ 12 4 0 0
16/ 14 1 1 0

Z_4 8/ 15 3 0 0
16/ 14 1 1 0

Z_8 28/ 9 5 0 0
20/ 11 2 1 0
8/ 10 0 2 0
12/ 12 4 0 0
12/ 14 1 1 0

Z_8 14/ 9 5 0 0
14/ 11 2 1 0
8/ 15 3 0 0
1/ 6 6 0 0
12/ 8 3 1 0
13/ 10 0 2 0
27/ 12 4 0 0
12/ 14 1 1 0

(50, 15) Z_4 24/ 9 3 1 0
18/ 13 4 0 0
16/ 15 1 1 0
4/ 4 7 0 0
14/ 6 4 1 0
24/ 8 1 2 0
10/ 10 0 0 1
8/ 10 5 0 0
24/ 12 2 1 0

Z_4 8/ 13 4 0 0
24/ 15 1 1 0
8/ 6 4 1 0
24/ 8 1 2 0
16/ 10 0 0 1
24/ 10 5 0 0

Z_8 24/ 9 3 1 0
6/ 13 4 0 0
2/ 6 4 1 0
24/ 8 1 2 0
22/ 10 0 0 1
4/ 10 5 0 0
28/ 12 2 1 0

Z_8 24/ 9 3 1 0
24/ 13 4 0 0
16/ 15 1 1 0
4/ 10 0 0 1
4/ 10 5 0 0

(52, 15) Z_2 24/ 11 5 0 0
8/ 13 2 1 0
4/ 4 5 1 0
20/ 8 1 0 1
16/ 8 6 0 0
24/ 12 0 2 0
20/ 14 4 0 0
20/ 26 0 0 0

Z_2 24/ 7 4 1 0
16/ 11 0 0 1
8/ 11 5 0 0
8/ 13 2 1 0
8/ 8 1 0 1
24/ 10 3 1 0
8/ 12 0 2 0
24/ 14 4 0 0
18/ 26 0 0 0

Degree	Group	Generator
	Z_2	24/ 11 5 0 0
		16/ 13 2 1 0
		16/ 8 6 0 0
		8/ 10 3 1 0
		12/ 12 0 2 0
		24/ 14 4 0 0
		31/ 26 0 0 0
	Z_8	14/ 11 0 0 1
		24/ 11 5 0 0
		4/ 13 2 1 0
		4/ 8 1 0 1
		24/ 10 3 1 0
		24/ 12 0 2 0
		16/ 14 4 0 0
		12/ 26 0 0 0
	Z_2	4/ 9 6 0 0
		28/ 13 0 2 0
		12/ 15 4 0 0
		4/ 6 7 0 0
		4/ 8 4 1 0
		4/ 10 1 2 0
		12/ 12 0 0 1
		24/ 14 2 1 0
		8/ 24 1 0 0
	Z_8	4/ 9 6 0 0
		24/ 11 3 1 0
		4/ 13 0 2 0
		4/ 15 4 0 0
		4/ 10 1 2 0
		12/ 12 0 0 1
		20/ 12 5 0 0
		20/ 14 2 1 0
		4/ 24 1 0 0

Degree	Group	Generator
	Z_4	4/ 7 4 1 0
		4/ 11 0 0 1
		24/ 11 5 0 0
		24/ 13 2 1 0
		8/ 10 3 1 0
		16/ 14 4 0 0
		14/ 26 0 0 0
	Z_8	2/ 7 4 1 0
		16/ 11 0 0 1
		20/ 11 5 0 0
		20/ 13 2 1 0
		4/ 12 0 2 0
		20/ 14 4 0 0
		10/ 26 0 0 0
	Z_2	8/ 9 6 0 0
		16/ 11 3 1 0
		16/ 13 0 2 0
		24/ 15 4 0 0
		4/ 8 4 1 0
		8/ 10 1 2 0
		12/ 12 0 0 1
		20/ 12 5 0 0
		8/ 14 2 1 0
		14/ 24 1 0 0
	Z_8	16/ 9 6 0 0
		16/ 11 3 1 0
		24/ 15 4 0 0
		4/ 12 0 0 1
		8/ 12 5 0 0
		6/ 24 1 0 0

Degree	Group	Generator
	Z_8	6/ 11 0 0 1
		20/ 11 5 0 0
		24/ 13 2 1 0
		2/ 4 5 1 0
		6/ 8 1 0 1
		12/ 8 6 0 0
		4/ 10 3 1 0
		14/ 14 4 0 0
		17/ 26 0 0 0
(54,15)	Z_2	28/ 9 6 0 0
		4/ 13 0 2 0
		4/ 15 4 0 0
		4/ 6 2 0 1
		28/ 6 7 0 0
		28/ 10 1 2 0
		20/ 12 0 0 1
		12/ 12 5 0 0
	Z_4	6/ 9 6 0 0
		20/ 11 3 1 0
		22/ 13 0 2 0
		10/ 15 4 0 0
		2/ 6 2 0 1
		2/ 6 7 0 0
		2/ 10 1 2 0
		26/ 12 0 0 1
		10/ 12 5 0 0
		16/ 14 2 1 0
		28/ 24 1 0 0
	Z_8	16/ 15 4 0 0
		4/ 12 5 0 0
		16/ 14 2 1 0
		14/ 24 1 0 0

The leading differential $d^{26}(2\gamma_1(M_1^{11}\bar{M}_2+10M_1^{14})) = \eta\sigma A[32,1]M_1$ determines tentative differentials by making the following assignments to monomials of $Z_{32}\gamma_1 \otimes H_*BP$ of degree 43: M_1^{14} is assigned 1; $M_1^{11}M_2$ is assigned 2; $M_1^8M_2^2$ is assigned 12; $M_1^4M_2M_3$ is assigned 8 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the new γ_1-leader is $4\gamma_1(M_1^{11}\bar{M}_2+2M_1^{14})$.

DEGREE	GROUP	GENERATOR
(28,15)	Z_4	4/ 11 1 0 0
		8/ 14 0 0 0
(30,15)	Z_8	16/ 15 0 0 0
		4/ 8 0 1 0
		4/ 12 1 0 0
(32,15)	Z_2	8/ 13 1 0 0
		8/ 10 2 0 0

(34,15)	Z_8	4/ 10	0 1 0
		28/ 14	1 0 0
(36,15)	Z_4	4/ 11	0 1 0
		8/ 15	1 0 0
		8/ 12	2 0 0
	Z_4	8/ 15	1 0 0
(38,15)	Z_8	8/ 13	2 0 0
		4/ 6	2 1 0
		16/ 10	3 0 0
(40,15)	Z_2	16/ 11	3 0 0
		16/ 13	0 1 0
		8/ 8	4 0 0
		24/ 10	1 1 0
	Z_2	8/ 11	3 0 0
		8/ 10	1 1 0
	Z_4	20/ 11	3 0 0
		28/ 13	0 1 0
		4/ 8	4 0 0
		4/ 10	1 1 0
		16/ 14	2 0 0
(42,15)	Z_4	4/ 11	1 1 0
		24/ 15	2 0 0
		4/ 6	5 0 0
		8/ 8	2 1 0
		8/ 12	3 0 0
		8/ 14	0 1 0
	Z_8	8/ 15	2 0 0
		4/ 8	2 1 0
		28/ 12	3 0 0
		12/ 14	0 1 0
	Z_8	4/ 11	1 1 0
		8/ 15	2 0 0
		8/ 12	3 0 0
		20/ 14	0 1 0
(44,15)	Z_2	24/ 13	3 0 0
		24/ 15	0 1 0
		8/ 6	3 1 0
		16/ 10	4 0 0
		16/ 12	1 1 0
	Z_4	12/ 9	2 1 0
		4/ 15	0 1 0
		4/ 6	3 1 0
		4/ 10	4 0 0
		12/ 12	1 1 0
	Z_4	8/ 15	0 1 0
	Z_8	16/ 13	3 0 0
		4/ 10	4 0 0
		4/ 12	1 1 0
(46,15)	Z_2	8/ 11	4 0 0
		8/ 13	1 1 0
		16/ 8	0 0 1
		16/ 10	2 1 0
	Z_8	4/ 4	4 1 0
		4/ 8	0 0 1
		12/ 8	5 0 0
		28/ 10	2 1 0
		4/ 14	3 0 0
	Z_8	8/ 11	4 0 0
		24/ 13	1 1 0
		4/ 8	0 0 1
		28/ 8	5 0 0
		8/ 10	2 1 0
		8/ 14	3 0 0
(48,15)	Z_2	8/ 9	5 0 0
		16/ 15	3 0 0
		8/ 6	6 0 0
		16/ 8	3 1 0
		16/ 12	4 0 0
	Z_2	24/ 9	5 0 0
		8/ 11	2 1 0
		16/ 15	3 0 0
		16/ 10	0 2 0
		24/ 12	4 0 0
		16/ 14	1 1 0
	Z_4	4/ 9	5 0 0
		20/ 11	2 1 0
		2/ 6	6 0 0
		24/ 8	3 1 0
		10/ 10	0 2 0
		30/ 12	4 0 0
		8/ 14	1 1 0
	Z_4	8/ 15	3 0 0
		16/ 14	1 1 0
	Z_8	28/ 9	5 0 0
		20/ 11	2 1 0
		8/ 10	0 2 0
		12/ 12	4 0 0
		12/ 14	1 1 0
(50,15)	Z_4	8/ 13	4 0 0
		24/ 15	1 1 0
		8/ 6	4 1 0
		24/ 8	1 2 0
		16/ 10	0 0 1
		24/ 10	5 0 0
	Z_4	24/ 9	3 1 0
		18/ 13	4 0 0
		16/ 15	1 1 0
		4/ 4	7 0 0
		14/ 6	4 1 0
		24/ 8	1 2 0
		10/ 10	0 0 1
		8/ 10	5 0 0
		24/ 12	2 1 0
	Z_8	24/ 9	3 1 0
		6/ 13	4 0 0
		2/ 6	4 1 0
		24/ 8	1 2 0
		22/ 10	0 0 1
		4/ 10	5 0 0
		28/ 12	2 1 0
	Z_8	24/ 9	3 1 0
		24/ 13	4 0 0
		16/ 15	1 1 0
		4/ 10	0 0 1
		4/ 10	5 0 0

(52,15)

Z_2	24/ 11	5	0	0
	8/ 13	2	1	0
	4/ 4	5	1	0
	20/ 8	1	0	1
	16/ 8	6	0	0
	24/ 12	0	2	0
	20/ 14	4	0	0
	20/ 26	0	0	0

Z_2	24/ 7	4	1	0
	16/ 11	0	0	1
	8/ 11	5	0	0
	8/ 13	2	1	0
	8/ 8	1	0	1
	24/ 10	3	1	0
	8/ 12	0	2	0
	24/ 14	4	0	0
	18/ 26	0	0	0

