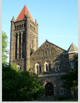
The C_2-equivariant analog of the subalgebra of $\mathcal{A}$ generated by Sq^1 and Sq^2

Homotopy theory:
tools and applications
The Crypto Goerss Fest
University of Illinois
at Urbana-Champaign
July 20, 2017



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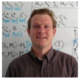
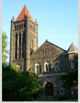
Joint work with

Bert Guillou, University of Kentucky

Mike Hill, UCLA



Dan Isaksen, Wayne State University



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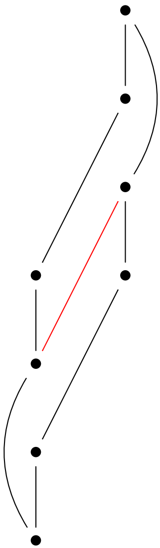

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Equivariant homotopy theory

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- S^V denotes its one point compactification,

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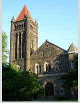

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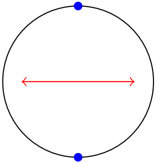
Here is a picture of S^σ, the twisted circle, whose fixed point set is S^0:



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The surprising property of the equivariant Hopf map

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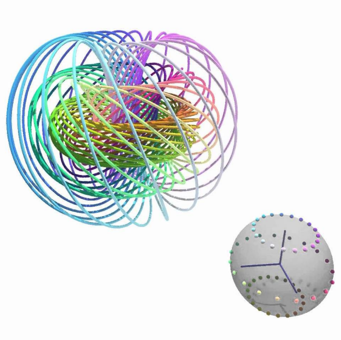
The surprising property of the equivariant Hopf map

Recall the Hopf map

$$\mathbb{C}^2 \supset S^3 \xrightarrow{\eta} \mathbb{C}P^1 = S^2.$$



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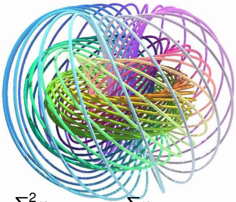
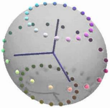


The composite map

$$S^7 \xrightarrow{\Sigma^4\eta} S^6 \xrightarrow{\Sigma^3\eta} S^5 \xrightarrow{\Sigma^2\eta} S^4 \xrightarrow{\Sigma\eta} S^3$$



is known to be null homotopic.

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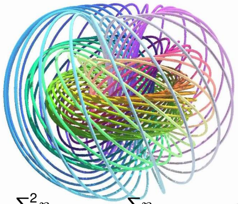

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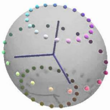

is known to be null homotopic.

Both source and target of η have a C_2-action induced by complex conjugation. The Hopf map η preserves it, so we get an equivariant map

$$S(2+2\sigma) \approx S^{1+2\sigma} \xrightarrow{\quad\overline{\eta}\quad} S^{1+\sigma}$$

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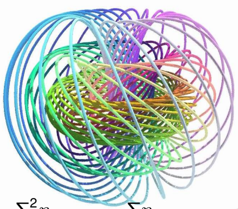

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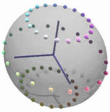

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$$S(2+2\sigma) \approx S^{1+2\sigma} \xrightarrow{\overline{\eta}} S^{1+\sigma}$$

and the induced map of fixed point sets is the degree 2 map

$$S(2) \approx S^1 \xrightarrow{[2]} \mathbb{R}P^1 = S^1.$$

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The surprising property of the equivariant Hopf map (continued)

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The surprising property of the equivariant Hopf map (continued)

Iterating the equivariant Hopf map $\overline{\eta} : S^{1+2\sigma} \to S^{1+\sigma}$ gives a diagram of equivariant maps and fixed point sets

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$$S^1 \xrightarrow{[2]} S^1 \xrightarrow{[2]} S^1 \xrightarrow{[2]} S^1 \xrightarrow{[2]} S^1$$

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The equivariant mod 2 homology of a point

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The equivariant mod 2 homology of a point

In equivariant stable homotopy theory we can speak of homology and homotopy groups graded over $RO(G)$, the real orthogonal representation ring of G.

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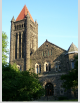

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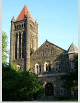

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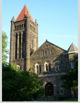
There are two elements of interest.



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- The inclusion map of the fixed point set (the north and south poles) $a : S^0 \to S^\sigma$

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- The inclusion map of the fixed point set (the north and south poles) $a : S^0 \to S^\sigma$ defines an element $a \in \pi^{C_2}_{-\sigma} S^{-0}$, and we use the same symbol for its mod 2 Hurewicz image.

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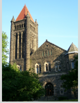
and we denote its generator by u.



