The C_2-equivariant analog of the subalgebra of $\mathscr{A}$ generated by Sq^1 and Sq^2

Homotopy theory:
tools and applications
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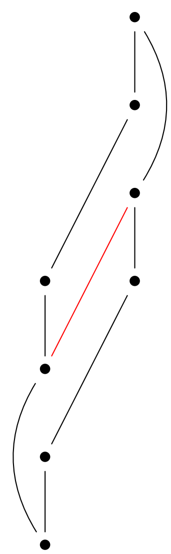
1.1



1.2

1 Equivariant homotopy theory

1.1 Some spheres with group action

Equivariant homotopy theory

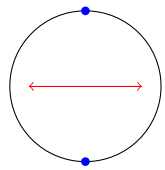
This talk is about equivariant homotopy theory. The group G in question will always by C_2, the group of order 2.

Every finite dimensional orthogonal representation V of G is isomorphic to $m+n\sigma$, where $m,n \geq 0$ are integers, and σ denotes the sign representation.

Given such a representation V,

- $S(V)$ denotes its unit sphere, which is underlain by S^{m+n-1}, and
- S^V denotes its one point compactification, which is underlain by S^{m+n}.

Here is a picture of S^{σ}, the twisted circle, whose fixed point set is S^0:



1.3

1.2 The Hopf map

The surprising property of the equivariant Hopf map

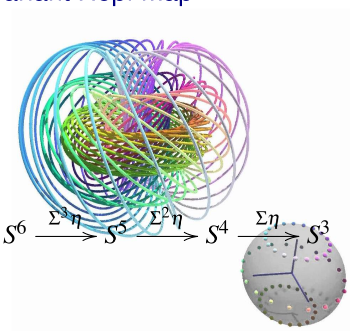
Recall the Hopf map

$$\mathbb{C}^2 \supset S^3 \xrightarrow{\eta} \mathbb{C}P^1 = S^2.$$

The composite map

$$S^7 \xrightarrow{\Sigma^4\eta} S^6 \xrightarrow{\Sigma^3\eta} S^5 \xrightarrow{\Sigma^2\eta} S^4 \xrightarrow{\Sigma\eta} S^3$$

is known to be null homotopic.



Both source and target of η have a C_2-action induced by complex conjugation. The Hopf map η preserves it, so we get an equivariant map

$$S(2+2\sigma) \approx S^{1+2\sigma} \xrightarrow{\overline{\eta}} S^{1+\sigma}$$

and the induced map of fixed point sets is the degree 2 map

$$S(2) \approx S^1 \xrightarrow{[2]} \mathbb{R}P^1 = S^1.$$

1.4

The surprising property of the equivariant Hopf map (continued)

Iterating the equivariant Hopf map $\overline{\eta} : S^{1+2\sigma} \to S^{1+\sigma}$ gives a diagram of equivariant maps and fixed point sets

$$\begin{array}{ccccccccc} S^{1+5\sigma} & \xrightarrow{\Sigma^{3\sigma}\overline{\eta}} & S^{1+4\sigma} & \xrightarrow{\Sigma^{2\sigma}\overline{\eta}} & S^{1+3\sigma} & \xrightarrow{\Sigma^{\sigma}\overline{\eta}} & S^{1+2\sigma} & \xrightarrow{\overline{\eta}} & S^{1+\sigma} \\ S^1 & \xrightarrow{[2]} & S^1 & \xrightarrow{[2]} & S^1 & \xrightarrow{[2]} & S^1 & \xrightarrow{[2]} & S^1 \end{array}$$

where Σ^σ denotes the twisted suspension $S^\sigma \wedge -$. The composite map of fixed point sets is essential since it is the degree 16 map. In fact, any iterate of $\overline{\eta}$ induces a nontrivial map on fixed points. This means that the stable equivariant Hopf map is not equivariantly nilpotent, unlike the classical stable Hopf map.