Z_2	24/ 11	5	0	0
	16/ 13	2	1	0
	16/ 8	6	0	0
	8/ 10	3	1	0
	12/ 12	0	2	0
	24/ 14	4	0	0
	31/ 26	0	0	0

Z_4	4/ 11	0	0	1
	24/ 11	5	0	0
	24/ 8	6	0	0
	16/ 14	4	0	0

Z_4	4/ 7	4	1	0
	8/ 11	5	0	0
	8/ 13	2	1	0
	8/ 12	0	2	0
	8/ 14	4	0	0
	20/ 26	0	0	0

Z_4	24/ 11	5	0	0
	16/ 13	2	1	0
	8/ 8	6	0	0
	8/ 10	3	1	0
	24/ 12	0	2	0
	24/ 14	4	0	0
	26/ 26	0	0	0

Z_8	12/ 11	5	0	0
	4/ 13	2	1	0
	2/ 4	5	1	0
	2/ 8	1	0	1
	4/ 8	6	0	0
	20/ 10	3	1	0
	16/ 12	0	2	0
	22/ 14	4	0	0
	7/ 26	0	0	0

(54,15)

Z_2	28/ 9	6	0	0
	4/ 13	0	2	0
	4/ 15	4	0	0
	4/ 6	2	0	1
	28/ 6	7	0	0
	28/ 10	1	2	0
	20/ 12	0	0	1
	12/ 12	5	0	0

Z_2	4/ 9	6	0	0
	28/ 13	0	2	0
	12/ 15	4	0	0
	4/ 6	7	0	0
	4/ 8	4	1	0
	4/ 10	1	2	0
	12/ 12	0	0	1
	24/ 14	2	1	0
	8/ 24	1	0	0

Z_2	8/ 9	6	0	0
	16/ 11	3	1	0
	16/ 13	0	2	0
	24/ 15	4	0	0
	4/ 8	4	1	0
	8/ 10	1	2	0
	12/ 12	0	0	1
	20/ 12	5	0	0
	8/ 14	2	1	0
	14/ 24	1	0	0

Z_4	6/ 9	6	0	0
	20/ 11	3	1	0
	22/ 13	0	2	0
	10/ 15	4	0	0
	2/ 6	2	0	1
	2/ 6	7	0	0
	2/ 10	1	2	0
	26/ 12	0	0	1
	10/ 12	5	0	0
	16/ 14	2	1	0
	28/ 24	1	0	0

Z_8	4/ 9	6	0	0
	24/ 11	3	1	0
	4/ 13	0	2	0
	4/ 15	4	0	0
	4/ 10	1	2	0
	12/ 12	0	0	1
	20/ 12	5	0	0
	20/ 14	2	1	0
	4/ 24	1	0	0

Z_8	16/ 9	6	0	0
	16/ 11	3	1	0
	24/ 15	4	0	0
	4/ 12	0	0	1
	8/ 12	5	0	0
	6/ 24	1	0	0

Z_8	16/ 15	4	0	0
	4/ 12	5	0	0
	16/ 14	2	1	0
	14/ 24	1	0	0

The leading differential $d^{28}(4\gamma_1(M_1^{11}\overline{M}_2+2M_1^{14}) = C[42]$ determines tentative differentials by making the following assignments to monomials of $Z_{32}\gamma_1 \otimes H_*BP$ of degree 43: M_1^{14} is assigned 1; $M_1^{11}M_2$ is assigned 2; $M_1^8M_2^2$ is assigned 12; $M_1^4M_2M_3$ is assigned 8 and all other monomials are assigned 0. The kernel of these differentials is given by the table below, and the new γ_1-leader is $16\gamma_1(M_1^8\overline{M}_3+M_1^{12}\overline{M}_2)$.

DEGREE	GROUP	GENERATOR
(30,15)	Z_2	16/ 8 0 1 0 16/ 12 1 0 0
(34,15)	Z_2	16/ 10 0 1 0 16/ 14 1 0 0
(36,15)	Z_2	16/ 15 1 0 0
(38,15)	Z_2	16/ 6 2 1 0
(40,15)	Z_2	16/ 11 3 0 0 16/ 13 0 1 0 8/ 8 4 0 0 24/ 10 1 1 0
(42,15)	Z_2	16/ 11 1 1 0 32/ 6 5 0 0 16/ 8 2 1 0 16/ 12 3 0 0
	Z_2	16/ 8 2 1 0 16/ 12 3 0 0 16/ 14 0 1 0
(44,15)	Z_2	16/ 10 4 0 0 16/ 12 1 1 0
	Z_2	16/ 15 0 1 0
(46,15)	Z_2	16/ 4 4 1 0 16/ 8 0 0 1 16/ 8 5 0 0 16/ 10 2 1 0 16/ 14 3 0 0
	Z_2	16/ 8 0 0 1 16/ 8 5 0 0
(48,15)	Z_2	16/ 9 5 0 0 16/ 15 3 0 0 4/ 6 6 0 0 16/ 8 3 1 0 4/ 10 0 2 0 4/ 12 4 0 0
	Z_2	16/ 14 1 1 0
	Z_2	16/ 15 3 0 0
(50,15)	Z_2	16/ 15 1 1 0 16/ 8 1 2 0 16/ 10 0 0 1 16/ 10 5 0 0
	Z_2	16/ 13 4 0 0 32/ 4 7 0 0 16/ 6 4 1 0 16/ 10 5 0 0
	Z_2	24/ 13 4 0 0 8/ 6 4 1 0 24/ 10 0 0 1 16/ 10 5 0 0 16/ 12 2 1 0
(52,15)	Z_2	8/ 11 5 0 0 16/ 13 2 1 0 8/ 10 3 1 0 20/ 12 0 2 0 24/ 14 4 0 0 17/ 26 0 0 0
	Z_2	24/ 11 5 0 0 8/ 13 2 1 0 4/ 4 5 1 0 20/ 8 1 0 1 16/ 8 6 0 0 24/ 12 0 2 0 20/ 14 4 0 0 20/ 26 0 0 0
	Z_2	16/ 7 4 1 0 16/ 11 0 0 1 16/ 11 5 0 0 16/ 8 6 0 0 16/ 10 3 1 0 16/ 12 0 2 0 16/ 14 4 0 0 16/ 26 0 0 0
	Z_2	16/ 8 6 0 0 16/ 10 3 1 0 16/ 12 0 2 0 16/ 14 4 0 0 12/ 26 0 0 0
(54,15)	Z_2	8/ 9 6 0 0 8/ 11 3 1 0 16/ 13 0 2 0 8/ 15 4 0 0 4/ 6 2 0 1 28/ 8 4 1 0 8/ 12 0 0 1 20/ 12 5 0 0 8/ 14 2 1 0 16/ 24 1 0 0
	Z_2	24/ 9 6 0 0 8/ 13 0 2 0 8/ 15 4 0 0 8/ 6 7 0 0 8/ 8 4 1 0 24/ 10 1 2 0 8/ 12 0 0 1 16/ 12 5 0 0
	Z_2	16/ 9 6 0 0 8/ 11 3 1 0 16/ 15 4 0 0 4/ 8 4 1 0 24/ 10 1 2 0 12/ 12 0 0 1 4/ 12 5 0 0 8/ 14 2 1 0 10/ 24 1 0 0
	Z_2	32/ 10 1 2 0 16/ 12 0 0 1 8/ 24 1 0 0
	Z_2	16/ 12 5 0 0 8/ 24 1 0 0

The leading differential $d^{28}(16\gamma_1 M_1^8\langle M_3\rangle) = 4C[42]M_1$ determines tentative differentials by making the following assignments to monomials of $Z_{32}\gamma_1 \otimes H_*BP$ of degree 45: M_1^{15} is assigned 1; $M_1^{12}M_2$ is assigned 2; $M_1^9M_2^2$ is assigned 12; $M_1^6M_2^3$ is assigned 8 and all other monomials are assigned 0. The table below gives the kernel of these tentative differentials, and the new γ_1-leader is $\gamma_1(M_1^{26}+\cdots)$.

DEGREE	GROUP	GENERATOR
(52,15)	Z_2	16/ 11 5 0 0
		24/ 13 2 1 0
		4/ 4 5 1 0
		20/ 8 1 0 1
		8/ 10 3 1 0
		20/ 12 0 2 0
		12/ 14 4 0 0
		7/ 26 0 0 0

DEGREE	GROUP	GENERATOR
(54,15)	Z_2	16/ 9 6 0 0
		16/ 11 3 1 0
		24/ 13 0 2 0
		4/ 6 2 0 1
		8/ 6 7 0 0
		8/ 8 4 1 0
		16/ 10 1 2 0
		12/ 12 0 0 1
		8/ 12 5 0 0
		16/ 14 2 1 0
		2/ 24 1 0 0

DEGREE 16: A[16]

The leading differential $d^8(A[16]M_1^6\bar{M}_2) = A[23]M_1^2\bar{M}_2$ determines tentative differentials which are a monomorphism on $Z_2A[16] \otimes H_*BP$ / $d^{10}(E^{10}_{*,7})$ in degrees less than 69. Thus there are no remaining elements.

DEGREE 18: C[18]

The leading differential $d^{16}(C[18](M_1^4\bar{M}_2^2+2M_1^7\bar{M}_2)) = \nu A[30]M_1^2$ determines tentative differentials by making the following assignments to monomials of $Z_8C[18] \otimes H_*BP$ of degree 38: $C[18]M_1^{10}$ is assigned 1 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the new C[18]-leader is $2C[18](M_1^4\bar{M}_3+3M_1^8\bar{M}_2)$.