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The equivariant mod 2 homology of a point (continued)

Dually we have

$$a \in H^{C_2}_{\sigma} \qquad \text{and} \qquad u \in H^{C_2}_{\sigma-1}.$$

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The equivariant mod 2 homology of a point (continued)

Dually we have

$$a \in H^{\sigma}_{C_2} \qquad \text{and} \qquad u \in H^{\sigma-1}_{C_2}.$$

In real motivic homotopy theory one has analogous elements

$$\rho \in H^{(1,1)}_{\mathbb{R}} \qquad \text{and} \qquad \tau \in H^{(0,1)}_{\mathbb{R}},$$

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In real motivic homotopy theory one has analogous elements

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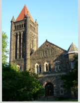
where the motivic bidegree (s, w) (for stem and weight) corresponds to the $RO(C_2)$ degree $s - w + w\sigma$.



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The equivariant mod 2 homology of a point (continued)

Dually we have

$$a \in H^{\sigma}_{C_2} \qquad \text{and} \qquad u \in H^{\sigma-1}_{C_2}.$$

In real motivic homotopy theory one has analogous elements

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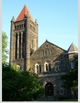

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$$NC = \Sigma\mathbb{Z}/2[a,u]/(a^\infty, u^\infty) = \bigoplus_{i,j>0} \mathbf{Z}/2\left\{\frac{w}{a^i u^j}\right\}$$

Here w has cohomogical degree 1, so

$$:= \left|\frac{w}{a^i u^j}\right| = 1 - i\sigma - j(\sigma - 1) = (1+j) - (i+j)\sigma.$$

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$$a w_{i+1,j} = w_{i,j} = u w_{i,j+1} \quad \text{and} \quad a^i w_{i,j} = u^j w_{i,j} = 0.$$

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Each $w_{i,j}$ is both a-divisible and u-divisible.

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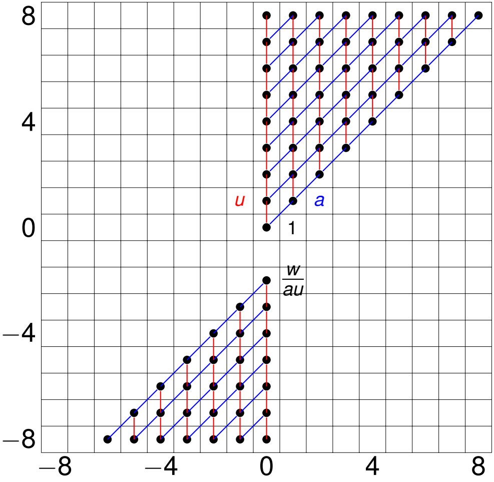
The equivariant mod 2 cohomology of a point



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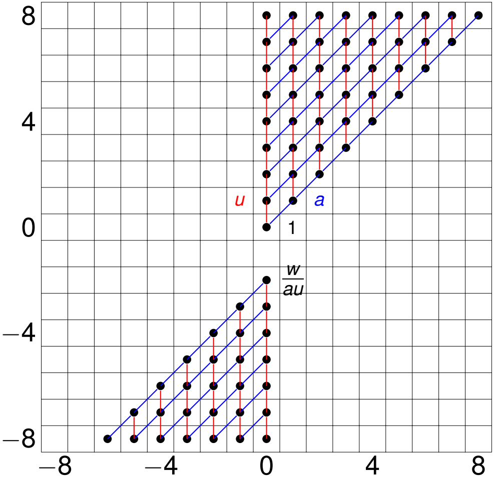

The point (x, y) above represents degree $x - y + y\sigma$.

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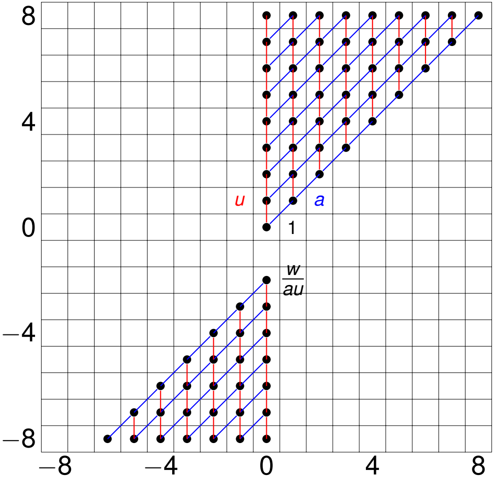

The point (x, y) above represents degree $x - y + y\sigma$.
Red and blue lines indicate multiplication by u and a.

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The Steenrod algebra

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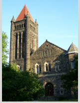
Igor Kriz and Po Hu



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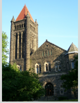

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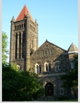

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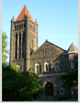

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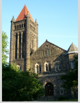
The Steenrod algebra (continued)



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The Steenrod algebra (continued)

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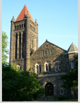

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$$Sq^k u = \begin{cases} u & \text{for } k = 0 \\ a & \text{for } k = 1 \\ 0 & \text{otherwise} \end{cases}$$

Its action on other elements is determined by the Cartan formula to be given below.

