Theorem (The class invariance conjecture)



The Chromatic Conjectures in Homotopy Theory

Doug Ravenel University of Rochester



INTERDISCIPLINARY COLLOQUIUM SERIES July 21, 2024

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Some topology

Homotopy groups of spheres The Adams-Novikov spectral sequence Morava K-theory

Smith-Toda complexes

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Bousfield equivalence

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Harmonic and dissonant spectra

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It is easier to classify them up to continuous deformation.

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It is easier to classify them up to continuous deformation.

Definition

Two maps $f_0, f_1 : X \to Y$ are homotopic, $f_0 \simeq f_1$, if there is a continuous map

 $h: X \times [0, 1] \to Y$ with $h(x, t) = f_t(x)$ for t = 0, 1.

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Homotopy is an equivalence relation among such maps,

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Homotopy is an equivalence relation among such maps, and we get a set [X, Y] of homotopy classes of maps from X to Y.

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Consider the *m*-dimensional sphere

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Consider the *m*-dimensional sphere

$$S^m = \left\{ (x_0,\ldots,x_m) \in \mathbb{R}^{m+1} : \sum x_i^2 = 1 \right\}.$$

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It turns out that the set

 $\pi_m Y := [S^m, Y]$ for Y path connected





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A FUNDAMENTAL PROBLEM OF HOMOTOPY THEORY: Determine the homotopy groups of spheres $\pi_m S^n$ for m, n > 0.

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Three developments circa 1970

1. Complex cobordism and the Adams-Novikov spectral sequence

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1. Complex cobordism and the Adams-Novikov spectral sequence

The Adams-Novikov spectral sequence is an algebraic machine for computing the homotopy groups of spheres.

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Four Fields medalists



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You can read about it in my first book, Complex cobordism and stable homotopy groups of spheres.

1. Complex cobordism and the Adams-Novikov spectral sequence (continued)

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1. Complex cobordism and the Adams-Novikov spectral sequence (continued)

Quillen discovered an intimate connection between the cobordism theory of complex manifolds (such as complex projective algebraic varieties),

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1. Complex cobordism and the Adams-Novikov spectral sequence (continued)

Quillen discovered an intimate connection between the cobordism theory of complex manifolds (such as complex projective algebraic varieties), and formal group laws. The latter are 2-variable power series over a ring *R* with certain properties.

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For each prime p there is a graded ring $BP_* = \mathbf{Z}_{(p)}[v_1, v_2, ...]$ with $|v_n| = 2(p^n - 1)$,

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For each prime *p* there is a graded ring $BP_* = Z_{(p)}[v_1, v_2, ...]$ with $|v_n| = 2(p^n - 1)$, over which a formal group law with a certain universal property is defined.

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For each prime *p* there is a graded ring $BP_* = \mathbf{Z}_{(p)}[v_1, v_2, ...]$ with $|v_n| = 2(p^n - 1)$, over which a formal group law with a certain universal property is defined. There is a functor that assigns to each space or spectrum *X* a module BP_*X over this ring. It is a crucial tool in this theory.

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2. Morava K-theory

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2. Morava K-theory

In the early 70's Jack Morava discovered the eponynumous spectra K(n).

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K(0) is rational cohomology.

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K(0) is rational cohomology. For each n > 0 and each prime p, there is a nonconnective complex oriented p-local spectrum K(n) with

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2. Morava K-theory

In the early 70's Jack Morava discovered the eponynumous spectra K(n). They are closely related to *BP*. I was lucky enough to spend a lot of time listening to him explain their inner

workings.



K(0) is rational cohomology. For each n > 0 and each prime p, there is a nonconnective complex oriented p-local spectrum K(n) with

$$\pi_* K(n) = \mathbb{Z}/p[v_n^{\pm 1}]$$
 where $|v_n| = 2(p^n - 1)$.

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It is related to height *n* formal group laws, and $K(n)_*(K(n))$ is related to the Morava stabilizer group \mathbb{G}_n . It is a *p*-adic Lie group and the automorphism group of a height *n* formal group law over a suitable field of characteristic *p*.

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3. Smith-Toda complexes

Let $1 \le n \le 3$ and $p \ge 2n + 1$.