DEGREE	GROUP	GENERATOR
(22,18)	Z_2	2/ 4 0 1 0
		6/ 8 1 0 0

DEGREE	GROUP	GENERATOR
(26,18)	Z_2	4/ 3 1 1 0
		2/ 0 2 1 0
		2/ 4 3 0 0

DEGREE	GROUP	GENERATOR
(28,18)	Z_4	4/ 5 3 0 0
		2/ 7 0 1 0
		2/ 11 1 0 0
		2/ 4 1 1 0

(30,18)	Z_2	2/ 5 1 1 0
(34,18)	Z_2	4/ 7 1 1 0
		2/ 4 2 1 0
		2/ 8 3 0 0
(36,18)	Z_4	2/ 5 2 1 0
		6/ 9 3 0 0
		4/ 15 1 0 0
		6/ 12 2 0 0
(38,18)	Z_2	2/ 6 2 1 0
		6/ 10 3 0 0
	Z_2	2/ 12 0 1 0
		6/ 16 1 0 0
(40,18)	Z_4	6/ 7 2 1 0
		2/ 11 3 0 0
		6/ 13 0 1 0
		2/ 17 1 0 0
		2/ 4 3 1 0
(42,18)	Z_2	4/ 5 3 1 0
		4/ 11 1 1 0
		6/ 8 2 1 0
		6/ 12 3 0 0
	Z_4	2/ 5 3 1 0
		4/ 11 1 1 0
(44,18)	Z_4	2/ 15 0 1 0
		2/ 19 1 0 0
		2/ 12 1 1 0
(46,18)	Z_2	2/ 4 4 1 0
		6/ 8 5 0 0
	Z_2	4/ 7 3 1 0
		4/ 13 1 1 0
	Z_2	2/ 13 1 1 0
(48,18)	Z_2	2/ 11 2 1 0
		6/ 15 3 0 0
		4/ 21 1 0 0
		2/ 8 3 1 0
(50,18)	Z_2	4/ 3 5 1 0
		2/ 0 6 1 0
		2/ 4 7 0 0
	Z_2	4/ 7 1 0 1
		2/ 4 0 3 0
		2/ 4 2 0 1
		2/ 4 7 0 0
		2/ 8 1 2 0
		6/ 12 2 1 0
	Z_2	4/ 7 1 0 1
		2/ 4 2 0 1
		2/ 4 7 0 0
		6/ 16 3 0 0
	Z_2	8/ 3 5 1 0
		4/ 15 1 1 0
		2/ 12 2 1 0
		2/ 16 3 0 0

The leading differential $d^{20}(2C[18](M_1^4\bar{M}_3+3M_1^8\bar{M}_2)) = (\sigma A[30]+A[37])M_1$ determines tentative differentials by making the following assignments to monomials of $Z_8C[18] \otimes H_*BP$ of degree 38: $C[18]M_1^{11}$ is assigned 1 and all other monomials are assigned 0. The kernel of these tentative differentials is given by the table below, and the new C[18]-leader is $4C[18](M_1^7\bar{M}_3+M_1^{11}\bar{M}_2)$.

DEGREE	GROUP	GENERATOR
(28,18)	Z_2	4/ 7 0 1 0
		4/ 11 1 0 0
(36,18)	Z_2	4/ 5 2 1 0
		4/ 9 3 0 0
(40,18)	Z_2	4/ 7 2 1 0
		4/ 11 3 0 0
		4/ 13 0 1 0
		4/ 17 1 0 0
(42,18)	Z_2	4/ 5 3 1 0
(44,18)	Z_2	4/ 15 0 1 0
		4/ 19 1 0 0
(46,18)	Z_2	4/ 7 3 1 0
		4/ 13 1 1 0
(48,18)	Z_2	2/ 11 2 1 0
		6/ 15 3 0 0
		4/ 21 1 0 0
		2/ 8 3 1 0
(50,18)	Z_2	4/ 7 1 0 1
		4/ 15 1 1 0
		2/ 4 2 0 1
		2/ 4 7 0 0
		2/ 12 2 1 0

The leading differential $d^{28}(4C[18](M_1^7\bar{M}_3+M_1^{11}\bar{M}_2)) = D[45]$ determines tentative differentials by making the following assignments to monomials of $Z_8C[18] \otimes H_*BP$ of degree 46: $C[18]M_1^{11}M_2$ is assigned 1 and all other monomials are assigned 0. The only element of degree less than 69 in the kernel of these differntials is $4C[18](M_1^7\bar{M}_2^3\bar{M}_3+M_1^{13}\bar{M}_2\bar{M}_3)$ which is the new C[18]-leader.

<u>DEGREE 21:</u> $\nu C[18]$

The leading differential $d^{12}(\nu C[18]M_1^3\bar{M}_2) = A[32,3]$ determines tentative differentials by making the following assignments to monomials of $Z_2\nu C[18] \otimes H_*BP$ of degree 12: $\nu C[18]M_1^3M_2$ is assigned 1 and the other two monomials are asssigned 0. The kernel of these differentials in degrees less than 68 is given by the table below, and the new $\nu C[18]$-leader is $\nu C[18]M_1^6\bar{M}_2\bar{M}_3$.

<u>DEGREE</u>	<u>BASIS</u>	<u>DEGREE</u>	<u>BASIS</u>	<u>DEGREE</u>	<u>BASIS</u>
(32,21)	6 1 1 0	(40,21)	4 3 1 0	(44,21)	6 3 1 0
(48,21)	14 1 1 0				

<u>DEGREE 23:</u> $A[23] = \sigma A[16]$

A[23] is defined by the leading differential $A[23] = d^6(2C[18]\bar{M}_2)$. In addition $\eta A[23] \neq 0$ and $d^8(A[16]M_1^6\bar{M}_2) = A[23]M_1^2\bar{M}_2$. Let $A(23,4) = Z_2A[23] \otimes$ B<2>. The following table is constructed from computer printouts. In each row of this table, except for the last row, the sum of the numbers in the third and fourth columns equals the number in the second column. Thus, the only nonzero element of $E^{10}_{*,23}$ in degrees less than 68 with a representative in $Z_2A[23] \otimes H_*BP$ is $A[23]M_1^6\bar{M}_2^3\bar{M}_3$ which is the new A[23]-leader.

DEGREE	DIM A(23,4)	DIM $d^6(E^{18})$	DIM $d^8(E^{16})$
(0,23)	1	1	0
(2,23)	0	0	0
(4,23)	1	1	0
(6,23)	1	1	0
(8,23)	1	1	0
(10,23)	1	0	1
(12,23)	2	2	0
(14,23)	2	2	0
(16,23)	2	2	0
(18,23)	3	2	1
(20,23)	3	3	0
(22,23)	3	2	1
(24,23)	4	3	1
(26,23)	4	3	1
(28,23)	5	5	0
(30,23)	6	5	1
(32,23)	6	5	1
(34,23)	7	5	2
(36,23)	8	7	1
(38,23)	8	6	2
(40,23)	9	7	2
(42,23)	11	9	2
(44,23)	11	10	0

DEGREE 23: $\nu C[20]$

The leading differential $d^{12}(4\nu C[20]M_1^3M_2) = A[14]C[20]$ determines tentative differentials which are a monomorphism on $Z_2(4\nu C[20]M_1^3M_2) \otimes B\langle 4\rangle$. There are no remaining elements.

DEGREE 28: $A[8]C[20]$

The leading differential $d^6(A[8]C[20]M_1\bar{M}_2) = \eta A[32,2]M_1$ determines tentative differentials which are a monomorphism on $Z_2A[8]C[20]\{M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$. The remaining elements in degrees less than 69 are $Z_2A[8]C[20]M_1^{13}\bar{M}_3 \oplus (Z_2A[8]C[20]M_1^2\bar{M}_2 \otimes B\langle 4\rangle)$, and the new $A[8]C[20]$-leader is $A[8]C[20]M_1^2\bar{M}_2$.

The leading differential $d^{18}(2\beta_1M_1^{11}M_2) = A[8]C[20]M_1^2\bar{M}_2$ determines tentative differentials with image $Z_2A[8]C[20]M_1^2\bar{M}_2 \otimes H_*BP$ in degrees less than 69. The only remaining element in degrees less than 69 is $A[8]C[20]M_1^{13}\bar{M}_3$.

DEGREE 30: A[30]

The leading differential $d^4(A[30]M_1^2) = \nu A[30]$ determines tentative differentials which are a monomorphism on $Z_2A[30]\{M_1^2, \bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle$. The remaining elements are

$$(Z_2A[30]\langle M_4\rangle \otimes Z_2[\langle M_1^4\rangle^2, \langle M_2^2\rangle^2, \langle M_3\rangle^2, \langle M_4\rangle^2, \{M_5\}, \ldots, \{M_n\}, \ldots]) \oplus \bigoplus_{\alpha,\beta,\gamma} Z_2A[30]\langle M_1^4\rangle^\alpha \langle M_2^2\rangle^\beta \langle M_3\rangle^\gamma \otimes B\langle 8\rangle$$

where the sum is taken over all $0 \le \alpha, \beta, \gamma \le 1$ with $0 < \alpha\beta\gamma$. The new A[30]-leader is $A[30]\langle M_1^4\rangle$.

The leading differential $d^8(A[30]\langle M_1^4\rangle) = \sigma A[30]$ determines tentative differentials which are a monomorphism on $\bigoplus_{\alpha,\beta,\gamma} Z_2A[30]\langle M_1^4\rangle^\alpha \langle M_2^2\rangle^\beta \langle M_3\rangle^\gamma \otimes B\langle 8\rangle$ where the sum is taken over all $0 \le \alpha, \beta, \gamma \le 1$ with $0 < \alpha\beta\gamma$. The remaining elements are $Z_2A[30]\langle M_4\rangle \otimes Z_2[\langle M_1^4\rangle^2, \langle M_2^2\rangle^2, \langle M_3\rangle^2, \langle M_4\rangle^2, \{M_5\}, \ldots, \{M_n\}, \ldots]$, and the new A[30]-leader is $A[30]\langle M_4\rangle$.

DEGREE 31: A[31]

The leading differential $d^4(A[31]M_1^2) = \nu A[31]$ determines tentative differentials which are a monomorphism on $Z_2A[31]\{M_1^2, M_1^3, M_1M_2, M_1^2M_2, M_1^3M_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2A[31]M_2 \otimes B\langle 4\rangle$, and the new A[31]-leader is $A[31]M_2$.

The leading differential $d^6(A[31]M_2) = A[36]$ determines tentative differentials which are a monomorphism on $Z_2A[36]M_2 \otimes B\langle 4\rangle$. There are no remaining elements.