1.5

2 The mod 2 homology of a point

The equivariant mod 2 homology of a point

In equivariant stable homotopy theory we can speak of homology and homotopy groups graded over $RO(G)$, the real orthogonal representation ring of G. We will now describe $H_\star^{C_2}(S^{-0};\mathbb{Z}/2)$, the equivariant mod 2 homology of the sphere spectrum S^{-0}. The cohomology group $H_{C_2}^\star(S^{-0};\mathbb{Z}/2)$ is isomorphic to it, but oppositely graded.

There are two elements of interest.

- The inclusion map of the fixed point set (the north and south poles) $a : S^0 \to S^\sigma$ defines an element $a \in \pi_{-\sigma}^{C_2} S^{-0}$, and we use the same symbol for its mod 2 Hurewicz image. We call a the polar generator. It is also called an Euler class.
- One can show that
$$H_1^{C_2}(S^\sigma; \mathbb{Z}/2) = H_{1-\sigma}^{C_2}(S^{-0}; \mathbb{Z}/2) = \mathbb{Z}/2,$$
and we denote its generator by u.

1.6

The equivariant mod 2 homology of a point (continued)

Dually we have
$$a \in H_{C_2}^{\sigma} \qquad \text{and} \qquad u \in H_{C_2}^{\sigma-1}.$$

In real motivic homotopy theory one has analogous elements
$$\rho \in H_{\mathbb{R}}^{(1,1)} \qquad \text{and} \qquad \tau \in H_{\mathbb{R}}^{(0,1)},$$
where the motivic bidegree (s,w) (for stem and weight) corresponds to the $RO(C_2)$ degree $s - w + w\sigma$. The element ρ is trivial image in complex motivic homotopy theory.

It is known that, for appropriate versions of the sphere spectrum S^{-0},
$$\mathbf{M}_{\mathbb{C}}^* := H_{\mathbb{C}}^*(S^{-0}; \mathbb{Z}/2) = \mathbb{Z}/2[\tau],$$
$$\mathbf{M}_{\mathbb{R}}^* := H_{\mathbb{R}}^*(S^{-0}; \mathbb{Z}/2) = \mathbb{Z}/2[\rho, \tau]$$
and
$$\mathbf{M}^* := H_{C_2}^*(S^{-0}; \mathbb{Z}/2) \supset \mathbb{Z}/2[a, u].$$

1.7

The equivariant mod 2 homology of a point (continued)

We have
$$\mathbf{M}^* H_{C_2}^*(S^{-0}; \mathbb{Z}/2) \supset \mathbb{Z}/2[a,u] \text{ with } a \in H_{C_2}^{\sigma} \text{ and } u \in H_{C_2}^{\sigma-1},$$
but there is an additional summand called the negative cone NC, namely
$$NC = \Sigma\mathbb{Z}/2[a,u]/(a^\infty, u^\infty) = \bigoplus_{i,j>0} \mathbf{Z}/2\left\{\frac{w}{a^i u^j}\right\}$$

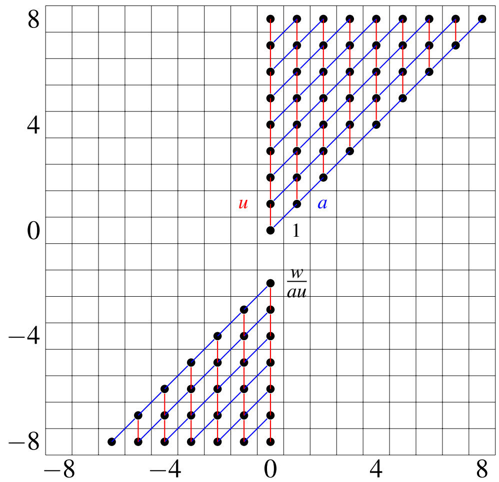
Here w has cohomogical degree 1, so
$$:= \left|\frac{w}{a^i u^j}\right| = 1 - i\sigma - j(\sigma - 1) = (1+j) - (i+j)\sigma.$$

We abbreviate this element by $w_{i,j}$. The fractional notation is meant to indicate that
$$a w_{i+1,j} = w_{i,j} = u w_{i,j+1} \quad \text{and} \quad a^i w_{i,j} = u^j w_{i,j} = 0.$$