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and a cofiber sequence

$$\Sigma^{|v_n|} V(n-1) \xrightarrow{w_n} V(n-1) \longrightarrow V(n)$$

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We know that $K(n)_*V(n-1) \neq 0$





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We know that $K(n)_*V(n-1) \neq 0$ and w_n is a K(n)-equivalence. These lead to the construction of the v_n -periodic families aka Greek letter elements

$$\begin{array}{ll} \alpha_t \in \pi_{t|v_1|-1} \mathbb{S} & \text{for } p \geq 3 \\ \beta_t \in \pi_{t|v_2|-2p} \mathbb{S} & \text{for } p \geq 5 \\ \gamma_t \in \pi_{t|v_3|-2p^2-2p+1} \mathbb{S} & \text{for } p \geq 7 \end{array}$$

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for $p \ge 3$ for $p \ge 5$ for $p \ge 7$

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$\alpha_t \in \pi_{t v_1 -1} \mathbb{S}$	for ${m ho} \geq$ 3
$\beta_t \in \pi_{t v_2 -2p}\mathbb{S}$	for $p \ge 5$
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These are nicely displayed in the E_2 -term of the Adams-Novikov spectral sequence.

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These are nicely displayed in the E_2 -term of the Adams-Novikov spectral sequence. In it there are similar families for all *n*. This brings us to

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Annals of Mathematics, 106 (1977), 469-516

Periodic phenomena in the Adams-Novikov spectral sequence

> By HAYNES R. MILLER, DOUGLAS C. RAVENEL, and W. STEPHEN WILSON



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This was our attempt to understand the work of Jack Morava.

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In that 1977 paper, Haynes Miller, Steve Wilson and I constructed the chromatic spectral sequence converging to the E_2 -term of the Adams-Novikov spectral sequence.

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We used the term chromatic because each column (value of *n*) displays periodic families of elements with varying frequencies,

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MRW was also motivated by several examples of periodic families of elements in the stable homotopy groups of spheres.

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MRW was also motivated by several examples of periodic families of elements in the stable homotopy groups of spheres. Each was constructed as follows.

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Periodic families

MRW was also motivated by several examples of periodic families of elements in the stable homotopy groups of spheres. Each was constructed as follows.

We have a finite complex V equipped with maps

$$S^{d+k} \xrightarrow{\Sigma^d i} \Sigma^d V \xrightarrow{v} V \xrightarrow{j} S^\ell$$





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with the following properties:

• d > 0 and all iterates of v are essential.

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- In the known examples, *i* was the inclusion of the bottom cell into V

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- In the known examples, *i* was the inclusion of the bottom cell into *V* and *j* was projection onto the top cell.

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 $S^{td+k} \xrightarrow{\Sigma^{td}_i} \Sigma^{dt} V \xrightarrow{v^t} V \xrightarrow{j} S^{\ell}$

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It was known that for each t > 0,

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It was known that for each t > 0, the composite

$$S^{td+k} \xrightarrow{\Sigma^{td}_i} \Sigma^{dt} V \xrightarrow{v^t} V \xrightarrow{j} S^\ell$$

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represented a nontrivial element in $\pi_{td+k-\ell}S$.

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That was in 1973.

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That was in 1973. To this day nobody has constructed V(4).

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That was in 1973. To this day nobody has constructed V(4). In each case there is a lower bound on the prime *p*. The Chromatic Conjectures



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That was in 1973. To this day nobody has constructed V(4). In each case there is a lower bound on the prime *p*. In 2010 Lee Nave showed that V((p + 1)/2) does not exist.

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In 1973 Toda constructed finite complexes he called V(n) with $\frac{BP_*V(n)}{BP_*} \cong BP_*/(p, v_1, \dots, v_n) \quad \text{for } 0 \le n \le 3$

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$$\Sigma^{2p^n-2}V(n-1) \xrightarrow{v_n} V(n-1) \longrightarrow V(n)$$
 for $1 \le n \le 3$.

where the map v_n is periodic.

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Are there more maps like this?

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Are there more maps like this? Are there more periodic families in π_*S ?

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Are there any periodic maps that are not detected by *BP*-theory?

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What would happen if we replace $I_n = (p, \dots, v_{n-1})$ by a smaller invariant regular ideal with *n* generators,

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Are there more maps like this? Are there more periodic families in π_*S ?

Are there any periodic maps that are not detected by *BP*-theory?

What would happen if we replace $I_n = (p, \dots, v_{n-1})$ by a smaller invariant regular ideal with *n* generators, and look for a self map inducing multiplication by some power of v_n ?

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Recall that $BP_* \cong \mathbb{Z}_{(p)}[v_1, v_2, \dots]$, where $|v_n| = 2(p^n - 1)$,

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Recall that $BP_* \cong \mathbb{Z}_{(p)}[v_1, v_2, ...]$, where $|v_n| = 2(p^n - 1)$, and $\Gamma := BP_*(BP) \cong BP_*[t_1, t_2, ...]$, with $|t_i| = 2(p^i - 1)$

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 $\Gamma := BP_*(BP) \cong BP_*[t_1, t_2, ...], \text{ with } |t_i| = 2(p^i - 1)$

which has a Hopf algebroid structure.

