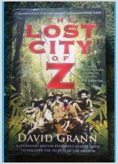
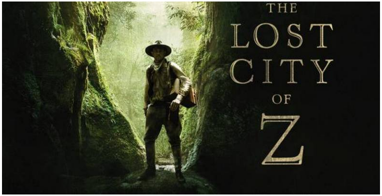


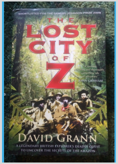
The Lost Telescope of Z

Electronic Computational Homotopy Theory Seminar

March 9, 2017



Doug Ravenel
University of Rochester



This talk began in discussions last summer with

Agnes Beaudry

Mark Behrens

Prasit Bhattacharya

Dominic Culver

Zhouli Xu

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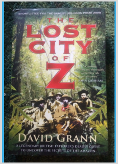

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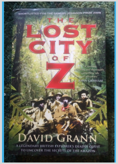

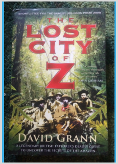

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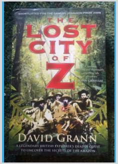

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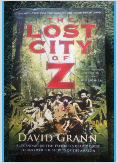

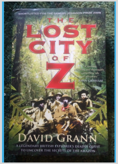

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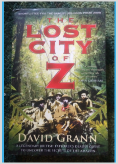

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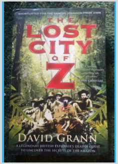

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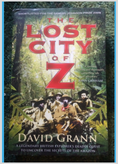
Z might have a motivic analog. This could lead to additional structure in its Adams spectral sequence.



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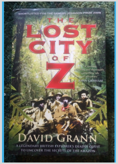

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LOCALIZATION WITH RESPECT TO CERTAIN PERIODIC HOMOLOGY THEORIES

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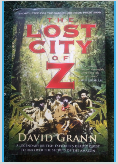

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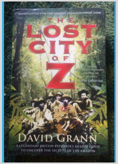

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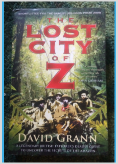

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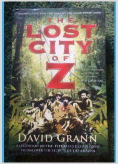

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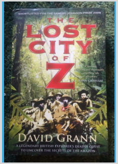

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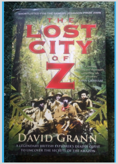

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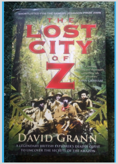

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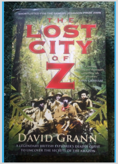

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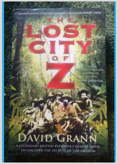

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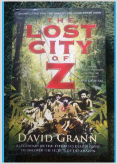

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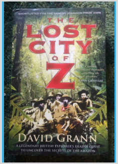

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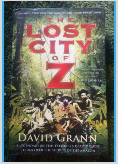
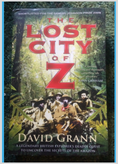


The triple loop space approach



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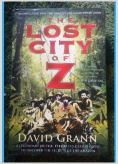

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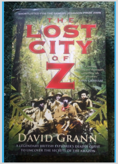

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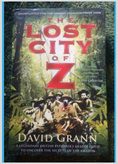

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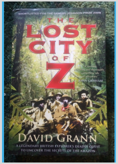

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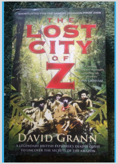

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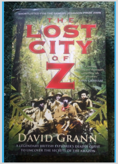

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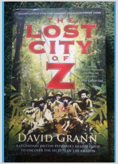

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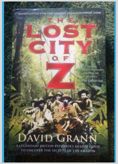

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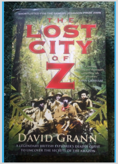

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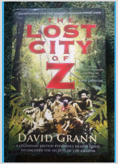

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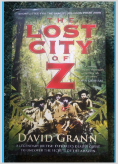

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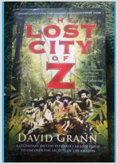

The triple loop space approach (continued)

The Bhattacharya-Egger spectrum Z has

$$H_* Z = \mathbf{Z}/2[\xi_1, \xi_2]/(\xi_1^8, \xi_2^4).$$

and a self map

$$\Sigma^6 Z \xrightarrow{v_2} Z \longrightarrow C_{v_2} = \text{cofiber}.$$

with

$$H_* C_{v_2} = \mathbf{Z}/2[\xi_1, \xi_2, \xi_3]/(\xi_1^8, \xi_2^4, \xi_3^2) = A(2)_*.$$

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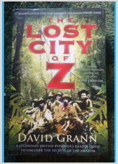
Unlike Y and Z, it is an associative ring spectrum.