Each $w_{i,j}$ is both a-divisible and u-divisible. 1.8

2.1 The equivariant mod 2 cohomology of a point

The equivariant mod 2 cohomology of a point



The point (x,y) above represents degree $x-y+y\sigma$.
Red and blue lines indicate multiplication by u and a. 1.9

3 The Steenrod algebra

The Steenrod algebra

Vladimir Voevodsky

Igor Kriz and Po Hu

The analog of the mod 2 Steenrod algebra $\mathscr{A}$ was described by Voevodsky in the motivic case and by Hu-Kriz in the equivariant case. The two answers are essentially the same. 1.10

The Steenrod algebra (continued)

One has squaring operations Sq^k for $k \geq 0$ whose degrees are

$$|Sq^k| = \begin{cases} i(1+\sigma) & \text{for } k=2i \\ i(1+\sigma)+1 & \text{for } k=2i+1. \end{cases}$$

As in the classical case, $Sq^0 = 1$. The algebra acts on the coefficient ring $\mathbf{M}$, acting trivially on a and w with

$$Sq^k u = \begin{cases} u & \text{for } k=0 \\ a & \text{for } k=1 \\ 0 & \text{otherwise} \end{cases}$$

Its action on other elements is determined by the Cartan formula to be given below. 1.11

The Steenrod algebra (continued)

Half of the Cartan formula is

$$Sq^{2i}(xy) = \sum_{0\le r\le i} Sq^{2r}(x)Sq^{2i-2r}(y) + u\sum_{0\le s<i} Sq^{2s+1}(x)Sq^{2i-2s-1}(y)$$

The factor of u in the second sum is needed for degree reasons.

The operation Sq^1 is a derivation with $Sq^1Sq^1 = 0$ as usual with $Sq^1Sq^{2i} = Sq^{2i+1}$ and $Sq^1u = a$. Applying it to both sides of the above gives the other half of the Cartan formula,

$$Sq^{2i+1}(xy) = \sum_{0\le j\le 2i+1} Sq^{j}(x)Sq^{2i+1-j}(y) + a\sum_{0\le s<i} Sq^{2s+1}(x)Sq^{2i-2s-1}(y).$$

Note that setting $u = 1$ and $a = 0$ reduces this to the classical Cartan formula. 1.12

The Steenrod algebra (continued)

For the Adem relations, let $0 < m < 2n$. The formula for Sq^mSq^n depends on the parity of $m+n$. When it is even we nearly have the classical relation,

$$Sq^mSq^n = \sum_{j=0}^{[m/2]} \binom{n-1-j}{m-2j} \left\{ \begin{array}{ll} u & \text{for } j \text{ odd} \\ & \text{and} \\ & m,n \text{ even} \\ 1 & \text{otherwise} \end{array} \right\} Sq^{m+n-j}Sq^j.$$

When $m+n$ is odd we have a more complicated formula,

$$Sq^mSq^n = \sum_{j=0}^{[m/2]} \binom{n-1-j}{m-2j} Sq^{m+n-j}Sq^j + a\sum_{j \text{ odd}} \left\{ \begin{array}{ll} \binom{n-1-j}{m-2j} & \text{for } m \text{ odd} \\ \binom{n-1-j}{m-2j-1} & \text{for } n \text{ odd} \end{array} \right\} Sq^{m+n-j-1}Sq^j.$$

As before, setting $u = 1$ and $a = 0$ reduces this to the classical Adem relation. The above are due to Jöel Riou, 2012. Voevodsky got it wrong. 1.13

The Steenrod algebra (continued)