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 $\Gamma := BP_*(BP) \cong BP_*[t_1, t_2, ...], \text{ with } |t_i| = 2(p^i - 1)$

which has a Hopf algebroid structure.

The E_2 -term of the Adams-Novikov spectral sequence converging to the *p*-local stable homotopy groups of spheres is

$$E_2^{s,t} = \operatorname{Ext}_{BP_*(BP)}^{s,t} (BP_*, BP_*),$$

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so this object is of great interest. It can be studied with the long exact sequence of $BP_*(BP)$ -comodules

$$0 \rightarrow BP_* \rightarrow M^0 \rightarrow M^1 \rightarrow M^2 \rightarrow M^3 \rightarrow \cdots,$$

the chromatic resolution.

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This leads to a trigraded chromatic spectral sequence converging to the bigraded Adams-Novikov E_2 -term, with

$$E_1^{n,s,t} = \operatorname{Ext}_{BP_*(BP)}^{s,t}(BP_*, M^n) \Rightarrow E_2^{n+s,t}$$

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For a fixed *n*, this group is related to the cohomology of the *n*th Morava stabilizer group,





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For a fixed *n*, this group is related to the cohomology of the *n*th Morava stabilizer group, which is the automorphism group of a certain formal group law of height *n*. It is also related to v_n -periodic phenomena in the stable homotopy groups of spheres.

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Again, we used the term chromatic because each column (value of *n*) displays periodic families of elements with varying frequencies,

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 $0 \rightarrow BP_* \rightarrow M^0 \rightarrow M^1 \rightarrow M^2 \rightarrow M^3 \rightarrow \cdots$

The comodules M^n are defined inductively as follows.

The Chromatic Conjectures



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The comodules M^n are defined inductively as follows.

• *M*⁰ is obtained from *BP*_{*} by inverting *p*.





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The comodules M^n are defined inductively as follows.

 M⁰ is obtained from BP_{*} by inverting p. This means there is a short exact sequence

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• For n > 0, M^n is obtained from N^n by inverting v_n .



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The comodules M^n are defined inductively as follows.

*M*⁰ is obtained from *BP*_{*} by inverting *p*. This means there is a short exact sequence



 For n > 0, Mⁿ is obtained from Nⁿ by inverting v_n. There is a short exact sequence

$$0 \longrightarrow N^{n} \xrightarrow{v_{n}^{-1}} M^{n} \xrightarrow{N^{n+1}} 0$$

$$|| \qquad || \qquad || \qquad || \qquad || \qquad || \qquad BP_{*}/(p^{\infty}, \dots, v_{n-1}^{\infty})$$

$$v_{n}^{-1}BP_{*}/(p^{\infty}, \dots, v_{n-1}^{\infty})$$

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The chromatic resolution

$$0 \rightarrow BP_* \rightarrow M^0 \rightarrow M^1 \rightarrow M^2 \rightarrow M^3 \rightarrow \cdots$$

is obtained by splicing together these short exact sequence for all $n \ge 0$.

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The chromatic resolution

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This construction is purely algebraic.

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The chromatic resolution

$$0 \rightarrow BP_* \rightarrow M^0 \rightarrow M^1 \rightarrow M^2 \rightarrow M^3 \rightarrow \cdots$$

is obtained by splicing together these short exact sequence for all $n \ge 0$.

This construction is purely algebraic. It takes place in the category of $BP_*(BP)$ -comodules.

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IS THERE A SIMILAR CONSTRUCTION IN THE STABLE HOMOTOPY CATEGORY?

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IS THERE A SIMILAR CONSTRUCTION, AND THE BEAUTIFUL ALGEBRA THAT GOES ALONG WITH IT, IN THE STABLE HOMOTOPY CATEGORY?

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IS THERE A SIMILAR CONSTRUCTION, AND THE BEAUTIFUL ALGEBRA THAT GOES ALONG WITH IT, IN THE STABLE HOMOTOPY CATEGORY?

OR IS IT JUST AN ARTIFACT OF COMPLEX COBORDISM THEORY?

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IS THERE A SIMILAR CONSTRUCTION, AND THE BEAUTIFUL ALGEBRA THAT GOES ALONG WITH IT, IN THE STABLE HOMOTOPY CATEGORY?

OR IS IT JUST AN ARTIFACT OF COMPLEX COBORDISM THEORY?

This question occupied me for several years.

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This paper appeared in 1984.

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This paper appeared in 1984.