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The Steenrod algebra (continued)

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The Steenrod algebra (continued)

Half of the Cartan formula is

$$Sq^{2i}(xy) = \sum_{0 \le r \le i} Sq^{2r}(x) Sq^{2i-2r}(y) + u \sum_{0 \le s < i} Sq^{2s+1}(x) Sq^{2i-2s-1}(y)$$

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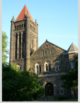
The factor of u in the second sum is needed for degree reasons.



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Note that setting $u = 1$ and $a = 0$ reduces this to the classical Cartan formula.

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The Steenrod algebra (continued)

For the Adem relations, let $0 < m < 2n$.

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The Steenrod algebra (continued)

For the Adem relations, let $0 < m < 2n$. The formula for $Sq^m Sq^n$ depends on the parity of $m + n$.

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$$\begin{aligned} Sq^m Sq^n = & \sum_{j=0}^{[m/2]} \binom{n-1-j}{m-2j} Sq^{m+n-j} Sq^j \\ & + a \sum_{j \text{ odd}} \left\{ \begin{array}{ll} \binom{n-1-j}{m-2j} & \text{for } m \text{ odd} \\ \binom{n-1-j}{m-2j-1} & \text{for } n \text{ odd} \end{array} \right\} Sq^{m+n-j-1} Sq^j . \end{aligned}$$

As before, setting $u = 1$ and $a = 0$ reduces this to the classical Adem relation. The above are due to Jöel Riou, 2012.

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The Steenrod algebra (continued)

For the Adem relations, let $0 < m < 2n$. The formula for $Sq^m Sq^n$ depends on the parity of $m+n$. When it is even we nearly have the classical relation,

$$Sq^m Sq^n = \sum_{j=0}^{[m/2]} \binom{n-1-j}{m-2j} \left\{ \begin{array}{ll} u & \text{for } j \text{ odd} \\ & \text{and} \\ & m, n \text{ even} \\ 1 & \text{otherwise} \end{array} \right\} Sq^{m+n-j} Sq^j .$$

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$$Sq^m Sq^n = \sum_{j=0}^{[m/2]} \binom{n-1-j}{m-2j} Sq^{m+n-j} Sq^j$$
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As before, setting $u = 1$ and $a = 0$ reduces this to the classical Adem relation. The above are due to Jöel Riou, 2012. Voevodsky got it wrong.

Bert Guillou
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The Steenrod algebra (continued)

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The Steenrod algebra (continued)

For example we have the usual

$$Sq^1 Sq^n = \begin{cases} Sq^{n+1} & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd,} \end{cases}$$

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$$Sq^2 Sq^n = \begin{cases} Sq^{n+2} + uSq^{n+1}Sq^1 & \text{for } n \equiv 0 \bmod 4 \\ Sq^{n+1}Sq^1 & \text{for } n \equiv 1 \bmod 4 \\ uSq^{n+1}Sq^1 & \text{for } n \equiv 2 \bmod 4 \\ Sq^{n+2} + Sq^{n+1}Sq^1 & \text{for } n \equiv 3 \bmod 4 \end{cases}$$

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and

$$Sq^3 Sq^n = \begin{cases} Sq^{n+3} + aSq^{n+1}Sq^1 & \text{for } n \equiv 0 \bmod 4 \\ Sq^{n+2}Sq^1 & \text{for } n \equiv 1 \bmod 4 \\ aSq^{n+1}Sq^1 & \text{for } n \equiv 2 \bmod 4 \\ Sq^{n+2}Sq^1 & \text{for } n \equiv 3 \bmod 4. \end{cases}$$

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Dan Isaksen
Doug Ravenel

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Mike Hill
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The subalgebra $\mathcal{A}^{C_2}(1)$

Bert Guillou
Mike Hill
Dan Isaksen
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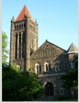
The subalgebra $\mathcal{A}^{C_2}(1)$

It follows that the subalgebra $\mathcal{A}^{C_2}(1)$ generated by Sq^1 and Sq^2 is a free **M**-module with the expected basis as shown here.



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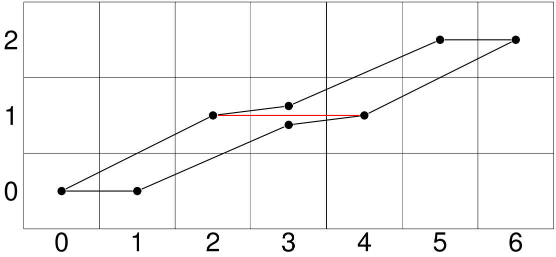

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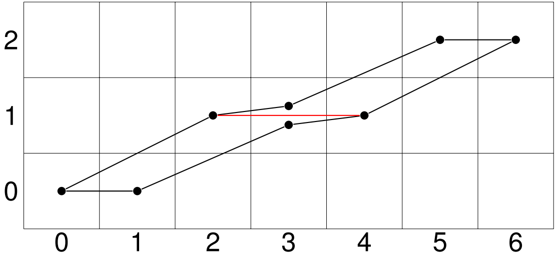

As before an element at (x, y) has degree $x - y + y\sigma$.