For example we have the usual

$$Sq^1Sq^n = \begin{cases} Sq^{n+1} & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd,} \end{cases}$$

$$Sq^2Sq^n = \begin{cases} Sq^{n+2} + uSq^{n+1}Sq^1 & \text{for } n \equiv 0 \bmod 4 \\ Sq^{n+1}Sq^1 & \text{for } n \equiv 1 \bmod 4 \\ uSq^{n+1}Sq^1 & \text{for } n \equiv 2 \bmod 4 \\ Sq^{n+2} + Sq^{n+1}Sq^1 & \text{for } n \equiv 3 \bmod 4 \end{cases}$$

and

$$Sq^3Sq^n = \begin{cases} Sq^{n+3} + aSq^{n+1}Sq^1 & \text{for } n \equiv 0 \bmod 4 \\ Sq^{n+2}Sq^1 & \text{for } n \equiv 1 \bmod 4 \\ aSq^{n+1}Sq^1 & \text{for } n \equiv 2 \bmod 4 \\ Sq^{n+2}Sq^1 & \text{for } n \equiv 3 \bmod 4. \end{cases}$$

1.14

3.1 The subalgebra $\mathscr{A}^{C_2}(1)$

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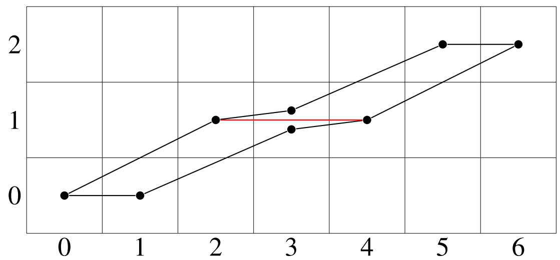
The subalgebra $\mathscr{A}^{C_2}(1)$

It follows that the subalgebra $\mathscr{A}^{C_2}(1)$ generated by Sq^1 and Sq^2 is a free **M**-module with the expected basis as shown here.

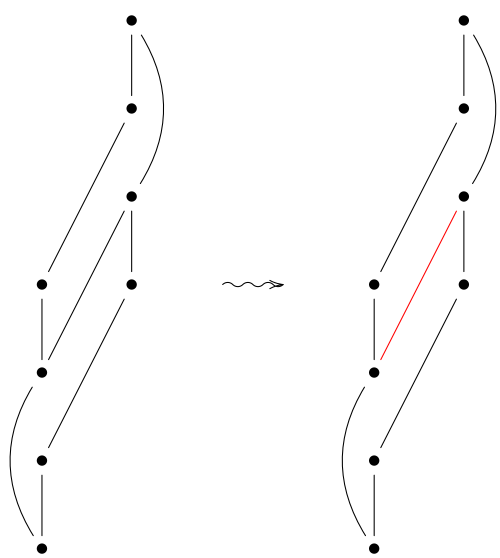


As before an element at (x,y) has degree $x-y+y\sigma$. Black lines of slopes 0 and $1/2$ indicate left multiplication by Sq^1 and Sq^2 respectively, with the Adem relation

$$Sq^2Sq^2 = uSq^3Sq^1 = uSq^1Sq^2Sq^1$$

indicated by the red line. 1.15

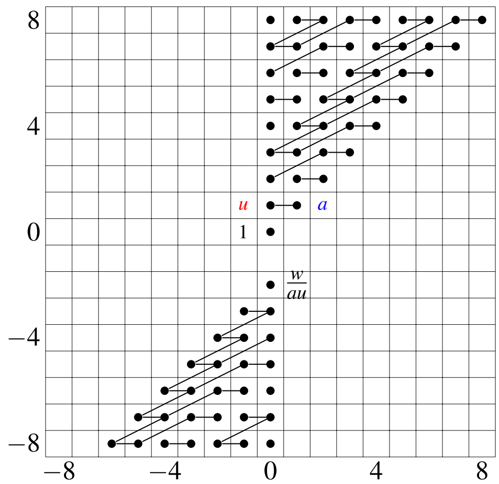
The subalgebra $\mathscr{A}^{C_2}(1)$ (continued)



1.16

The subalgebra $\mathscr{A}^{C_2}(1)$ (continued)

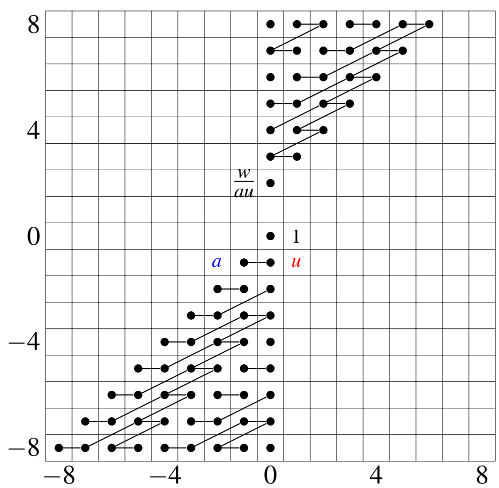
This chart shows the action of $\mathscr{A}^{C_2}(1)$ on $H^{\star}_{C_2}(S^{-0})$.