DEGREE 31: $\eta A[30]$

The leading differential $d^{18}(\sigma^2 M_1^4 M_2^2) = \eta A[30]M_1 + A[31]M_1$ determines tentative differentials with cokernel in degrees less than 68 given by the table below. The new $\eta A[30]$-leader is $\eta A[30]M_1\bar{M}_2$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(8,31)	1 1 0 0	(10,31)	5 0 0 0	(12,31)	3 1 0 0
(14,31)	1 2 0 0		7 0 0 0	(16,31)	1 0 1 0
	5 1 0 0	(18,31)	3 2 0 0	(20,31)	1 3 0 0
	3 0 1 0		7 1 0 0	(22,31)	1 1 1 0
	5 2 0 0	(24,31)	3 3 0 0		5 0 1 0
	9 1 0 0	(26,31)	3 1 1 0		7 2 0 0
	13 0 0 0	(28,31)	1 2 1 0		5 3 0 0
	7 0 1 0		11 1 0 0	(30,31)	5 1 1 0
	9 2 0 0		15 0 0 0	(32,31)	1 5 0 0
	3 2 1 0		7 3 0 0		9 0 1 0
	13 1 0 0	(34,31)	1 3 1 0		5 4 0 0
	7 1 1 0		11 2 0 0		
(36.31)	1 1 2 0		3 5 0 0		5 2 1 0
	9 3 0 0		11 0 1 0		15 1 0 0

The leading differential $d^6(\eta A[30]M_1\bar{M}_2) = A[36]M_1$ determines tentative differentials which are a monomorphism on $Z_2\eta A[30]\{M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$. The remaining elements of $Z_2\eta A[30] \otimes H_*BP$ in degrees less than 68 are given by the table below, and the new $\eta A[30]$-leader is $\eta A[30]M_1^5$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(10,31)	5 0 0 0	(14,31)	1 2 0 0		7 0 0 0
(16,31)	1 0 1 0 5 1 0 0	(18,31)	3 2 0 0	(20,31)	3 0 1 0 7 1 0 0
(22,31)	5 2 0 0	(24,31)	5 0 1 0 9 1 0 0	(26,31)	7 2 0 0
	13 0 0 0	(28,31)	1 2 1 0 5 3 0 0		7 0 1 0 11 1 0 0

(30,31)	9 2 0 0		15 0 0 0	(32,31)	3 2 1 0 7 3 0 0
	9 0 1 0 13 1 0 0	(34,31)	5 4 0 0		11 2 0 0
(36,31)	5 2 1 0 9 3 0 0		11 0 1 0 15 1 0 0		

The leading differential $d^{10}(\eta A[30]M_1^5) = A[40,1]$ determines tentative differntials with kernel in degrees less than 68 equal to $Z_2\eta A[30]\{M_1M_2^2, M_1^9M_2^2\}$, and the new $\eta A[30]$-leader is $\eta A[30]M_1M_2^2$.

The leading differential $d^{14}(\eta A[30]M_1M_2^2) = C[44]$ determines the tentative differential $d^{14}(\eta A[30]M_1^9M_2^2) = C[44]M_1^8$. There are no remaining elements in degrees less than 68.

<u>DEGREE 32:</u> A[32,1]

The leading differential $d^{24}(\eta^2\sigma M_1^5\widetilde{M}_3) = A[32,1]$ determines tentative differentials with cokernel in degrees less than 69 given by the table below. This table takes into account that $\eta A[32,1] \neq 0$. The A[32,1]-leader is $A[32,1]M_1^4$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(8,32)	4 0 0 0	(10,32)	2 1 0 0	(12,32)	6 0 0 0
(14,32)	4 1 0 0	(16,32)	2 2 0 0	(18,32)	0 3 0 0
	2 0 1 0		6 1 0 0	(20,32)	0 1 1 0
	4 2 0 0	(22,32)	2 3 0 0		4 0 1 0
(24,32)	2 1 1 0		6 2 0 0		12 0 0 0
(26,32)	0 2 1 0		4 3 0 0		6 0 1 0
	10 1 0 0	(28,32)	4 1 1 0		14 0 0 0
(30,32)	2 2 1 0		6 3 0 0		8 0 1 0
	12 1 0 0	(32,32)	0 3 1 0		4 4 0 0
	6 1 1 0		10 2 0 0		

(34,32)	2 0 0 1	2 5 0 0	4 2 1 0
	8 3 0 0	10 0 1 0	14 1 0 0
(36,32)	2 3 1 0	4 0 2 0	6 4 0 0
	8 1 1 0	12 2 0 0	

The leading differential $d^6(A[32,1]M_1^2\overline{M}_2) = A[37]M_1^2$ determines tentative differentials which are a monomorphism on $Z_2A[32,1]M_1^2\overline{M}_2 \otimes B<4>$. The remaining elements of $Z_2A[32,1] \otimes B<4>$ in degrees less than 69 are given by the table below, and the A[32,1]-leader remains $A[32,1]M_1^4$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(8,32)	4 0 0 0	(12,32)	6 0 0 0	(14,32)	4 1 0 0
(16,32)	2 2 0 0	(18,32)	0 3 0 0		2 0 1 0 6 1 0 0
(20,32)	0 1 1 0		4 2 0 0	(22,32)	4 0 1 0
(24,32)	6 2 0 0		12 0 0 0	(26,32)	0 2 1 0
	4 3 0 0		6 0 1 0 10 1 0 0	(28,32)	4 1 1 0
	14 0 0 0	(30,32)	2 2 1 0 6 3 0 0		8 0 1 0
	12 1 0 0	(32,32)	0 3 1 0		4 4 0 0
	10 2 0 0	(34,32)	2 0 0 1 2 5 0 0 10 0 1 0		4 2 1 0
	8 3 0 0		10 0 1 0 14 1 0 0	(36,32)	4 0 2 0
	6 4 0 0		8 1 1 0		12 2 0 0

The leading differential $d^8(A[32,1]M_1^4) = \sigma A[32,1]$ determines tentative differentials with kernel given by the table below. The new A[32,1]-leader is $A[32,1](\overline{M}_2^3+M_1^2\overline{M}_3+M_1^6\overline{M}_2)$.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(18,32)	0 3 0 0 2 0 1 0 6 1 0 0	(26,32)	0 2 1 0 6 0 1 0 10 1 0 0	(30,32)	8 0 1 0

(34,32)	2 0 0 1	8 3 0 0
	2 5 0 0	10 0 1 0
	10 0 1 0	14 1 0 0

<u>DEGREE 32:</u> A[32,2]

The leading differential $d^{22}(2\beta_1 M_1^{11})$ = A[32,2] determines tentative differentials with image in degrees less than 69 equal to Z_2A[32,2]$\{1,M_1^2\} \otimes$ B<2>. Since ηA[32,2] $\neq$ 0, the remaining elements are Z_2A[32,2]$\{\bar{M}_2, M_1^2\bar{M}_2\} \otimes$ B<4>, and the A[32,2]-leader is A[32,2]$\bar{M}_2$.

The leading differential d^4(A[32,2]$\bar{M}_2$) = νA[32,2]M_1 = ηA[14]C[20]M_1 determines tentative differential which are a monomorphism on Z_2A[32,2]$\{\bar{M}_2, M_1^2\bar{M}_2\} \otimes$ B<4>. There are no remaining elements of Z_2A[32,2] $\otimes H_*BP$ in degrees less than 69.

<u>DEGREE 32:</u> A[32,3]

The leading differential $d^{12}(\nu C[18]M_1^3\bar{M}_2)$ = A[32,3] determines tentative differentials with image Z_2A[32,3] $\otimes$ B<4>. The remaining elements are Z_2A[32,3]$\{M_1, M_1^2, M_1^3, M_2, M_1M_2, M_1^2M_2, M_1^3M_2\} \otimes$ B<4>, and the A[32,3]-leader is A[32,3]M_1.

The leading differential d^4(A[32,3]M_1^2) = νA[32,3] determines tentative differentials which are a monomorphism on Z_2A[32,3]$\{M_1^2, M_1^3, M_1M_2, M_1^2M_2, M_1^3M_2\} \otimes$ B<4>. The remaining elements are Z_2A[32,3]$\{M_1, M_2\} \otimes$ B<4>, and the A[32,3]-leader remains A[32,3]M_1.

The leading differential d^6(A[32,3]M_2) = A[37] determines tentative differentials which are a monomorphism on Z_2A[32,3]$M_2 \otimes$ B<4>. The remaining elements are Z_2A[32,3]$M_1 \otimes$ B<4>, and the A[32,3]-leader remains A[32,3]M_1.

The leading differential d^8(A[32,3]M_1^5) = A[39,1]M_1 determines tentaive differentials with kernel in degrees less than 69 given by the table below. The A[32,3]-leader remains A[32,3]M_1.

DEGREE	BASIS	DEGREE	BASIS	DEGREE	BASIS
(2,32)	1 0 0 0	(18,32)	9 0 0 0	(26,32)	1 4 0 0 13 0 0 0
(30,32)	1 0 2 0 9 2 0 0	(32,32)	1 0 0 1 1 5 0 0 9 0 1 0	(34,32)	17 0 0 0

The leading differential $d^{24}(\eta^2\sigma M_1^{13}) = A[32,3]M_1$ determines tentative differentials with image in degrees less than 69 equal to all the elements in the above table. Thus, there are no remaining elements.

DEGREE 33: $\eta A[32,1]$

Since $\eta^2 A[32,1]$ is nonzero, the only element of $E^4_{*,33}$ with a representative in $Z_2\eta A[32,1] \otimes H_*BP$ is zero.

DEGREE 33: $\eta A[32,2]$

The leading differential $d^2(A[32,2]M_1) = \eta A[32,2]$ determines tentative differentials with image $Z_2\eta A[32,1] \otimes B<2>$. The remaining elements are $Z_2\eta A[32,2]M_1 \otimes B<2>$, and the $\eta A[32,2]$-leader is $\eta A[32,2]M_1$.

The leading differential $d^6(A[8]C[20]M_1\bar{M}_2) = \eta A[32,2]M_1$ determines tentative differentials with image $Z_2\eta A[32,2]\{M_1, M_1^3\} \otimes B<4>$. The remaining elements are $Z_2\eta A[32,2]\{M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B<4>$, and the new $\eta A[32,2]$-leader is $\eta A[32,2]M_1\bar{M}_2$.

The leading differential $d^8(\eta A[32,2]M_1\bar{M}_2) = C[20]^2$ determines tentative differentials which are a monomorphism on $Z_2\eta A[32,2]\{M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B<4>$. Thus, there are no remaining elements.

DEGREE 33: $\nu A[30]$

The leading differential $d^4(A[30]M_1^2) = \nu A[30]$ determines tentative differentials with image $Z_2\nu A[30]\{1,M_1,M_2\} \otimes B<4>$. There are also tentative d^{16}-differentials determined by the leading differential $d^{16}(C[18](M_1^4\bar{M}_2^2+2M_1^7\bar{M}_2)) = \nu A[30]M_1^2$. In addition the leading differential $d^{12}(\nu A[30]M_1^6) = 2C[44]$ determines tentative differentials with image K. These tentative differentials are determined by assigning 1 to every monomial of degree 45 of $Z_2\nu A[30] \otimes H_*BP$. Let $'E^2_{*,33} = Z_2\nu A[30] \otimes H_*BP$. In the following table, the numbers in the last three columns add up to the numbers in the second column. Therefore, the only element of degree less than 68 in $E^{18}_{*,33}$ with a representative in $Z_2\nu A[30] \otimes H_*BP$ is zero.