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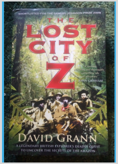

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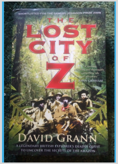
$$\Sigma^{2(2^n-1)}y(n) \xrightarrow{\ v_n\ } y(n)$$

inducing an isomorphism in $K(n)_*(-)$, the nth Morava K-theory.



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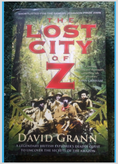

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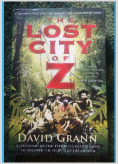

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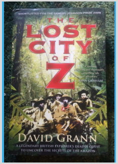

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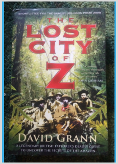
We have ways to study the homotopy groups of both of them.

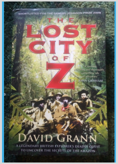


The construction of $y(n)$

Consider the diagram

$$\begin{array}{ccc} S^1 & \xrightarrow{\quad f \quad} & BO \\ & {\scriptstyle i}\searrow \quad \nearrow{\scriptstyle g} & \\ & \Omega^2 S^3 & \end{array}$$

where



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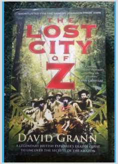

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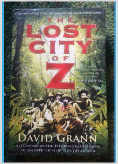

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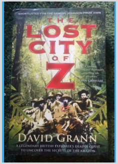

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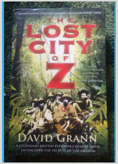
$$H_*\Omega^2 S^3 = \mathbf{Z}/2[u_1, u_2, \dots] \qquad \text{with } |u_n| = 2^n - 1 = |\xi_n|.$$



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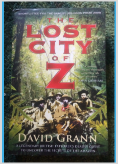

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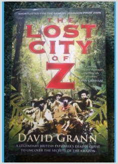

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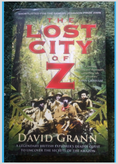

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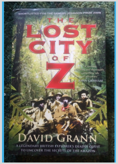

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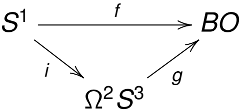

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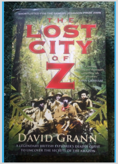

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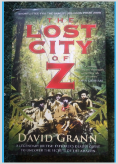

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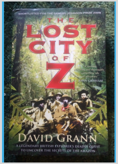

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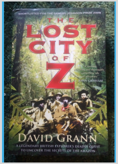

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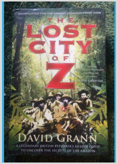

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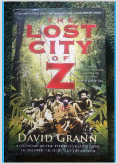

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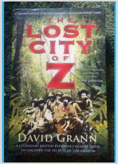

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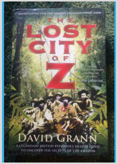

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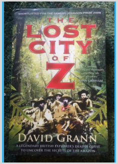

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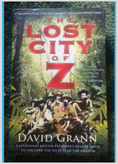
we get maps of spectra

$$\Sigma^\infty S^{|v_n|} \to \Sigma^\infty \Omega^3 S^{2^{n+1}+1} \to y(n) \to H\mathbf{Z}/2.$$



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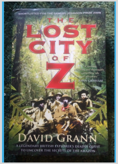

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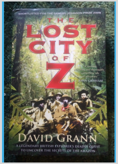
where the map $S^{|v_n|} \to \Omega^3 S^{2^{n+1}+1}$ is the inclusion of the bottom cell. Since $y(n)$ is the Thom spectrum for a loop map, it is an associative ring spectrum.



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$$\Omega J_{2^n-1}S^2 \longrightarrow \Omega^2 S^3 \xrightarrow{\ g\ } BO.$$

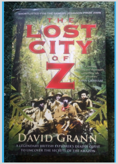
From James' 2-local fiber sequence

$$\Omega^3 S^{2^{n+1}+1} \to \Omega J_{2^n-1}S^2 \to \Omega^2 S^3$$

we get maps of spectra

$$\Sigma^\infty S^{|v_n|} \to \Sigma^\infty \Omega^3 S^{2^{n+1}+1} \to y(n) \to H\mathbf{Z}/2.$$

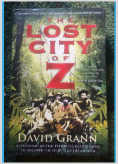
where the map $S^{|v_n|} \to \Omega^3 S^{2^{n+1}+1}$ is the inclusion of the bottom cell. Since $y(n)$ is the Thom spectrum for a loop map, it is an associative ring spectrum. The composite map above leads to the desired v_n-self map of $y(n)$.