LOCALIZATION WITH RESPECT TO CERTAIN PERIODIC HOMOLOGY THEORIES

By DOUGLAS C. RAVENEL*

This paper represents an attempt, only partially successful, to get at what appear to be some deep and hitherto unexamined properties of the stable homotopy category. This work was motivated by the discovery of the pervasive manifestation of various types of periodicity in the E_2 -term of the Adams-Novikov spectral sequence converging to the stable homotopy groups of spheres. In section 3 of [34] and section 8 of [41], we introduced the chromatic spectral sequence, which converges to the above E_2 -term. Unlike most spectral sequences, its input is in some sense more interesting than its output, as the former displays many appealing patterns which are somewhat hidden in the latter (see section 8 of [41] for a more detailed discussion). It is not so much a computational aid (although it has been used [34] for computing the Novikov 2-line) as a conceptual tool for understanding certain qualitative aspects of the Novikov E_2 -term.

Since the Novikov E2-term is a reasonably good approximation to sta-

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It would be nice if each short exact sequence above were the BP_* homology of a cofiber sequence of spectra.

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This was easy enough for n = 0.





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This was easy enough for n = 0. We knew then how to invert a prime *p* homotopically. The resulting N^1 is the Moore spectrum for the group $\mathbb{Q}/\mathbb{Z}_{(p)}$. But how would we invert v_1 to do the next step?



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As luck would have it, Bousfield localization had just been invented!





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Pete Bousfield 1941-2020





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As luck would have it, Bousfield localization had just been invented!



Pete Bousfield 1941-2020

Topology, Vol. 18, pp. 257-281 Pergamon Press Ltd., 1979. Printed in Great Britain

THE LOCALIZATION OF SPECTRA WITH RESPECT TO HOMOLOGY

A. K. BOUSFIELD[†]

(Received 3 January 1979)

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Suppose we have a generalized homology theory represented by a spectrum *E*.

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Suppose we have a generalized homology theory represented by a spectrum E. We say a spectrum Z is E-local if,

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Suppose we have a generalized homology theory represented by a spectrum *E*. We say a spectrum *Z* is *E*-local if, whenever $f : A \rightarrow B$ is an *E*_{*}-equivalence, that is a map inducing an isomorphism $E_*A \rightarrow E_*B$,

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Suppose we have a generalized homology theory represented by a spectrum *E*. We say a spectrum *Z* is *E*-local if, whenever $f: A \rightarrow B$ is an *E*_{*}-equivalence, that is a map inducing an isomorphism *E*_{*}*A* \rightarrow *E*_{*}*B*, then the induced map

$$f^*: [B, Z] \rightarrow [A, Z]$$

is also an isomorphism.

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is also an isomorphism.

Theorem (Bousfield localization of spectra 1979)

For a given E there is a coaugmented functor L_E such that for any spectrum X, $L_E X$ is E-local and the map $X \rightarrow L_E X$ is an E_* -equivalence. The Chromatic Conjectures



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For a given E there is a coaugmented functor L_E such that for any spectrum X, $L_E X$ is E-local and the map $X \to L_E X$ is an E_* -equivalence.

It turns out that when *E* and *X* are both connective, then $L_E X$ can be described in arithmetic terms.

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It turns out that when *E* and *X* are both connective, then $L_E X$ can be described in arithmetic terms. It is either obtained from *X* by inverting some set of primes,

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It turns out that when *E* and *X* are both connective, then $L_E X$ can be described in arithmetic terms. It is either obtained from *X* by inverting some set of primes, or it is the *p*-adic completion for a single prime *p*.

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It turns out that when *E* and *X* are both connective, then $L_E X$ can be described in arithmetic terms. It is either obtained from *X* by inverting some set of primes, or it is the *p*-adic completion for a single prime *p*.

Things can be much more interesting when either E or X (or both) fail to be connective.

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Bousfield localization (continued)

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Bousfield localization (continued)

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WHAT IF OUR HYPOTHETICAL SPECTRUM M_n COULD BE OBTAINED FROM THE INDUCTIVELY CONSTRUCTED N_n BY SOME FORM OF BOUSFIELD LOCALIZATION?

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Bousfield localization (continued)

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Definition

Two spectra E and E' are Bousfield equivalent





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It follows that the maximal Bousfield class is that of the sphere spectrum $\mathbb{S},$

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It is easy to check that wedges and smash products of Bousfield classes are well defined, that is we can define

 $\langle E \rangle \lor \langle F \rangle := \langle E \lor F \rangle$ and $\langle E \rangle \land \langle F \rangle := \langle E \land F \rangle$.