DEGREE	DIM $'E^2_{*,33}$	DIM $d^4(E^4_{*,30})$	DIM $d^{16}(E^{16}_{*,18})$	DIM K
(0,33)	1	1	0	0
(2,33)	1	1	0	0
(4,33)	1	0	1	0
(6,33)	2	1	1	0
(8,33)	2	1	1	0
(10,33)	2	1	1	0
(12,33)	3	1	1	1
(14,33)	4	3	0	1
(16,33)	4	2	1	1
(18,33)	5	2	1	2
(20,33)	6	2	2	2
(22,33)	6	3	1	2
(24,33)	7	3	2	2
(26,33)	8	4	1	3
(28,33)	9	4	2	3
(30,33)	11	6	1	4
(32,33)	12	5	3	4
(34,33)	13	5	4	4

DEGREE 34: B[34]

The leading differential $d^{24}(\beta_1 M_1^6\bar{M}_2^2) = B[34]$ determines tentative differentials which have cokernel in $Z_2 \otimes [Z_4B[34] \otimes H_*BP]$ in degrees less than 69 given by the table below. The B[34]-leader is $B[34]\bar{M}_2$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(6,34)	0 1 0 0	(10,34)	2 1 0 0	(12,34)	3 1 0 0
	6 0 0 0	(14,34)	0 0 1 0		4 1 0 0

	7 0 0 0	(16,34)	5 1 0 0	(18,34)	0 3 0 0
	2 0 1 0		6 1 0 0	(20,34)	0 1 1 0
	1 3 0 0		4 2 0 0		7 1 0 0
(22,34)	1 1 1 0		2 3 0 0		4 0 1 0
	8 1 0 0	(24,34)	2 1 1 0		3 3 0 0
	5 0 1 0		6 2 0 0	(26,34)	0 2 1 0
	3 1 1 0		4 3 0 0		6 0 1 0
	7 2 0 0		10 1 0 0	(28,34)	1 2 1 0
	4 1 1 0		5 3 0 0		7 0 1 0
	11 1 0 0		14 0 0 0	(30,34)	0 5 0 0
	2 2 1 0		5 1 1 0		6 3 0 0
	8 0 1 0		12 1 0 0		15 0 0 0
(32,34)	0 3 1 0		3 2 1 0		6 1 1 0
	7 3 0 0		13 1 0 0	(34,34)	0 1 2 0
	1 3 1 0		2 5 0 0		4 2 1 0
	7 1 1 0		8 3 0 0		10 0 1 0
	14 1 0 0				

The leading differential $d^6(B[34]M_1^3) = A[39,3]$ determines tentative differentials by assigning 1 to $B[34]M_1^3$ and 0 to $B[34]M_2$. The kernel of these tentative differentials on the remaining elements of $Z_2 \otimes (Z_4B[34] \otimes H_*BP)$ in degrees less than 69 is given by the table below. The new B[34]-leader is $B[34]M_1^6$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(12,34)	6 0 0 0	(14,34)	0 0 1 0 4 1 0 0		7 0 0 0 4 1 0 0
(16,34)	5 1 0 0	(18,34)	2 0 1 0 6 1 0 0	(20,34)	1 3 0 0
	4 2 0 0	(22,34)	1 1 1 0		4 0 1 0 8 1 0 0
(24,34)	5 0 1 0		6 2 0 0	(26,34)	0 2 1 0 4 3 0 0

	7 2 0 0	6 0 1 0	(28,34)	1 2 1 0
	4 3 0 0	10 1 0 0		
	7 0 1 0	5 3 0 0	(30,34)	2 2 1 0
	11 1 0 0			6 3 0 0
	4 1 1 0	14 0 0 0		
	5 1 1 0	8 0 1 0		15 0 0 0
		12 1 0 0		12 1 0 0
(32,34)	3 2 1 0	13 1 0 0		
	7 3 0 0			
	0 3 1 0			
(34,34)	1 3 1 0	4 2 1 0		10 0 1 0
		8 3 0 0		14 1 0 0

The leading differential $d^{12}(B[34]M_1^6) = 2D[45]$ determines tentative differentials by assigning 1 to $B[34]M_1^6$ and 0 to all other monomials of $Z_2 \otimes (Z_4 \otimes B[34] \otimes H_{12}BP)$. There is no point to computing the kernel of these tentative differentials now because there will be a hidden differential $d^8(B[34]M_1^4M_2) = \eta A[40,1]M_1^3$ whose tentative differentials must be computed first. Thus, we postpone both of these computations to Section 7.6. The new B[34]-leader is $B[34]M_1^4M_2$.

<u>DEGREE 34:</u> $2B[34] = \eta^2 A[32,1]$

The leading differentials $d^2(\eta A[32,1]M_1) = 2B[34]$, $d^{20}(\gamma_1 M_1^{11}) = 2B[34]M_1$ and $d^{24}(2\beta_1 M_1^7\bar{M}_2^3) = 2B[34]M_1\bar{M}_2$ determine tentative differentials with image in $Z_2(2B[34]) \otimes H_*BP$. In addition the leading differential $d^{12}(2B[34]M_1^3\bar{M}_2) = A[45,1]$ determines tentative differentials which are a monomorphism on $Z_2(2B[34]M_1^3\bar{M}_2) \otimes B\langle 4\rangle$. The table below shows that the only nonzero elements of $E^{26}_{*,34}$ with representatives in $Z_2(2B[34]) \otimes H_*BP$ of degree less than 69 are $2B[34]M_1^5\bar{M}_2\bar{M}_3$ and $2B[34]M_1\bar{M}_2^3\bar{M}_3$. The new 2B[34]-leader is $2B[34]M_1^5\bar{M}_2\bar{M}_3$.

DEGREE	DIM $2E^4_{*,34}$	DIM $d^{20}(E^{20}_{*,15})$	DIM $d^{24}(E^{24}_{*,11})$	DIM $d^{12}(2E^{12}_{*,34})$
(0,34)	0	0	0	0
(2,34)	1	1	0	0
(4,34)	0	0	0	0
(6,34)	1	1	0	0
(8,34)	1	0	1	0
(10,34)	1	1	0	0
(12,34)	1	0	0	1
(14,34)	2	2	0	0
(16,34)	2	1	1	0
(18,34)	2	2	0	0
(20,34)	3	1	1	1
(22,34)	3	2	1	0
(24,34)	3	1	1	1
(26,34)	4	3	0	1
(28,34)	4	2	1	1
(30,34)	5	4	0	0
(32,34)	6	3	2	1
(34,34)	6	4	0	1

DEGREE 34: $\nu A[31]$

The leading differential $d^4(A[31]M_1^2) = \nu A[31]$ determines tentative differentials with image $Z_2\nu A[31]\{1, M_1M_1^2, M_2, M_1M_2\} \otimes B\langle 4\rangle$ and cokernel $Z_2\nu A[31]\{M_1^3, M_1^2M_2, M_1^3M_2\}$. The $\nu A[31]$-leader is $\nu A[31]M_1^3$.

The leading differntial $d^6(\nu A[31]M_1^3) = A[39,1]$ determines tentative differentials which are a monomorphism on $Z_2\nu A[31]\{M_1^3, M_1^2M_2, M_1^3M_2\}$. Thus, there are no remaining elements.

DEGREE 34: $A[14]C[20]$

The leading differential $d^{12}(4\nu C[20]M_1^3M_2) = A[14]C[20]$ determines tentative differentials with image $Z_2A[14]C[20] \otimes B\langle 4\rangle$. Since $\eta A[14]C[20] \neq 0$, the remaining elements are $Z_2A[14]C[20]\{M_1^2, \bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle$, and the $A[14]C[20]$-leader is $A[14]C[20]M_1^2$.

The leading differential $d^{24}(4\beta_1M_1^{11}M_2) = A[14]C[20]M_1^2$ determines tentative differentials with image equal to $Z_2A[14]C[20]\{M_1^2, \bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle$ in degrees less than 69. Thus, there are no remaining elements.

DEGREE 35: $\nu A[32,3]$

The leading differential $d^4(A[32,3]M_1^2) = \nu A[32,3]$ determines tentative differentials with image $Z_2\nu A[32,3]\{1,M_1,M_1^2,M_2,M_1M_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2\nu A[32,3]\{M_1^3,M_1^2M_2,M_1^3M_2\} \otimes B\langle 4\rangle$, and the $\nu A[32,3]$-leader is $\nu A[32,3]M_1^3$.

The leading differential $d^4(\nu A[32,3]M_1^3) = \eta\sigma A[30]M_1$ determines tentative differentials which are a monomorphism on $Z_2\nu A[32,3]\{M_1^3,M_1^2M_2,M_1^3M_2\} \otimes B\langle 4\rangle$. Thus, there are no remaining elements.

DEGREE 35: $\eta A[14]C[20]$

The leading differential $d^2(A[14]C[20]M_1) = \eta A[14]C[20]$ determines tentative differentials with image $Z_2\eta A[14]C[20] \otimes B\langle 2\rangle$ and cokernel $Z_2\eta A[14]C[20]M_1 \otimes B\langle 2\rangle$. The $\eta A[14]C[20]$-leader is $\eta A[14]C[20]M_1$.

The leading differential $d^4(A[32,2]\bar{M}_2) = \eta A[14]C[20]M_1$ determines tentative differentials with image $Z_2\eta A[14]C[20]\{M_1,M_1^3\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2\eta A[14]C[20]\{M_1\bar{M}_2,M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$, and the new $\eta A[14]C[20]$-leader is $\eta A[14]C[20]M_1\bar{M}_2$.

The leading differential $d^6(\eta A[14]C[20]M_1\bar{M}_2) = 2C[20]^2M_1$ determines tentative differentials which are a monomorphism on $Z_2\eta A[14]C[20]\{M_1\bar{M}_2,M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$. There are no remaining elements.

DEGREE 36: $A[36]$

The leading differential $d^6(A[31]M_2) = A[36]$ determines tentative differentials with image $Z_2A[36] \otimes B\langle 4\rangle$. The remaining elements are $Z_2A[36]\{M_1,M_1^2,M_1^3,M_2,M_1M_2,M_1^2M_2,M_1^3M_2\} \otimes B\langle 4\rangle$, and the $A[36]$-leader is $A[36]M_1$.