Doug Ravenel

The Adams-Novikov spectral sequence for $L_{K(n)}y(n)$

Let $Y(n)$ denote the telescope associated with $y(n)$.



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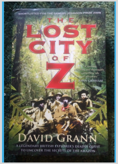

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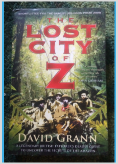
The Adams-Novikov E_2-term for $L_{K(n)}y(n)$ is

$$E_2 = \mathbf{Z}/2[v_n^{\pm 1}, v_{n+1}, \dots v_{2n}] \otimes E(h_{n+i,j} \colon 1 \le i \le n, 0 \le j < n)$$



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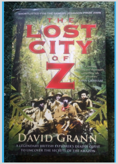

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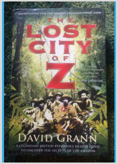

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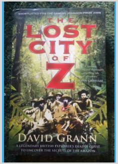
where $h_{n+i,j} = [t_{n+i}^{2^j}]$. The second factor is an exterior algebra on n^2 generators. This E_2-term is finitely generated as a module over the ring

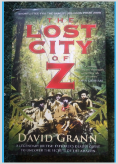
$$R(n) = \mathbf{Z}/2[v_n^{\pm 1}, v_{n+1}, \dots v_{2n}].$$



The Adams spectral sequences for $y(n)$ and $Y(n)$

Since

$$H_* y(n) = \mathbf{Z}/2[\xi_1, \xi_2, \ldots, \xi_n],$$



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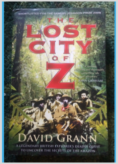

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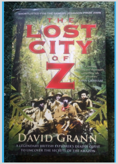

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This leads to an Adams E_1-term of the form

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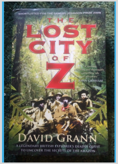

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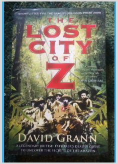
where, for such i and j,

$$v_{n+i-1} = [\xi_{n+i}] \in E_1^{1,2^{n+i}-1},$$

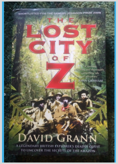
$$h_{n+i,j} = [\xi_{n+i}^{2^{j+1}}] \in E_1^{1,2^j(2^{n+i}-1)}$$

and $$d(v_{2n+i}^{2^j}) = \sum_{0\leq k<i} v_{n+k}^{2^j} h_{n+i+j-k,n+k} = v_n^{2^j} h_{n+i+j,n} + \ldots$$



Localizing the Adams spectral sequence for $y(n)$

The Adams spectral sequence for a spectrum X is based on an Adams resolution,



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$$X = X_0 \leftarrow X_1 \leftarrow X_2 \leftarrow X_3 \leftarrow \dots$$

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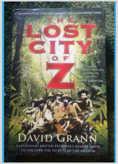

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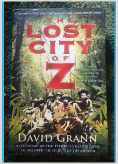

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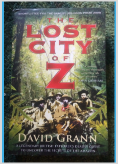
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\Sigma^{-12}X_0 & \leftarrow & \Sigma^{-12}X_1 & \leftarrow & \Sigma^{-12}X_2 & \leftarrow & \Sigma^{-12}X_3 & \leftarrow & \Sigma^{-12}X_4 & \leftarrow & \Sigma^{-12}X_5 & \leftarrow & \dots \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & &
\end{array}$$



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\end{array}$$

This leads to a localized Adams spectral sequence converging to the homotopy of

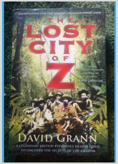
$$Y(n) = v_n^{-1} y(n).$$

Localizing the Adams spectral sequence for $y(n)$ (continued)

This localization converts

$$E_1 = P(v_n, v_{n+1}, \ldots) \otimes P(h_{n+i,j} \colon i > 0, j \geq 0)$$

converging to $\pi_* y(n)$



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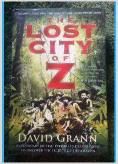
converging to $\pi_* y(n)$ to

$$E_2 = P(v_n^{\pm 1}, v_{n+1}, \ldots, v_{2n}) \otimes P(h_{n+i,j} \colon i > 0, 0 \leq j < n)$$

converging to $\pi_* Y(n)$.