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Theorem (Formal properties of Bousfield classes)

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• If $W \to X \to Y \xrightarrow{f} \Sigma W$ is a cofiber sequence,

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If f is smash nilpotent

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 $\langle X \rangle \leq \langle W \rangle \lor \langle Y \rangle.$

- If f is smash nilpotent (meaning that f^{∧k} : Y^{∧k} → (ΣW)^{∧k} is null for some k), then ⟨X⟩ = ⟨W⟩ ∨ ⟨Y⟩.
- For a self-map $\Sigma^d X \xrightarrow{v} X$, let C_v denote its cofiber and let \widehat{X} denote the homotopy colimit (mapping telescope) of

$$X \xrightarrow{v} \Sigma^{-d} X \xrightarrow{v} \Sigma^{-2d} X \xrightarrow{v} \cdots$$

Then $\langle X \rangle = \langle \widehat{X} \rangle \lor \langle C_{v} \rangle$ and $\langle \widehat{X} \rangle \land \langle C_{v} \rangle = \langle * \rangle$.

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Theorem (Some Bousfield equivalence classes)

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Theorem (Some Bousfield equivalence classes)

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where \mathbb{SQ} is the rational Moore spectrum and \mathbb{S}/p is the mod p Moore spectrum.





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where \mathbb{SQ} is the rational Moore spectrum and \mathbb{S}/p is the mod p Moore spectrum.

$$\langle BP \rangle \geq \langle H/p \rangle \lor \bigvee_{n > 0} \langle K(n) \rangle,$$

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Bousfield equivalence (continued)

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$$\langle E(n) \rangle = \langle E_n \rangle = \bigvee_{0 \le i \le n} \langle K(i) \rangle$$

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Bousfield equivalence (continued)

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 $\langle E(n) \rangle = \langle E_n \rangle = \bigvee_{0 \leq i \leq n} \langle K(i) \rangle.$

In each case, the smash product of any two of the wedge summands on the right is contractible.

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The localization functor L_E is determined by the Bousfield class $\langle E \rangle$.

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The localization functor L_E is determined by the Bousfield class $\langle E \rangle$. When $\langle E \rangle \ge \langle F \rangle$,

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The localization functor L_E is determined by the Bousfield class $\langle E \rangle$. When $\langle E \rangle \ge \langle F \rangle$, there is a natural transformation $L_E \Rightarrow L_F$.

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For a fixed prime p, let $L_n = L_{E(n)}$.

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For a fixed prime p, let $L_n = L_{E(n)}$. Then for any spectrum X we get a diagram

$$X \to L_{\infty}X \cdots \to L_nX \to L_{n-1}X \to \cdots \to L_1X \to L_0X.$$

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This the chromatic tower of *X*.

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This the chromatic tower of *X*. Here L_{∞} denotes localization with respect to the Bousfield class

$$\bigvee_{n\geq 0} \langle K(n)\rangle$$

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$$X \to L_{\infty}X \cdots \to L_nX \to L_{n-1}X \to \cdots \to L_1X \to L_0X.$$

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$$X \to L_{\infty}X \dots \to L_nX \to L_{n-1}X \to \dots \to L_1X \to L_0X.$$

This raises some questions:





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$$X \to L_{\infty}X \cdots \to L_nX \to L_{n-1}X \to \cdots \to L_1X \to L_0X.$$

This raises some questions:

• When is the map $X \to L_{\infty}X$ an equivalence?

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$$X \to L_{\infty}X \cdots \to L_nX \to L_{n-1}X \to \cdots \to L_1X \to L_0X.$$

This raises some questions:

When is the map X → L_∞X an equivalence? When it is, we say X is harmonic.

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$$X \to L_{\infty}X \cdots \to L_nX \to L_{n-1}X \to \cdots \to L_1X \to L_0X.$$

This raises some questions:

When is the map X → L_∞X an equivalence? When it is, we say X is harmonic. We call L_∞X the harmonic localization of X.

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This raises some questions:

When is the map X → L_∞X an equivalence? When it is, we say X is harmonic. We call L_∞X the harmonic localization of X. We say X is dissonant when L_∞X ≃ *.





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$$X \to L_{\infty}X \cdots \to L_nX \to L_{n-1}X \to \cdots \to L_1X \to L_0X.$$

This raises some questions:

- When is the map X → L_∞X an equivalence? When it is, we say X is harmonic. We call L_∞X the harmonic localization of X. We say X is dissonant when L_∞X ≃ *.
- When is the map $X \rightarrow \text{holim}L_n X$ an equivalence?

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$$X \to L_{\infty}X \cdots \to L_nX \to L_{n-1}X \to \cdots \to L_1X \to L_0X.$$

This raises some questions:

- When is the map X → L_∞X an equivalence? When it is, we say X is harmonic. We call L_∞X the harmonic localization of X. We say X is dissonant when L_∞X ≃ *.
- When is the map X → holimL_nX an equivalence? This is the chromatic convergence question.