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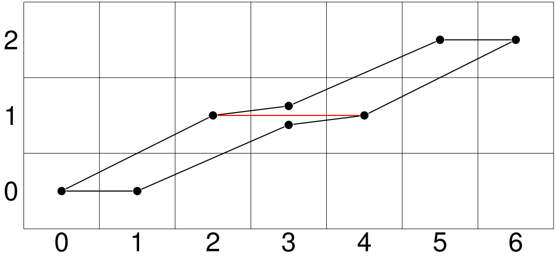

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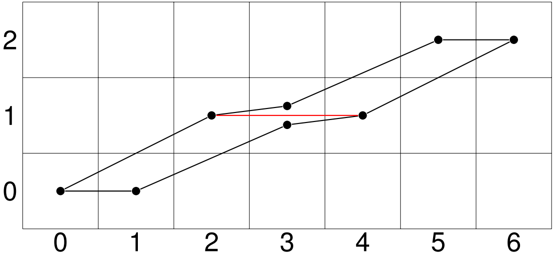

As before an element at (x, y) has degree $x - y + y\sigma$. Black lines of slopes 0 and $1/2$ indicate left multiplication by Sq^1 and Sq^2 respectively, with the Adem relation

$$Sq^2 Sq^2 = u Sq^3 Sq^1 = u Sq^1 Sq^2 Sq^1$$

indicated by the red line.

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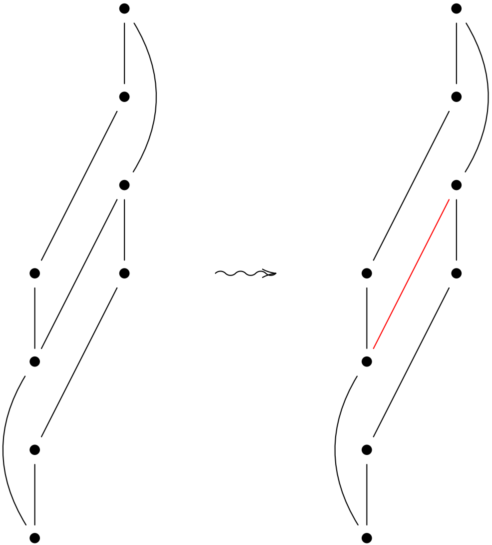
The subalgebra $\mathcal{A}^{C_2}(1)$ (continued)



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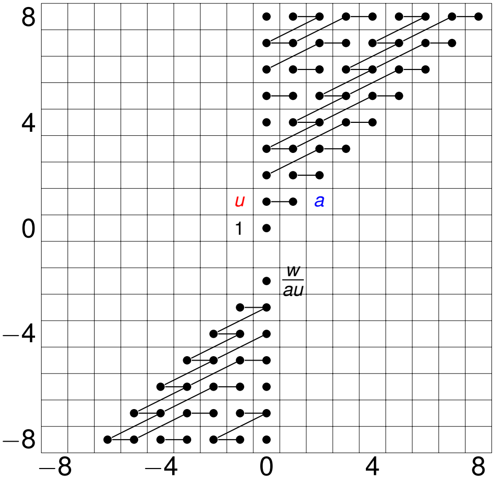
The subalgebra $\mathcal{A}^{C_2}(1)$ (continued)

This chart shows the action of $\mathcal{A}^{C_2}(1)$ on $H^{\star}_{C_2}(S^{-0})$.



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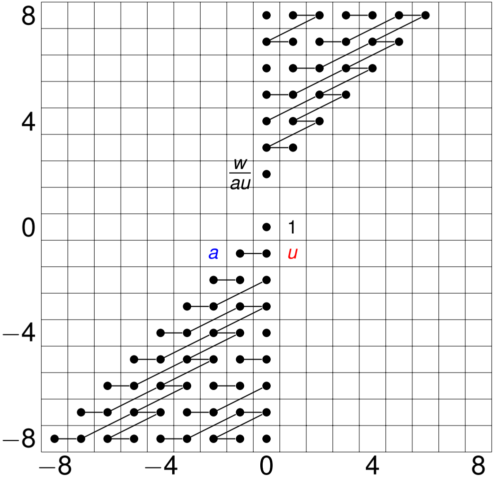
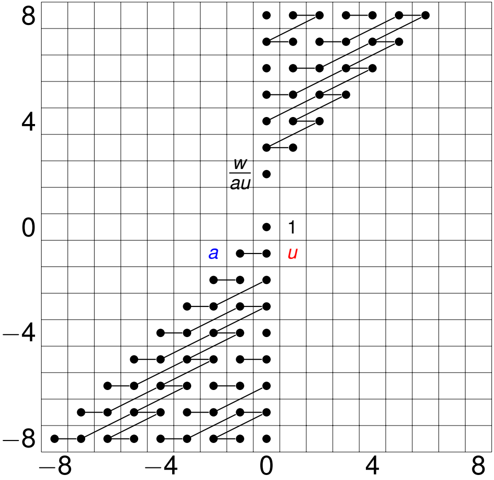
The subalgebra $\mathcal{A}^{C_2}(1)$ (continued)

This chart shows the action of $\mathcal{A}^{C_2}(1)$ on the oppositely graded $H_\star^{C_2}(S^{-0})$.