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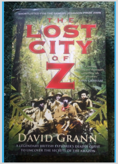
converging to $\pi_* Y(n)$. For $n = 2$ this reads

$$E_2 = P(v_2^{\pm 1}, v_3, v_4) \otimes P(h_{2+i,0}, h_{2+i,1}\colon i > 0).$$

It is likely that for $i > 0$ there are Adams differentials

$$\begin{aligned} d_2 h_{4+i,0} &= v_2 h_{2+i,1}^2 \\ d_4 h_{3+i,1} &= v_2 h_{2+i,0}^4. \end{aligned}$$



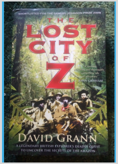
Localizing the Adams spectral sequence for $y(n)$ (continued)

In the localized Adams spectral sequence for $Y(2)$ we have

$$E_2 = P(v_2^{\pm 1}, v_3, v_4) \otimes P(h_{2+i,0}, h_{2+i,1} \colon i > 0).$$

with likely differentials

$$d_2 h_{4+i,0} = v_2 h_{2+i,1}^2 \qquad \text{and} \qquad d_4 h_{3+i,1} = v_2 h_{2+i,0}^4.$$



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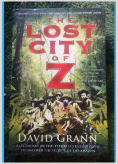
This would leave

$$E_5 = E_\infty = P(v_2^{\pm 1}, v_3, v_4) \otimes E(h_{3,0}, h_{3,1}, h_{4,0}) \otimes E(b_{3,0}, b_{4,0}, b_{5,0}, \dots)$$

where $b_{i,0} = h_{i,0}^2$. This is infinitely generated over the ring

$$R(2) = P(v_2^{\pm 1}, v_3, v_4)$$



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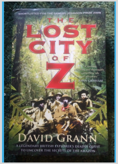
$$R(2) = P(v_2^{\pm 1}, v_3, v_4)$$

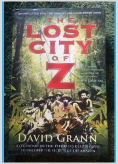
while $\pi_* L_{K(2)} y(2)$ is finitely generated over it.



Disproving the Telescope Conjecture for $n \geq 2$?

We have just seen that, if all goes according to plan, the Adams-Novikov spectral sequence shows that

$$\pi_* L_{K(2)} y(2) = P(v_2^{\pm 1}, v_3, v_4) \otimes E(h_{3,0}, h_{3,1}, h_{4,0}, h_{4,1})$$



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while the localized Adams spectral sequence shows that

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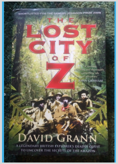

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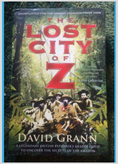

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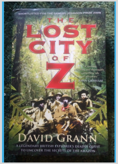

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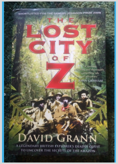

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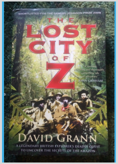

Doug Ravenel

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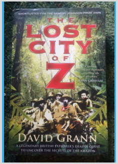

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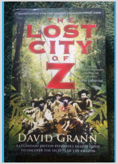
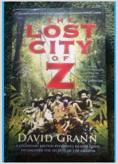


Going equivariant

If this approach is to succeed, we need some more structure in the localized Adams spectral sequence for $Y(n)$.



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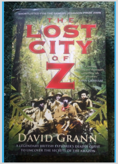

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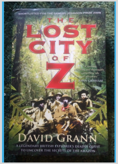
$$\begin{array}{ccccc} S^1 & \xrightarrow{\;i\;} & \Omega^2 S^3 & \xrightarrow{\;g\;} & BO \\ & & \uparrow & & \\ & & \Omega J_{2^n-1} S^2 & & \end{array}$$



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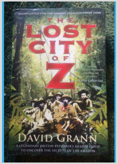
We can add another space and get

$$\begin{array}{ccccc} S^1 & \xrightarrow{i} & \Omega^2 S^3 & \xrightarrow{g} & BO \\ & & \uparrow & & \uparrow \\ & & \Omega J_{2^n-1} S^2 & \to & \Omega(SU(k+1)/SO(k+1)) \end{array} \qquad \text{for } k \gg 0.$$



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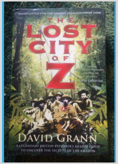

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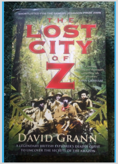
The map a_k is related to Bott's proof of his Periodicity Theorem.