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- When is the map X → L_∞X an equivalence? When it is, we say X is harmonic. We call L_∞X the harmonic localization of X. We say X is dissonant when L_∞X ≃ *.
- When is the map X → holimL_nX an equivalence? This is the chromatic convergence question.
- Can we describe BP_*L_nX in terms of BP_*X ?

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$$X \to L_{\infty}X \cdots \to L_nX \to L_{n-1}X \to \cdots \to L_1X \to L_0X.$$

This raises some questions:

- When is the map X → L_∞X an equivalence? When it is, we say X is harmonic. We call L_∞X the harmonic localization of X. We say X is dissonant when L_∞X ≃ *.
- When is the map X → holimL_nX an equivalence? This is the chromatic convergence question.
- Can we describe *BP*_{*}*L*_n*X* in terms of *BP*_{*}*X*? This is the localization question.



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Recall that L_{∞} denotes localization with respect to the Bousfield class

 $\bigvee \langle K(n) \rangle.$ *n*>0



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Recall that L_{∞} denotes localization with respect to the Bousfield class

$$\bigvee_{\geq 0} \langle K(n) \rangle.$$

A *p*-local spectrum is harmonic if $X \simeq L_{\infty} X$.

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Recall that L_{∞} denotes localization with respect to the Bousfield class

 $\bigvee_{n\geq 0} \langle K(n)\rangle.$

A *p*-local spectrum is harmonic if $X \simeq L_{\infty}X$. It is dissonant if $L_{\infty}X \simeq *$, meaning that $K(n)_*X = 0$ for all *n*.





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A *p*-local spectrum is harmonic if $X \simeq L_{\infty}X$. It is dissonant if $L_{\infty}X \simeq *$, meaning that $K(n)_*X = 0$ for all n. It follows from the definitions that there are no essential maps from a dissonant spectrum to a harmonic one.

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In the 1984 paper I showed that

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A *p*-local spectrum is harmonic if $X \simeq L_{\infty}X$. It is dissonant if $L_{\infty}X \simeq *$, meaning that $K(n)_*X = 0$ for all n. It follows from the definitions that there are no essential maps from a dissonant spectrum to a harmonic one.

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Every p-local finite spectrum is harmonic.

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A *p*-local spectrum is harmonic if $X \simeq L_{\infty}X$. It is dissonant if $L_{\infty}X \simeq *$, meaning that $K(n)_*X = 0$ for all *n*. It follows from the definitions that there are no essential maps from a dissonant spectrum to a harmonic one.

In the 1984 paper I showed that

- Every *p*-local finite spectrum is harmonic.
- A *p*-local connective spectrum X is harmonic when BP_{*}X has finite projective dimension as a BP_{*}-module.

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A *p*-local spectrum is harmonic if $X \simeq L_{\infty}X$. It is dissonant if $L_{\infty}X \simeq *$, meaning that $K(n)_*X = 0$ for all *n*. It follows from the definitions that there are no essential maps from a dissonant spectrum to a harmonic one.

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A *p*-local spectrum is harmonic if $X \simeq L_{\infty}X$. It is dissonant if $L_{\infty}X \simeq *$, meaning that $K(n)_*X = 0$ for all *n*. It follows from the definitions that there are no essential maps from a dissonant spectrum to a harmonic one.

In the 1984 paper I showed that

- Every *p*-local finite spectrum is harmonic.
- A *p*-local connective spectrum *X* is harmonic when *BP*_{*}*X* has finite projective dimension as a *BP*_{*}-module.
- The mod *p* Eilenberg-Mac Lane spectrum *H*/*p* is dissonant. The same is true for any spectrum whose homotopy groups are all torsion and bounded above.



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A *p*-local spectrum *X* is chromatically convergent if it is equivalent to the homotopy limit of the diagram

$$\cdots \rightarrow L_n X \rightarrow L_{n-1} X \rightarrow \cdots \rightarrow L_1 X \rightarrow L_0 X.$$

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In 2014 Tobias Barthel proved a *p*-local connective spectrum *X* is chromatically convergent when BP_*X has finite projective dimension as a BP_* -module.

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In 2014 Tobias Barthel proved a *p*-local connective spectrum *X* is chromatically convergent when BP_*X has finite projective dimension as a BP_* -module. Such spectra were previously known to be harmonic.

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Recall one of the original questions of this lecture: Does the chromatic resolution (leading to the chromatic spectral sequence of Miller-R-Wilson) have a geometric underpinning?





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It turns out that $L_n BP$ is easy to analyze,



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It turns out that $L_n BP$ is easy to analyze, and this makes it easy to understand the spectrum $X \wedge L_n BP$.

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It turns out that $L_n BP$ is easy to analyze, and this makes it easy to understand the spectrum $X \wedge L_n BP$.