The leading differential $d^6(\eta A[30]M_1\bar{M}_2) = A[36]M_1$ determines tentative differentials with image $Z_2A[36]\{M_1,M_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2A[36]\{M_1^2,M_1^3,M_1M_2,M_1^2M_2,M_1^3M_2\} \otimes B\langle 4\rangle$, and the A[36]-leader is $A[36]M_1^2$.

The leading differential $d^4(A[36]M_1^3) = \eta B[38]M_1$ determines tentative differentials which are a monomorphism on $Z_2A[36]\{M_1^3,M_1^2M_2,M_1^3M_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2A[36]\{M_1^2,M_1M_2\} \otimes B\langle 4\rangle$, and the A[36]-leader remains $A[36]M_1^2$.

The leading differential $d^6(A[36]M_1M_2) = \eta A[40,1]M_1$ determines tentative differentials which are a monomorphism on $Z_2A[36]M_1M_2 \otimes B\langle 4\rangle$. The remaining elements are $Z_2A[36]M_1^2 \otimes B\langle 4\rangle$, and the A[36]-leader remains $A[36]M_1^2$.

The leading differential $d^{25}(2\beta_1M_1^6\overline{M}_2^3) = A[36]M_1^2$ determines tentative differentials with cokernel in degrees less than 69 given by the table below. The new A[36]-leader is $A[36]M_1^2\overline{M}_3$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(18,36)	2 0 1 0	(24,36)	6 2 0 0	(26,36)	6 0 1 0
(30,36)	2 2 1 0				

DEGREE 37: A[37] and $\sigma A[30]$

The leading differential $d^6(A[32,3]M_2) = A[37]$ determines tentative differentials with image $Z_2A[37] \otimes B\langle 4\rangle$. The remaining elements are $(Z_2A[37]M_1 \otimes B\langle 2\rangle) \oplus (Z_2A[37]\{M_1^2,\overline{M}_2,M_1^2\overline{M}_2\} \otimes B\langle 4\rangle)$, and the A[37]-leader is $A[37]M_1$.

The leading differential $d^8(A[30]M_1^4) = \sigma A[30]$ determines tentative differentials with image in degrees less than 68 equal to

$$Z_2\sigma A[30]\{1,M_1^2,\overline{M}_2,M_2^2,\overline{M}_3,M_1^8,M_1^2\overline{M}_3+\overline{M}_2M_2^2,M_1^{10},M_1^8\overline{M}_2,M_1^{12}+M_2^4,M_2^2\overline{M}_3+M_1^{10}\overline{M}_2,\langle M_3\rangle^2,M_1^2M_2^4+M_1^{14},$$
$$M_1^8M_2^2,\overline{M}_2M_2^4+\overline{M}_2M_1^{12},\langle M_4\rangle,M_1^8\overline{M}_3\}.$$

The remaining elements in degrees less than 68 are

$(Z_2\sigma A[30]M_1 \otimes B\langle 2\rangle) \oplus (Z_2\sigma A[30]\{\langle M_1^4\rangle, M_1^2\bar{M}_2, \langle M_2^2\rangle, \langle M_3\rangle, M_1^2\langle M_2^2\rangle, M_1^2\langle M_3\rangle, M_1^6\bar{M}_2, \bar{M}_2\langle M_3\rangle,$
$M_1^2\bar{M}_2^3, M_1^4\langle M_3\rangle, M_1^6M_2^2, M_1^{12}, M_1^6\langle M_3\rangle, M_1^2\bar{M}_2\langle M_3\rangle, M_1^{10}\bar{M}_2, M_1^4\bar{M}_2^3, M_1^4\bar{M}_2\bar{M}_3, M_1^{14}, M_1^2M_2^2\bar{M}_3, M_1^6\bar{M}_2^3, \bar{M}_2^5\})$.

The $\sigma A[30]$-leader is $\sigma A[30]M_1$.

Note that $\eta\sigma A[30] = \eta A[37] \neq 0$. In addition the leading differential $d^{18}(2C[18](M_1^4\bar{M}_3+3M_1^8\bar{M}_2) = (\sigma A[30]+A[37])M_1$ determines tentative differentials with image in $Z_2(\sigma A[30]+A[37])M_1 \otimes B\langle 2\rangle$. The cokernel of these tentative differentials in degrees less than 68 is given by $Z_2(A[37]+\sigma A[30])M_1^3\bar{M}_2 \otimes B\langle 4\rangle$ as well as the elements listed in the table below. The remaining elements in $(Z_2A[37] \oplus Z_2\sigma A[30]) \otimes B\langle 2\rangle$ of degrees less than 68 are
$(Z_2A[37]\{M_1^2, \bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle)$
$\oplus\ (Z_2\sigma A[30]\{\langle M_1^4\rangle, M_1^2\bar{M}_2, \langle M_2^2\rangle, \langle M_3\rangle, M_1^2\langle M_2^2\rangle, M_1^2\langle M_3\rangle, M_1^6\bar{M}_2, \bar{M}_2\langle M_3\rangle, M_1^2\bar{M}_2^3, M_1^4\langle M_3\rangle, M_1^6M_2^2,$
$M_1^{12}, M_1^6\langle M_3\rangle, M_1^2\bar{M}_2\langle M_3\rangle, M_1^{10}\bar{M}_2, M_1^4\bar{M}_2^3, M_1^4\bar{M}_2\bar{M}_3, M_1^{14}, M_1^2M_2^2\bar{M}_3, M_1^6\bar{M}_2^3, \bar{M}_2^5\})$.

The new A[37]-leader is $A[37]M_1^2$, and the new $\sigma A[30]$-leader is $\sigma A[30]M_1^4$.

DEGREE	BASIS	DEGREE	BASIS
(14,37)	$(\sigma A[30]+A[37])M_1^7$	(16,37)	$(\sigma A[30]+A[37])M_1^5\bar{M}_2$
(20,37)	$(\sigma A[30]+A[37])M_1\bar{M}_2^3$	(22,37)	$(\sigma A[30]+A[37])M_1^5\bar{M}_2^2$
(24,37)	$(\sigma A[30]+A[37])M_1^5\bar{M}_3$	(26,37)	$(\sigma A[30]+A[37])M_1^7\langle M_2^2\rangle$
(28,37)	$(\sigma A[30]+A[37])M_1^5\bar{M}_2^3$		$(\sigma A[30]+A[37])M_1^7\langle M_3\rangle$
	$(\sigma A[30]+A[37])M_1\bar{M}_2^2\langle M_3\rangle$		
(30,37)	$(\sigma A[30]+A[37])M_1^{15}$		$(\sigma A[30]+A[37])M_1^5\bar{M}_2\langle M_3\rangle$

The leading differential $d^6(A[32,1]M_1^2\bar{M}_2) = A[37]M_1^2$ determines tentative differentials with image $Z_2A[37]M_1^2 \otimes B\langle 4\rangle$. The remaining elements from $Z_2A[37] \otimes B\langle 2\rangle$ are $Z_2A[37]\{\bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle$, and the new A[37]-leader is $A[37]\bar{M}_2$. The leading differential $d^4(A[37]\bar{M}_2) = \eta A[39,3]M_1 + \eta\sigma A[32,1]M_1$ determines tentative differentials which are a monomorphism on $(Z_2A[37]\{\bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle)$ $\oplus\ (Z_2(A[37]+\sigma A[30])M_1^3\bar{M}_2$. There are no remaining elements from $Z_2A[37] \otimes B\langle 2\rangle$. The leading differentials $d^8(\sigma A[30]M_1^4) = \sigma^2A[30] = 4C[44]$ and $d^8(A[37]M_1^4) = 0$

determine tentative differentials which are a monomorphism on the remaining elements of $Z_2(A[37]+\sigma A[30])M_1 \otimes B\langle 2\rangle$ given by the table above and on

$$(Z_2\sigma A[30]\{\langle M_1^4\rangle, M_1^2\bar{M}_2, \langle M_2^2\rangle, \langle M_3\rangle, M_1^2\langle M_2^2\rangle, M_1^2\langle M_3\rangle, M_1^6\bar{M}_2, \bar{M}_2\langle M_3\rangle, M_1^2\bar{M}_2^3, M_1^4\langle M_3\rangle, M_1^6M_2^2,$$
$$M_1^{12}, M_1^6\langle M_3\rangle, M_1^2\bar{M}_2\langle M_3\rangle, M_1^{10}\bar{M}_2, M_1^4\bar{M}_2^3, M_1^4\bar{M}_2\bar{M}_3, M_1^{14}, M_1^2M_2^2\bar{M}_3, M_1^6\bar{M}_2^3, \bar{M}_2^5\}).$$

There are no remaining elements in degrees less than 68 from

$(Z_2(A[37]+\sigma A[30])M_1 \otimes B\langle 2\rangle) \oplus (Z_2\sigma A[30] \otimes B\langle 2\rangle)$.

DEGREE 38: B[38]

The leading differential $d^{24}(\gamma_1 M_1^{12}) = B[38]$ defines B[38], and $\eta B[38] \neq 0$. Therefore the tentative differentials defined by this leading d^{24}-differential have image in $(Z_4B[38] \otimes B\langle 2\rangle) \oplus (Z_2(2B[38]M_1) \otimes B\langle 2\rangle)$. The cokernel of these tentative differntials is given by the table below. The B[38]-leader is $B[38]M_1^6$.

DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR
(12,38)	Z_2	6 0 0 0	(14,38)	Z_4	4 1 0 0	(18,38)	Z_4	6 1 0 0
(20,38)	Z_2	0 1 1 1		Z_2 2/	7 1 0 0	(22,38)	Z_4	4 0 1 0
(24,38)	Z_2	2 1 1 0		Z_2	6 2 0 0	(26,38)	Z_4	4 3 0 0
	Z_4	6 0 1 0	(28,38)	Z_2 2/	7 0 1 0		Z_2	14 0 0 0
	Z_4	4 1 1 0	(30,38)	Z_4	6 3 0 0		Z_4	12 1 0 0

DEGREE 38: $\eta A[37] = \eta\sigma A[30]$

The leading differential $d^2(A[37]M_1) = \eta A[37]$ determines tentative differentials with image $Z_2\eta A[37] \otimes B\langle 2\rangle$ and cokernel $Z_2\eta A[37]M_1 \otimes B\langle 2\rangle$. The $\eta A[37]$-leader is $\eta A[37]M_1$.