The subalgebra $\mathcal{A}^{C_2}(1)$ (continued)

This chart shows the action of $\mathcal{A}^{C_2}(1)$ on the oppositely graded $H_\star^{C_2}(S^{-0})$.



In this case Steenrod operations lower the stem degree.

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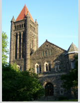
The dual equivariant Steenrod algebra



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Recall that the classical dual Steenrod algebra $\mathcal{A}_*$ is a Hopf algebra over $\mathbb{Z}/2$,

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Recall that the classical dual Steenrod algebra $\mathcal{A}_*$ is a Hopf algebra over $\mathbb{Z}/2$, namely

$$\mathcal{A}_* = \mathbb{Z}/2[\xi_1, \xi_2, \ldots], \quad \text{where } |\xi_i| = 2^i - 1,$$

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$$\Delta(\xi_n) = \sum_{0 \le i \le n} \xi_{n-i}^{2^i} \otimes \xi_i, \quad \text{where } \xi_0 = 1.$$

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$$\mathcal{A}_* = \mathbb{Z}/2[\tau_0, \tau_1, \ldots ; \xi_1, \xi_2, \ldots]/(\xi_i + \tau_{i-1}^2 : i > 0)$$

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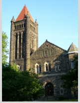

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The dual equivariant Steenrod algebra (continued)

$$\mathcal{A}_* = \mathbb{Z}/2[\tau_0, \tau_1, \ldots ; \xi_1, \xi_2, \ldots]/(\tau_i^2 + \xi_{i+1} : i \geq 0).$$

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Instead of being a Hopf algebra over $\mathbb{Z}/2$, it is a Hopf algebroid over $\mathbf{M}_\star$,

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$$\eta_R(a) = a \quad \text{and} \quad \eta_R(u) = u + a\tau_0 =: \overline{u}.$$

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Setting $a = 0$ and $u = 1$ gives us the description of $\mathcal{A}_*$ above.

Bert Guillou
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The quotient $\mathcal{A}^{C_2}(1)_\star$

$$\mathcal{A}^{C_2}_\star = \mathbf{M}_\star[\tau_0, \tau_1, \ldots; \xi_1, \xi_2, \ldots]/(\tau_i^2 + \overline{u}\xi_{i+1} + a\tau_{i+1} : i \geq 0).$$

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One could try to compute the group

$$\operatorname{Ext}^{*,\star}_{\mathcal{A}_\star^{C_2}}(\mathbf{M}_\star, \mathbf{M}_\star),$$

but this is very complicated.

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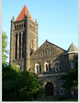

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Classically we have

$$\begin{aligned}\mathcal{A}(1)_* &= \mathcal{A}_*/(\tau_0^4, \tau_1^2, \tau_2, \ldots ; \xi_1^2, \xi_2, \ldots) \\ &= \mathbb{Z}/2[\tau_0, \tau_1, \xi_1]/(\tau_0^2 + \xi_1, \tau_1^2, \xi_1^2).\end{aligned}$$

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The quotient $\mathcal{A}^{C_2}(1)_\star$

$$\mathcal{A}^{C_2}_\star = \mathbf{M}_\star[\tau_0, \tau_1, \dots; \xi_1, \xi_2, \dots]/(\tau_i^2 + \overline{u}\xi_{i+1} + a\tau_{i+1} \colon i \geq 0).$$

One could try to compute the group

$$\mathrm{Ext}^{*,\star}_{\mathcal{A}^{C_2}_\star}(\mathbf{M}_\star, \mathbf{M}_\star),$$

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Equivariantly we have

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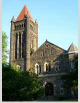

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Inverting the element a

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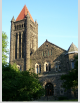
Suppose we formally invert a, which is the algebraic counterpart to passing to geometric fixed points. This will kill NC because each element in it is a-torsion.