Theorem (The localization conjecture)

For any spectrum X,

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Theorem (The localization conjecture)

For any spectrum X,

 $BP \wedge L_n X \simeq X \wedge L_n BP.$

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In particular, when $E(n-1)_*X = 0$, $BP_*L_nX = v_n^{-1}BP_*X$.

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It follows that the chromatic resolution can be realized as desired.

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It turns out that the functor L_n satisfies a stronger condition, conjectured in 1984, proved with Hopkins a few years later, and reported in the orange book.

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The chromatic tower Harmonic and dissonant spectra Chromatic convergence The chromatic resolution and the chromatic tower Some conjectures The nilpotence and periodicit theorems

Theorem (The smash product conjecture)

For any spectrum X, $L_n X \cong X \wedge L_n S$.

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Theorem (The smash product conjecture)

For any spectrum X, $L_n X \cong X \wedge L_n S$.

when your localization functor satisfies $L_E X = X \otimes_{\mathbb{S}} L_E \mathbb{S}$



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I ended the 1984 paper with a list of conjectures,





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Nilpotence Theorem (Devinatz-Hopkins-Smith 1988)

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Nilpotence Theorem (Devinatz-Hopkins-Smith 1988)

For a finite spectrum X, a map v : Σ^dX → X is nilpotent iff MU_{*}(v) is nilpotent.

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- Por a finite spectrum X, a map v : Σ^dX → X is nilpotent iff MU_{*}(v) is nilpotent.
- Por a finite spectrum X, a map g : X → Y is smash nilpotent if the map MU ∧ g is null homotopic.

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Nilpotence Theorem (Devinatz-Hopkins-Smith 1988)

- **9** For a finite spectrum X, a map $v : \Sigma^d X \to X$ is nilpotent iff $MU_*(v)$ is nilpotent.
- For a finite spectrum X, a map $g: X \to Y$ is smash nilpotent if the map $MU \wedge g$ is null homotopic.
- Let R be a connective ring spectrum of finite type, and let $h: \pi_* R \to MU_* R$ be the Hurewicz map.



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- Let R be a connective ring spectrum of finite type, and let $h: \pi_* R \to MU_* R$ be the Hurewicz map. Then $\alpha \in \pi_* R$ is nilpotent when $h(\alpha) = 0$.

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- S Let R be a connective ring spectrum of finite type, and let h : π_{*}R → MU_{*}R be the Hurewicz map. Then α ∈ π_{*}R is nilpotent when h(α) = 0.

🦪 Let

$$W \longrightarrow X \longrightarrow Y \xrightarrow{f} \Sigma W$$

be a cofiber sequence of finite spectra with $MU_*(f) = 0$.



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Some conjectures

I ended the 1984 paper with a list of conjectures, all but one of which (the telescope conjecture) were proved within a 15 years, most by Mike Hopkins and various collaborators. I have already mentioned some of them. I will state some more of them here as theorems.

Nilpotence Theorem (Devinatz-Hopkins-Smith 1988)

- **7** For a finite spectrum X, a map $v : \Sigma^d X \to X$ is nilpotent iff $MU_*(v)$ is nilpotent.
- **2** For a finite spectrum X, a map $g : X \rightarrow Y$ is smash nilpotent if the map MU \land g is null homotopic.
- S Let R be a connective ring spectrum of finite type, and let h : π_{*}R → MU_{*}R be the Hurewicz map. Then α ∈ π_{*}R is nilpotent when h(α) = 0.

🦪 Let

$$W \longrightarrow X \longrightarrow Y \xrightarrow{f} \Sigma W$$

be a cofiber sequence of finite spectra with $MU_*(f) = 0$. Then $\langle X \rangle = \langle W \rangle \lor \langle Y \rangle$.

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If it were the case that $\langle MU \rangle = \langle \mathbb{S} \rangle$,

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If it were the case that $\langle MU \rangle = \langle \mathbb{S} \rangle$, or if $\langle BP \rangle = \langle \mathbb{S}_{(p)} \rangle$ for each prime p,

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If it were the case that $\langle MU \rangle = \langle \mathbb{S} \rangle$, or if $\langle BP \rangle = \langle \mathbb{S}_{(p)} \rangle$ for each prime *p*, then the Nilpotence Theorem would follow immediately.

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However $\langle BP \rangle < \langle \mathbb{S}_{(p)} \rangle$,

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However $\langle BP \rangle < \langle \mathbb{S}_{(p)} \rangle$, meaning there are BP_* -acyclic *p*-local spectra that are not contractible.