The leading differential $d^4(\nu A[32,3]M_1^3) = \eta A[37]M_1$ determines tentative differentials with image $Z_2\eta A[37]\{M_1, M_1^3, M_1\bar{M}_2\} \otimes B\langle 4\rangle$. The remainng elements are $Z_2\eta A[37]M_1^3\bar{M}_2 \otimes B\langle 4\rangle$, and the new $\eta A[37]$-leader is $\eta A[37]M_1^3\bar{M}_2$.

DEGREE 39: A[39,1]

The leading differential $d^6(\nu A[31]M_1^3) = A[39,1]$ determines tentative differentials with image $Z_2A[39,1]\{1,M_1^2,\bar{M}_2\} \otimes B\langle 4\rangle$. The cokernel of these tentative differentials is $Z_2A[39,1]\{M_1,M_1^3,M_1\bar{M}_2,M_1^2\bar{M}_2,M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$, and the A[39,1]-leader is $A[39,1]M_1$.

The leading differential $d^8(A[32,3]M_1^5) = A[39,1]M_1$ determines tentative differentials. The remaining elements in degrees less than 70 are $Z_2A[39,1]\{M_1^2M_2,M_1^3M_2\} \otimes B\langle 4\rangle$ as well as the elements listed in the table below. The new A[39,1]-leaders are $A[39,1]M_1^5$ and $A[39,1]M_1^2M_2$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(10,39)	5 0 0 0	(14,39)	7 0 0 0	(16,39)	5 1 0 0
(18,39)	3 2 0 0	(20,39)	3 0 1 0	(22,39)	5 2 0 0
	1 1 1 0	(24,39)	5 0 1 0		3 3 0 0
(26,39)	7 2 0 0		13 0 0 0	(28,39)	7 0 1 0
	5 3 0 0	(30,39)	5 1 1 0		15 0 0 0
	6 3 0 0				

DEGREE 39: A[39,2]

The leading differential $d^6(A[14]C[20]\bar{M}_2) = A[39,2]$ determines tentative differentials with image $Z_2A[39,2]\{1,M_1^2\} \otimes B\langle 4\rangle$. Since $\eta A[39,2] \neq 0$, the remaining elements are $Z_2A[39,2]\{\bar{M}_2,M_1^2\bar{M}_2\} \otimes B\langle 4\rangle$ and the A[39,2]-leader is $A[39,2]\bar{M}_2$. The leading differential $d^4(A[39,2]\bar{M}_2) = \eta^2C[20]^2M_1$ determines tentative differentials which are a monomorphism on $Z_2A[39,2]\{\bar{M}_2,M_1^2\bar{M}_2\} \otimes B\langle 4\rangle$. There are no remaining elements.

DEGREE 39: A[39,3]

The leading differential $d^6(B[34]M_1^3)$ = A[39,3] determines tentative differentials with image Z_2A[39,3]$\{1,M_1^2,\bar{M}_2\}$ ⊗ B<4>. Since ηA[39,3] ≠ 0, the remaining elements are Z_2A[39,3]$M_1^2\bar{M}_2$ ⊗ B<4>, and the A[39,3]-leader is A[39,3]$M_1^2\bar{M}_2$.

DEGREE 39: ηB[38]

The leading differential $d^2(B[38]M_1)$ = ηB[38] determines tentative differntials with image Z_2ηB[38] ⊗ B<2>. The remaining elements are Z_2ηB[38]M_1 ⊗ B<2>, and the ηB[38]-leader is ηB[38]M_1.

The leading differential $d^4(A[36]M_1^3)$ = ηB[38]M_1 determines tentative differentials with image Z_2ηB[38]$\{M_1,M_1^3,M_1\bar{M}_2\}$ ⊗ B<4>. The remaining elements are Z_2ηB[38]$M_1^3\bar{M}_2$ ⊗ B<4>, and the new ηB[38-leader is ηB[38]$M_1^3\bar{M}_2$.

DEGREE 39: σA[32,1]

The leading differential $d^8(A[32,1]M_1^4)$ = σA[32,1] determines tentative differntials with image in Z_2σA[32,1] ⊗ B<2> since ησA[32,1] ≠ 0. The cokernel of these tentative differentials in degrees less than 68 is given by the table below. The σA[32,1]-leader is σA[32,1]$M_1^4\bar{M}_2$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(14,39)	4 1 0 0	(18,39)	6 1 0 0	(20,39)	4 2 0 0
(22,39)	4 0 1 0	(24,39)	6 2 0 0	(26,39)	4 3 0 0
	6 0 1 0	(28,39)	4 1 1 0		

DEGREE 40: A[40,1]

The leading differential $d^{10}(\eta A[30]M_1^5)$ = A[40,1] determines tentative differentials with image in $Z_2A[40,1] \otimes B\langle 2\rangle$ because $\eta A[40,1] \neq 0$. The cokernel of these differentials in degrees less than 69 is given by the table below, and the A[40,1]-leader is $A[40,1]M_1^6$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(12,40)	6 0 0 0	(14,40)	4 1 0 0	(18,40)	6 1 0 0
(20,40)	4 2 0 0		0 1 1 0	(22,40)	2 3 0 0
(24,40)	6 2 0 0		2 1 1 0	(26,40)	6 0 1 0
	4 3 0 0	(28,40)	14 0 0 0		4 1 1 0

DEGREE 40: A[40,2]

The leading differential $d^{26}(4\gamma_1 M_1^{13}) = A[40,2]$ determines tentative differentials with image in $Z_2A[40,2] \otimes B\langle 2\rangle$ because $\eta A[40,2] \neq 0$. The cokernel of these tentative differentials in degrees less than 69 equals $Z_2A[40,2]\{M_1^6\bar{M}_2, M_1^6\bar{M}_3\}$. The A[40,2]-leader is $A[40,2]M_1^6\bar{M}_2$.

DEGREE 40: $C[20]^2$

The leading differential $d^8(\eta A[32,2]M_1\bar{M}_2) = C[20]^2$ determines tentative differentials with image $Z_2C[20]^2\{1, M_1^2\} \otimes B\langle 4\rangle$. Since $\eta C[20]^2 \neq 0$, the remaining elements are $Z_2C[20]^2\{\bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle$, and the $C[20]^2$-leader is $C[20]^2\bar{M}_2$.

The leading differential $d^6(C[20]^2\bar{M}_2) = A[45,2]$ determines tentative differentials which are a monomorphism on $Z_2C[20]^2\{\bar{M}_2, M_1^2\bar{M}_2\} \otimes B\langle 4\rangle$. There are no remaining elements.

DEGREE 40: $2C[20]^2 = \eta A[39,2]$

The leading differential $d^2(A[39,2]M_1) = \eta A[39,2]$ determines tentative differentials with image $Z_2(2C[20]^2) \otimes B\langle 2\rangle$. The remaining elements are $Z_2(2C[20]^2M_1) \otimes B\langle 2\rangle$, and the $2C[20]^2$-leader is $2C[20]^2M_1$.

The leading differential $d^6(\eta A[14]C[20]M_1\bar{M}_2) = 2C[20]^2M_1$ determines tentative differentials with image $Z_2(2C[20]^2)\{M_1, M_1^3\} \otimes B\langle 4\rangle$. The remainig elements are $Z_2(2C[20]^2)\{M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$, and the new $2C[20]^2$-leader is $2C[20]^2M_1\bar{M}_2$.

<u>DEGREE 40:</u> $\eta A[39,3]$

The leading differential $d^2(A[39,3]M_1) = \eta A[39,3]$ determines tentative differntials with image $Z_2\eta A[39,3] \otimes B\langle 2\rangle$. The remaining elements ae $Z_2\eta A[39,3]M_1 \otimes B\langle 2\rangle$, and the $\eta A[39,3]$-leader $\eta A[39,3]M_1$.

The leading differential $d^4(A[37]\bar{M}_2) = \eta A[39,3]M_1 + \eta\sigma A[32,1]M_1$ determines tentative differentials with image $Z_2\eta(A[39,3]+\sigma A[32,1])\{M_1, M_1^3, M_1\bar{M}_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2(\eta A[39,3]+\eta\sigma A[32,1])M_1^3\bar{M}_2 \otimes B\langle 4\rangle$, and the new $\eta A[39,3]$-leader is $(\eta A[39,3]+\eta\sigma A[32,1])M_1^3\bar{M}_2$.

<u>DEGREE 40:</u> $\eta\sigma A[32,1]$

The leading differential $d^2(\sigma A[32,1]M_1) = \eta\sigma A[32,1]$ determines tentative differentials with image $Z_2\eta\sigma A[32,1] \otimes B\langle 2\rangle$. The remaining elements are $Z_2\eta\sigma A[32,1]M_1 \otimes B\langle 2\rangle$, and the $\eta\sigma A[32,1]$-leader is $\eta\sigma A[32,1]M_1$.

The leading differential $d^{26}(2\gamma_1(M_1^{11}\bar{M}_2+2M_1^{14})) = \eta\sigma A[32,1]M_1$ determines tentative differentials with image in degrees less than 60 euqal to all the elements of $Z_2\eta\sigma A[32,1]M_1 \otimes B\langle 4\rangle$ except $Z_2\eta\sigma A[32,1]\{M_1^5\langle M_3\rangle, M_1\langle M_2^2\rangle\langle M_3\rangle\}$. Thus, the remaining elements in degrees less than 69 are $(Z_2\eta\sigma A[32,1]\{M_1^3, M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle) \oplus (Z_2\eta\sigma A[32,1]\{M_1^5\langle M_3\rangle, M_1\langle M_2^2\rangle\langle M_3\rangle\})$. The new $\eta\sigma A[32,1]$-leader is $\eta\sigma A[32,1]M_1^3$.

The leading differential $d^6(\eta\sigma A[32,1]M_1^3) = 4D[45]$ determines tentative differentials which are a monomorphism on $Z_2\eta\sigma A[32,1]\{M_1^3, M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$. The remaining elements in degrees less than 69 are $Z_2\eta\sigma A[32,1]\{M_1^5\langle M_3\rangle, M_1\langle M_2^2\rangle\langle M_3\rangle\}$, and the new $\eta\sigma A[32,1]$-leader is $\eta\sigma A[32,1]M_1^5\langle M_3\rangle$.

DEGREE 41: $\eta A[40,1]$

The leading differential $d^2(A[40,1]M_1) = \eta A[40,1]$ determines tentative differentials with image $Z_2\eta A[40,1] \otimes B\langle 2\rangle$. The remaining elements are $Z_2\eta A[40,1]M_1 \otimes B\langle 2\rangle$, and the $\eta A[40,1]$-leader is $\eta A[40,1]M_1$.