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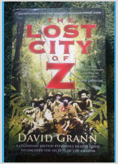

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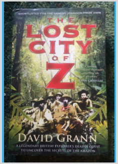

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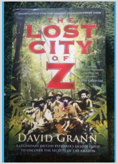

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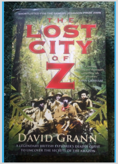

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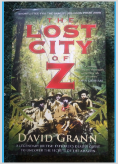

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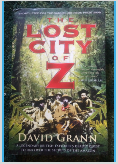

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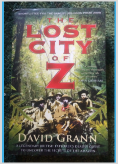

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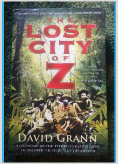

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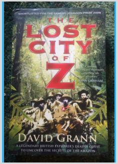

Going equivariant (continued)

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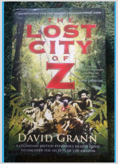

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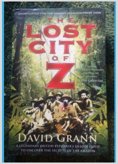

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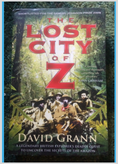

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with Thom spectra indicated on the right. Taking 2-local fibers of the vertical maps in the square gives

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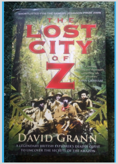

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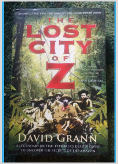
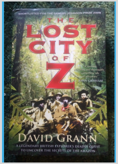
\end{array}$$

The two fibers have the same connectivity when $k = 2^{n+1} - 2$.



Going equivariant (continued)

$$\begin{array}{ccc}
\Omega^{1+\rho} S^{1+2\rho} & \xrightarrow{\quad g \quad} & BU_{\mathbf{R}} \\
\uparrow & & \uparrow a_{|v_n|} \\
\Omega^{\rho} J_{2^n-1} S^{2\rho} & \xrightarrow{\quad g_n \quad} & \Omega^{\sigma} SU(1+|v_n|)_{\mathbf{R}} \\
\uparrow & & \uparrow \\
\Omega^{2+\rho} S^{1+2^{n+1}\rho} & \longrightarrow & \Omega^{\rho}(SU/SU(1+|v_n|))_{\mathbf{R}}
\end{array}$$



Going equivariant (continued)

$$
\begin{array}{ccc}
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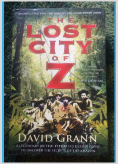
\uparrow & & \uparrow \\
\Omega^{2+\rho} S^{1+2^{n+1}\rho} & \longrightarrow & \Omega^{\rho}(SU/SU(1+|v_n|))_{\mathbf{R}}
\end{array}
$$

It follows that we have a map $y(n) \to w(|v_n|)$ inducing a monomorphism in mod 2 homology,



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Going equivariant (continued)

$$\begin{array}{ccc}
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It follows that we have a map $y(n) \to w(|v_n|)$ inducing a monomorphism in mod 2 homology, and therefore maps

$$S^{|v_n|} \to \Omega^3 S^{2^{n+1}+1} \to y(n) \to w(|v_n|),$$

Going equivariant (continued)

$$\begin{array}{ccc}
\Omega^{1+\rho}S^{1+2\rho} & \xrightarrow{\quad g \quad} & BU_{\mathbf{R}} \\
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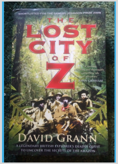
$$S^{|v_n|} \to \Omega^3 S^{2^{n+1}+1} \to y(n) \to w(|v_n|),$$

where $w(k)$ is the geometric fixed point set of the Thom spectrum $X(k)_{\mathbf{R}}$.



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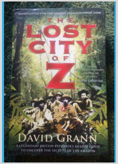

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Going equivariant (continued)

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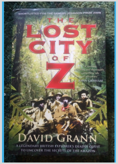
where $w(k)$ is the geometric fixed point set of the Thom spectrum $X(k)_{\mathbf{R}}$. The above composite leads to a telescope $W(|v_n|)$ which is the geometric fixed point spectrum of the telescope for a map

$$\Sigma^{(1+|v_n|)\rho-1}X(|v_n|)_{\mathbf{R}} \to X(|v_n|)_{\mathbf{R}}.$$



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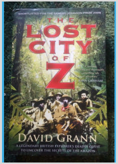
$$\Sigma^{(1+|v_n|)\rho-1} X(|v_n|)_{\mathbf{R}} \to X(|v_n|)_{\mathbf{R}}.$$

The underlying spectrum of this telescope is contractible



Going equivariant (continued)

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where $w(k)$ is the geometric fixed point set of the Thom spectrum $X(k)_{\mathbf{R}}$. The above composite leads to a telescope $W(|v_n|)$ which is the geometric fixed point spectrum of the telescope for a map

$$\Sigma^{(1+|v_n|)\rho-1}X(|v_n|)_{\mathbf{R}} \to X(|v_n|)_{\mathbf{R}}.$$

The underlying spectrum of this telescope is contractible because the underlying map is known to be nilpotent.