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$$0 \to NC \to \mathbf{M} \to a^{-1}\mathbf{M} = a^{-1}\mathbf{M}^{\mathbb{R}} \to \mathbf{M}^{\mathbb{R}}/(a^\infty) \to 0.$$

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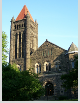
$$\tau_1 = a^{-1}(\tau_0^2 + \overline{u}\xi_1).$$



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It follows that

$$\begin{aligned} a^{-1}\mathcal{A}^{C_2}(1)_\star &= a^{-1}\mathbf{M}_\star[\tau_0, \tau_1, \xi_1]/(\tau_0^2 + \overline{u}\xi_1 + a\tau_1, \tau_1^2, \xi_1^2). \\ &= a^{-1}\mathbf{M}_\star^{\mathbb{R}}[\tau_0, \xi_1]/(\tau_0^4, \xi_1^2). \end{aligned}$$

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Inverting the element a (continued)

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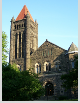

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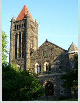
and the right unit is

$$u \mapsto u + a\tau_0 \quad \text{and} \quad u^2 \mapsto u^2 + a^2\tau_0^2 = u^2 + a^2(\overline{u}\xi_1 + a\tau_1).$$



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The resulting Ext group is easily seen to be

$$a^{-1}\mathrm{Ext}^{*,\star}_{\mathcal{A}_\star^{C_2}(1)}(\mathbf{M}_\star, \mathbf{M}_\star) = \mathbb{Z}/2[a^{\pm 1}, u^4][h_1],$$

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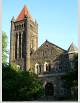

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Killing the element *a*

Again we have

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$$\mathbf{M}_\star/(a) = \mathbf{M}_\star^{\mathbb{R}}/(a) = \mathbf{M}_\star^{\mathbb{C}} = \mathbb{Z}/2[u].$$

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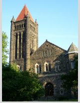

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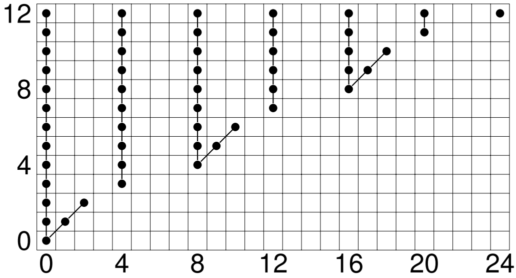

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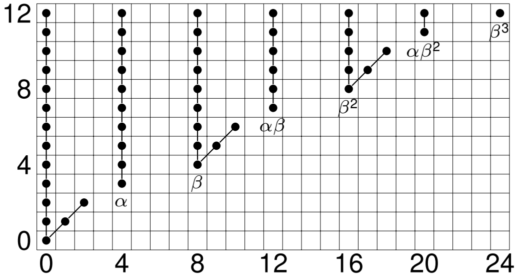
Killing the element *a* (continued)



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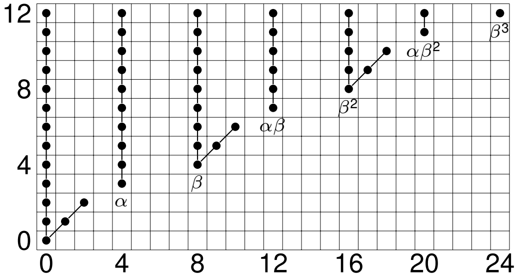
Killing the element a (continued)



As a ring with generators and relations, we have

$$\mathrm{Ext}_{\mathcal{A}(1)_*} = \mathbb{Z}/2[h_0, h_1, \alpha, \beta]/(h_0 h_1, h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

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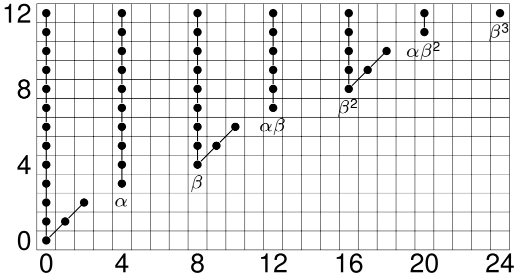
Killing the element *a* (continued)



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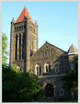
$$\alpha = \langle h_1^2,\ h_1,\ h_0\rangle \in \mathrm{Ext}^{3,7},$$

and $\beta = \langle h_1^2,\ h_1,\ h_1^2,\ h_1\rangle \in \mathrm{Ext}^{4,12}$.



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Killing the element a (continued)

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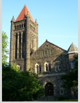
The complex motivic answer is only slightly different.

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$



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Mike Hill
Dan Isaksen
Doug Ravenel

Killing the element *a* (continued)

As a ring with generators and relations, we have

$$\mathrm{Ext}_{\mathcal{A}(1)_*} = \mathbb{Z}/2[h_0, h_1, \alpha, \beta]/(h_0 h_1, h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

where $\qquad h_0 = [\tau_0] \in \mathrm{Ext}^{1,1}, \qquad h_1 = [\xi_1] \in \mathrm{Ext}^{1,2},$

$$\alpha = \langle h_1^2,\ h_1,\ h_0\rangle \in \mathrm{Ext}^{3,7},$$

$$\text{and} \quad \beta = \langle h_1^2,\ h_1,\ h_1^2,\ h_1\rangle \in \mathrm{Ext}^{4,12}.$$

The complex motivic answer is only slightly different.