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In fact there are connective *p*-local spectra T(m) for $m \ge 0$ with

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 $BP_*T(m) \cong BP_*[t_1, t_2, \dots, t_m]$ (so $T(0) = \mathbb{S}_{(p)}$)

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In fact there are connective *p*-local spectra T(m) for $m \ge 0$ with

 $BP_*T(m) \cong BP_*[t_1, t_2, \dots t_m]$ (so $T(0) = \mathbb{S}_{(p)}$)

and strict Bousfield inequalities

 $\langle T(0) \rangle > \langle T(1) \rangle > \langle T(2) \rangle \cdots > \langle BP \rangle.$

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Nilpotence Theorem (Devinatz-Hopkins-Smith 1988)

Por a finite spectrum X, a map f : Σ^d X → X is nilpotent iff MU_{*}(f) is nilpotent.

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Nilpotence Theorem (Devinatz-Hopkins-Smith 1988)

● For a finite spectrum X, a map $f : \Sigma^d X \to X$ is nilpotent iff $MU_*(f)$ is nilpotent.

This means that such a map can be periodic (the opposite of being nilpotent) only if it detected as such by *MU*-homology.

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Nilpotence Theorem (Devinatz-Hopkins-Smith 1988)

Por a finite spectrum X, a map f : Σ^dX → X is nilpotent iff MU_{*}(f) is nilpotent.

This means that such a map can be periodic (the opposite of being nilpotent) only if it detected as such by *MU*-homology. In the *p*-local case, the internal properties of *MU*-theory imply that *f* must induce a nontriivial isomorphism in some Morava K-theory $K(n)_*$.

Periodicity Theorem (Hopkins-Smith 1998)

Let X be a p-local finite spectrum of chromatic type n,

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Periodicity Theorem (Hopkins-Smith 1998)

Let X be a p-local finite spectrum of chromatic type n, meaning that $K(n-1)_*X = 0$, but $K(n)_*X \neq 0$.

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Given a second such map $w : \Sigma^e X \to X$, there are positive integers i and j

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Given a second such map $w : \Sigma^e X \to X$, there are positive integers i and j such that id = je and $v^i = w^j$.

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Given a second such map $w : \Sigma^e X \to X$, there are positive integers *i* and *j* such that id = je and $v^i = w^j$. In other words, *v* is assymptotically unique.

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It follows that the cofiber of v (or of any of its iterates) is a p-local finite spectrum of chromatic type n + 1.

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A pleasant consequence of the Nilpotence Theorem is the following.

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A pleasant consequence of the Nilpotence Theorem is the following.

Theorem (The class invariance conjecture)

The Bousfield class of a p-local finite spectrum X is determined by its chromatic type, i.e., the smallest n for which $K(n)_*X \neq 0$.

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A pleasant consequence of the Nilpotence Theorem is the following.

Theorem (The class invariance conjecture)

The Bousfield class of a p-local finite spectrum X is determined by its chromatic type, i.e., the smallest n for which $K(n)_*X \neq 0$. In particular if H_*X is not all torsion, then $\langle X \rangle = \langle \mathbb{S}_{(p)} \rangle$.

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Suppose X is a p-local finite spectrum of chromatic type n.

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Suppose *X* is a *p*-local finite spectrum of chromatic type *n*. The Periodicity Theorem says that it has a v_n self-map $v : \Sigma^d X \to X$.

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Suppose *X* is a *p*-local finite spectrum of chromatic type *n*. The Periodicity Theorem says that it has a v_n self-map $v : \Sigma^d X \to X$. Let \widehat{X} be the associated mapping telescope, meaning the homotopy colimit of

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$$X \xrightarrow{v} \Sigma^{-d} X \xrightarrow{v} \Sigma^{-2d} X \xrightarrow{v} \cdots$$

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$$X \xrightarrow{v} \Sigma^{-d} X \xrightarrow{v} \Sigma^{-2d} X \xrightarrow{v} \cdots$$

Note that it is independent of the choice of v. Since v is a K(n)-equivalence and therefore an E(n)-equivalence, we have maps

 $X \longrightarrow \widehat{X} \longrightarrow L_n X$

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Is there more?

Periodic familie:

The chromatic resolution

Bousfield localization

Bousfield equivalence

The chromatic tower

Harmonic and dissonant spectra

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Some conjectures

The nilpotence and periodicity theorems The telescone conjecture

Telescope conjecture

For any p-local spectrum X of chromatic type n, the map $\lambda : \hat{X} \to L_n X$ is an equivalence.

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This is trivially true for n = 0, and for n = 1 it was proved around 1980 by Mahowald for p = 2 and by Miller for p odd.

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Something happened there that led me to think I could disprove the conjecture for $n \ge 2$.

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San Francisco earthquake of October 17, 1989

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Jeremy, Tomer, myself, Ishan and Robert at Oxford University, June 9, 2023. Photo by Matteo Barucco. The Chromatic Conjectures



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THANK YOU!



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