The leading differential $d^6(A[36]M_1M_2) = \eta A[40,1]M_1$ determines tentative differentials with image $Z_2\eta A[40,1]M_1 \otimes B\langle 4\rangle$. The remaining elements are $Z_2\eta A[40,1]\{M_1^3, M_1\bar{M}_2, M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$, and the new $\eta A[40,1]$-leader is $\eta A[40,1]M_1^3$.

DEGREE 41: $\eta A[40,2]$ and $\eta C[20]^2$

Since $\eta^2 A[40,2]$ and $\eta^2 C[20]^2$ are nonzero, the only element of $E^4_{*,41}$ with a representative in $(Z_2\eta A[40,2] \oplus Z_2\eta C[20]^2) \otimes H_*BP$ is zero.

DEGREE 42: $C[42]$

The leading differential $d^{28}(4\gamma_1(M_1^{11}+\bar{M}_2+2M_1^{14})) = C[42]$ defines an element of order four in E^{28} and determines tentative differentials whose cokernel in $Z_4 \otimes (Z_8C[42] \otimes H_*BP)$ in degrees less than 69 is given by the table below. The $C[42]$-leader is $C[42]M_1^2$.

DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR
(4,42)	Z_2	2 0 0 0	(6,42)	Z_4	0 1 0 0	(8,42)	Z_2	1 1 0 0
(10,42)	Z_4	2 1 0 0	(12,42)	Z_2	6 0 0 0		Z_4	3 1 0 0

(14,42)	Z_4	4 1 0 0	(16,42)	Z_2	1 0 1 0		Z_2	2 2 0 0
(18,42)	Z_2	2 0 1 0 6 1 0 0		Z_4	0 3 0 0		Z_4	6 1 0 0
(20,42)	Z_2	1 3 0 0		Z_2	10 0 0 0		Z_4	0 1 1 0
	Z_4	7 1 0 0	(22,42)	Z_2	1 1 1 0		Z_4	2 3 0 0
	Z_4	8 1 0 0	(24,42)	Z_2	5 0 1 0		Z_2	6 2 0 0
	Z_4	2 1 1 0		Z_4	3 3 0 0	(26,42)	Z_2	6 0 1 0 10 1 0 0
	Z_4	3 1 1 0		Z_4	4 3 0 0		Z_4	10 1 0 0

The leading differential $d^4(C[42]M_1^2) = \nu C[42] = 8D[45]$ determines tentative differentials which are a monomorphism on $Z_2 \otimes ((Z_8C[42])\{M_1^2, M_1^3, M_1M_2, M_1^2M_2, M_1^3M_2\} \otimes B\langle 4\rangle)$. The kernel of these tentative differentials in degrees less than 69 on elements with representatives in $Z_4 \otimes (Z_8C[42] \otimes H_*BP)$ is given by the table below. The new C[42]-leader is $2C[42]\bar{M}_2$.

DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR	DEGREE	GROUP	GENERATOR
(6,42)	Z_2 2/	0 1 0 0	(10,42)	Z_2 2/	2 1 0 0	(12,42)	Z_2 2/	3 1 0 0
(14,42)	Z_2 2/	4 1 0 0	(18,42)	Z_2 2/	0 3 0 0		Z_2 2/	6 1 0 0
(20,42)	Z_2 2/	0 1 1 0		Z_2 2/	7 1 0 0	(22,42)	Z_2 2/	2 3 0 0
	Z_2 2/	8 1 0 0	(24,42)	Z_2 2/	2 1 1 0		Z_2 2/	3 3 0 0
(26,42)	Z_2 2/	3 1 1 0		Z_2 2/	4 3 0 0		Z_2 2/	10 1 0 0

<u>DEGREE 42:</u> $4C[42] = \eta^2 A[40,2]$

The leading differential $d^2(\eta A[40,2]M_1) = 4C[42]$ determines tentative differentials with image $Z_2(4C[42]) \otimes B\langle 2\rangle$. The remaining elements are $Z_2(4C[42]M_1) \otimes B\langle 2\rangle$, and the 4C[42]-leader is $4C[42]M_1$.

The leading differential $d^{28}(16\gamma_1 M_1^8 \bar{M}_3) = 4C[42]M_1$ determines tentative differentials with image in degrees less than 69 equal to $Z_2(4C[42])M_1 \otimes B\langle 2\rangle$. Thus, there are no remaining elements.

DEGREE 42: $\eta^2C[20]^2$

The leading differential $d^2(\eta C[20]^2M_1) = \eta^2C[20]^2$ determines tentative differentials with image $Z_2\eta^2C[20]^2 \otimes B\langle 2\rangle$. The remaining elements are $Z_2\eta^2C[20]^2M_1 \otimes B\langle 2\rangle$, and the $\eta^2C[20]^2$-leader is $\eta^2C[20]^2M_1$.
The leading differntial $d^4(A[39,2]\bar{M}_2) = \eta^2C[20]^2M_1$ determines tentative differentials with image $Z_2(\eta^2C[20]^2)\{M_1,M_1^3\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2(\eta^2C[20]^2)\{M_1\bar{M}_2,M_1^3\bar{M}_2\} \otimes B\langle 4\rangle$, and the new $\eta^2C[20]^2$-leader is $\eta^2C[20]^2M_1\bar{M}_2$.

DEGREE 44: C[44]

The leading differential $d^{14}(\eta A[30]M_1M_2^2) = C[44]$ determines tentative differentials with image in $Z_2 \otimes (Z_8C[44] \otimes B\langle 2\rangle)$ since $\eta C[44] \neq 0$. The cokernel of these tentative differentials in degrees less than 69 is given by the table below. The C[44]-leader is $C[44]M_1^2$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(4,44)	2 0 0 0	(6,44)	0 1 0 0	(8,44)	4 0 0 0
(10,44)	2 1 0 0	(12,44)	0 2 0 0		6 0 0 0
(14,44)	0 0 1 0		4 1 0 0	(16,44)	2 2 0 0
(18,44)	0 3 0 0		2 0 1 0		6 1 0 0
(20,44)	0 1 1 0		4 2 0 0		10 0 0 0
(22,44)	2 3 0 0		4 0 1 0		8 1 0 0
(34,44)	2 1 1 0		12 0 0 0		0 4 0 0

DEGREE 44: 2C[44]

The leading differential $d^{12}(\nu A[30]M_1^6) = 2C[44]$ determines tentative differentials with cokernel in degrees less than 69 given by the table below. The 2C[44]-leader is $2C[44]M_1^6$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(12,44)	6 0 0 0	(14,44)	7 0 0 0	(16,44)	5 1 0 0
(18,44)	6 1 0 0	(20,44)	7 1 0 0		4 2 0 0
(22,44)	5 2 0 0		2 3 0 0	(24,44)	5 0 1 0
	6 2 0 0		3 3 0 0		

DEGREE 44: $4C[44] = \sigma^2 A[30]$

The leading differential $d^8(\sigma A[30]M_1^4) = 4C[44]$ determines tentative differentials whose cokernel in degrees less than 69 is given by the table below. The 4C[44]-leader is $4C[44]M_1^7$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(14,44)	7 0 0 0	(18,44)	6 1 0 0	(20,44)	7 1 0 0
(22,44)	4 0 1 0				

DEGREE 45: A[45,1]

The leading differential $d^{12}(2B[34]M_1^3\overline{M}_2) = A[45,1]$ determines tentative differentials with image $Z_2A[45,1] \otimes B\langle 4\rangle$. Since $\eta A[45,1] \neq 0$, the remaining elements are $Z_2A[45,1]\{M_1^2, \overline{M}_2, M_1^2\overline{M}_2\} \otimes B\langle 4\rangle$. The A[45,1]-leader is $A[45,1]M_1^2$.

DEGREE 45: A[45,2]

The leading differential $d^6(C[20]^2\overline{M}_2) = A[45,2]$ determines tentative differentials with image $Z_2A[45,2]\{1,M_1^2\} \otimes B\langle 4\rangle$. Since $\eta A[45,2] \neq 0$, the remaining elements are $Z_2A[45,2]\{\overline{M}_2, M_1^2\overline{M}_2\} \otimes B\langle 4\rangle$, and the A[45,2]-leader is $A[45,2]\overline{M}_2$.

DEGREE 45: D[45]

The leading differential $d^{28}(4C[18](M_1^7\overline{M}_3+M_1^{11}\overline{M}_2)) = D[45]$ determines tentative differentials with image in $Z_2 \otimes (Z_{16}D[45] \otimes B\langle 2\rangle)$ since $\eta D[45] \neq 0$. The cokernel of these tentative differentials in degrees less than 68 is given by the table below. The D[45]-leader is $D[45]M_1^2$.

DEGREE	GENERATOR	DEGREE	GENERATOR	DEGREE	GENERATOR
(4,45)	0 2 0 0	(6,45)	0 1 0 0	(10,45)	2 1 0 0
(12,45)	6 0 0 0	(14,45)	4 1 0 0	(16,45)	2 2 0 0
(18,45)	0 3 0 0		6 1 0 0	(20,45)	0 1 1 0
	10 0 0 0	(22,45)	2 3 0 0		8 1 0 0

DEGREE 45: 2D[45]

The leading differential $d^{12}(B[34]M_1^6) = 2D[45]$ determines tentative differentials whose cokernel in degrees less than 67 will be computed in Section 7.6 because of the hidden differential mentioned in the discussion of $d^{12}(B[34]M_1^6)$ above. The 2D[45]-leader is $2D[45]M_1^2$.

DEGREE 45: 4D[45]

The leading differential $d^6(\eta\sigma A[32,1]M_1^3) = 4D[45]$ determines tentative differentials with image $Z_2 \otimes (Z_4(4D[45])\{1, M_1, M_2\} \otimes B\langle 4\rangle)$. The remainig elements are $Z_2 \otimes (Z_4 D[45]\{M_1^2, M_1^3, M_1M_2, M_1^2M_2, M_1^3M_2\} \otimes B\langle 4\rangle)$, and the 4D[45]-leader is $4D[45]M_1^2$.

DEGREE 45: $8D[45] = \nu C[42]$

The leading differential $d^4(C[42]M_1^2) = 8D[45]$ determines tentative differentials with image $Z_2(8D[45])\{1, M_1, M_1^2, M_2, M_1M_2\} \otimes B\langle 4\rangle$. The remaining elements are $Z_2(8D[45])\{M_1^3, M_1^2M_2, M_1^3M_2\} \otimes B\langle 4\rangle$, and the 8D[45]-leader is $8D[45]M_1^3$.

DEGREE 45: $\eta C[44]$

Since $\eta^2 C[44] \neq 0$, the only element of $E^4_{*,45}$ with a representative in $Z_2\eta C[44] \otimes H_*BP$ is zero.