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

where $\qquad h_0 = [\tau_0] \in \mathrm{Ext}^{1,1}, \qquad h_1 = [\xi_1] \in \mathrm{Ext}^{1,1+\sigma},$

$$\alpha = \langle u h_1^2,\ h_1,\ h_0\rangle \in \mathrm{Ext}^{3,5+2\sigma},$$

$$\text{and} \quad \beta = \langle u h_1^2,\ h_1,\ u h_1^2,\ h_1\rangle \in \mathrm{Ext}^{4,8+4\sigma}.$$

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Killing the element *a* (continued)

As a ring with generators and relations, we have

$$\mathrm{Ext}_{\mathcal{A}(1)_*} = \mathbb{Z}/2[h_0, h_1, \alpha, \beta]/(h_0 h_1, h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

where $\quad h_0 = [\tau_0] \in \mathrm{Ext}^{1,1}, \qquad h_1 = [\xi_1] \in \mathrm{Ext}^{1,2},$

$$\alpha = \langle h_1^2,\ h_1,\ h_0 \rangle \in \mathrm{Ext}^{3,7},$$

$$\text{and} \quad \beta = \langle h_1^2,\ h_1,\ h_1^2,\ h_1 \rangle \in \mathrm{Ext}^{4,12}.$$

The complex motivic answer is only slightly different.

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

where $\quad h_0 = [\tau_0] \in \mathrm{Ext}^{1,1}, \qquad h_1 = [\xi_1] \in \mathrm{Ext}^{1,1+\sigma},$

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$$\text{and} \quad \beta = \langle u h_1^2,\ h_1,\ u h_1^2,\ h_1 \rangle \in \mathrm{Ext}^{4,8+4\sigma}.$$

Note that while $u h_1^3 = 0$, all powers of h_1 itself are nontrivial,

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Killing the element a (continued)

As a ring with generators and relations, we have

$$\mathrm{Ext}_{\mathcal{A}(1)_*} = \mathbb{Z}/2[h_0, h_1, \alpha, \beta]/(h_0 h_1, h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

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and $\quad \beta = \langle h_1^2,\ h_1,\ h_1^2,\ h_1\rangle \in \mathrm{Ext}^{4,12}.$

The complex motivic answer is only slightly different.

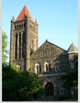
$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

where $\qquad h_0 = [\tau_0] \in \mathrm{Ext}^{1,1}, \qquad h_1 = [\xi_1] \in \mathrm{Ext}^{1,1+\sigma},$

$$\alpha = \langle u h_1^2,\ h_1,\ h_0\rangle \in \mathrm{Ext}^{3,5+2\sigma},$$

and $\quad \beta = \langle u h_1^2,\ h_1,\ u h_1^2,\ h_1\rangle \in \mathrm{Ext}^{4,8+4\sigma}.$

Note that while $u h_1^3 = 0$, all powers of h_1 itself are nontrivial, as was the case when we inverted a.



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Killing the element a (continued)

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

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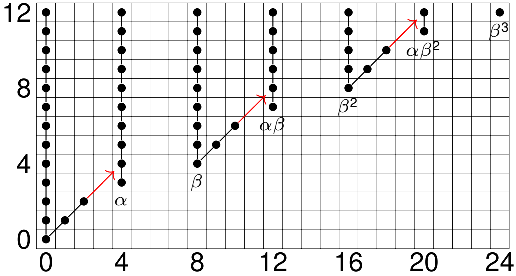
Killing the element a (continued)

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, {\color{red}u}h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

Here is an illustrative chart.



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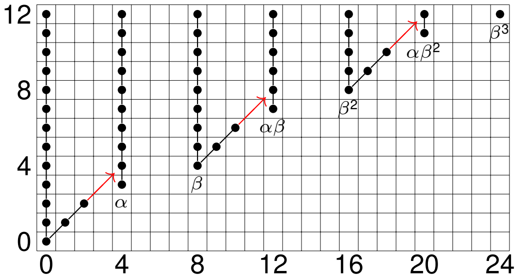
Killing the element a (continued)

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, {\color{red}u} h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

Here is an illustrative chart.



Each red arrow is shorthand for a diagonal tower of elements related by h_1 and killed by ${\color{red}u}$.

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Killing the element a (continued)

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

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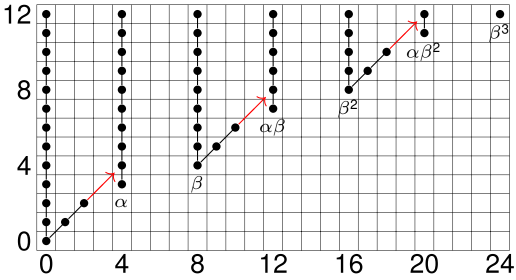

Each red arrow is shorthand for a diagonal tower of elements related by h_1 and killed by u. The elements in black are u-torsion free.

Killing the element a (continued)

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

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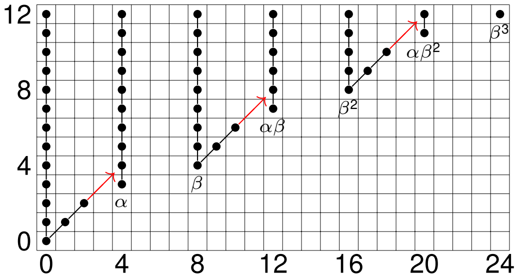

Each red arrow is shorthand for a diagonal tower of elements related by h_1 and killed by u. The elements in black are u-torsion free. As before, an element in $\mathrm{Ext}^{f,x+y\sigma}$ is shown at $(x+y-f, f)$.

Killing the element *a* (continued)

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1 \alpha, \alpha^2 + h_0^2 \beta),$$

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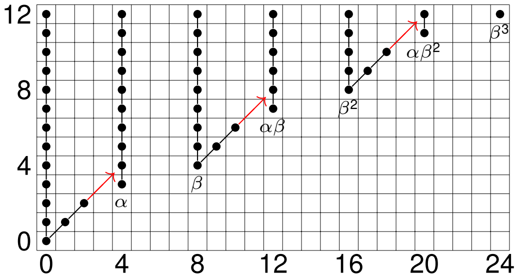

Each red arrow is shorthand for a diagonal tower of elements related by h_1 and killed by u. The elements in black are u-torsion free. As before, an element in $\mathrm{Ext}^{f, x+y\sigma}$ is shown at $(x+y-f, f)$. For example, the elements

$$h_1^4 \in \mathrm{Ext}^{4,4+4\sigma} \qquad \text{and} \qquad h_0\alpha \in \mathrm{Ext}^{4,6+2\sigma}$$

would both appear at $(4, 4)$.

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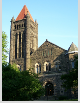
The polar spectral sequence

$$\mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0 h_1, u h_1^3, h_1 \alpha, \alpha^2 + h_0^2 \beta),$$



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Filtering $\mathbf{M}_\star^{\mathbb{R}}$

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Filtering $\mathbf{M}^{\mathbb{R}}_\star$ (which is $\mathbf{M}_\star$ without the negative cone)

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Filtering $\mathbf{M}_\star^{\mathbb{R}}$ (which is $\mathbf{M}_\star$ without the negative cone) by powers of a,

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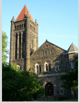
Filtering $\mathbf{M}_\star^{\mathbb{R}}$ (which is $\mathbf{M}_\star$ without the negative cone) by powers of a, the polar filtration,



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Filtering $\mathbf{M}_\star^{\mathbb{R}}$ (which is $\mathbf{M}_\star$ without the negative cone) by powers of a, the polar filtration, yields the polar spectral sequence

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The polar spectral sequence

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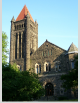
Filtering $\mathbf{M}_\star^{\mathbb{R}}$ (which is $\mathbf{M}_\star$ without the negative cone) by powers of a, the polar filtration, yields the polar spectral sequence

$$\mathbb{Z}/2[a] \otimes \mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} \Longrightarrow \mathrm{Ext}_{\mathcal{A}^{\mathbb{R}}(1)_\star}.$$



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$$\mathbb{Z}/2[a] \otimes \mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} \Longrightarrow \mathrm{Ext}_{\mathcal{A}^{\mathbb{R}}(1)_\star}.$$

It has three differentials:

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The polar spectral sequence

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$$\mathbb{Z}/2[a] \otimes \mathrm{Ext}_{\mathcal{A}^{\mathbb{C}}(1)_\star} \Longrightarrow \mathrm{Ext}_{\mathcal{A}^{\mathbb{R}}(1)_\star}.$$

It has three differentials:

$$d_1(u) = ah_0, \qquad d_2(u^2) = a^2uh_1 \quad \text{and} \quad d_3(u^3h_1^2) = a^3\alpha.$$

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The polar spectral sequence

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This leads to a ring with 9 generators and 22 relations.

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The polar spectral sequence

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This leads to a ring with 9 generators and 22 relations.
The answer for the negative cone is even more complicated.

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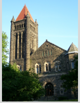

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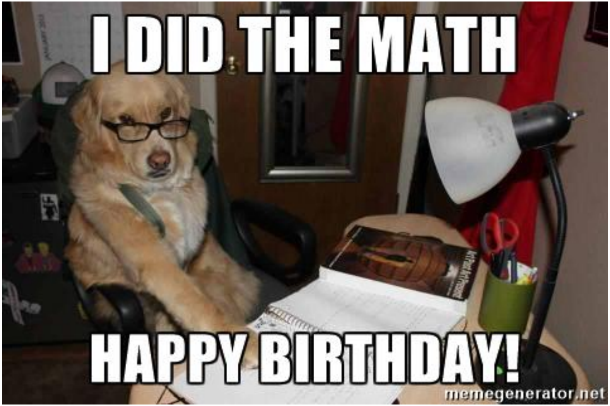

HAPPY BIRTHDAY, PAUL!