1.17

The subalgebra $\mathscr{A}^{C_2}(1)$ (continued)

This chart shows the action of $\mathscr{A}^{C_2}(1)$ on the oppositely graded $H_\star^{C_2}(S^{-0})$.



In this case Steenrod operations lower the stem degree. 1.18

4 The dual equivariant Steenrod algebra

The dual equivariant Steenrod algebra

Recall that the classical dual Steenrod algebra $\mathscr{A}_*$ is a Hopf algebra over $\mathbb{Z}/2$, namely

$$\mathscr{A}_* = \mathbb{Z}/2[\xi_1, \xi_2, \ldots], \quad \text{where } |\xi_i| = 2^i - 1,$$

with coproduct

$$\Delta(\xi_n) = \sum_{0 \le i \le n} \xi_{n-i}^{2^i} \otimes \xi_i, \quad \text{where } \xi_0 = 1.$$

We will rewrite this as

$$\mathscr{A}_* = \mathbb{Z}/2[\tau_0, \tau_1, \ldots; \xi_1, \xi_2, \ldots]/(\xi_i + \tau_{i-1}^2 \colon i > 0)$$

where $|\xi_i| = 2(2^i - 1)$ and $|\tau_i| = 1 + |\xi_i|$, with a similar coproduct. Thus we are renaming the original ξ_i as τ_{i-1}, and using the symbol ξ_i to denote the square of the original ξ_i.

The dual Steenrod algebra at an odd prime has a similar description with $\tau_i^2 = 0$. 1.19

The dual equivariant Steenrod algebra (continued)

$$\mathscr{A}_* = \mathbb{Z}/2[\tau_0, \tau_1, \dots; \xi_1, \xi_2, \dots]/(\tau_i^2 + \xi_{i+1} : i \geq 0).$$

The equivariant dual Steenrod algebra $\mathscr{A}_\star^{C_2}$ has a similar description.

Instead of being a Hopf algebra over $\mathbb{Z}/2$, it is a Hopf algebroid over $\mathbf{M}_\star$, the oppositely graded dual of the ring $\mathbf{M}^\star$ described earlier. There is a right unit map η_R with

$$\eta_R(a) = a \quad \text{and} \quad \eta_R(u) = u + a\tau_0 =: \overline{u}.$$

The degrees of the generators are

$$|\xi_i| = (1+\sigma)(2^i - 1) \quad \text{and} \quad |\tau_i| = 1 + |\xi_i|.$$

The multiplicative relations are

$$\tau_i^2 = a\tau_{i+1} + \overline{u}\xi_{i+1} \quad \text{for} \quad i \geq 0.$$

Setting $a = 0$ and $u = 1$ gives us the description of $\mathscr{A}_*$ above. 1.20

5 $\mathscr{A}^{C_2}(1)_\star$

The quotient $\mathscr{A}^{C_2}(1)_\star$

$$\mathscr{A}_\star^{C_2} = \mathbf{M}_\star[\tau_0, \tau_1, \dots; \xi_1, \xi_2, \dots]/(\tau_i^2 + \overline{u}\xi_{i+1} + a\tau_{i+1} : i \geq 0).$$

One could try to compute the group

$$\mathrm{Ext}^{*,\star}_{\mathscr{A}_\star^{C_2}}(\mathbf{M}_\star, \mathbf{M}_\star),$$

but this is very complicated. One can start by replacing $\mathscr{A}^{C_2}$ by the subalgebra $\mathscr{A}^{C_2}(1)$ generated by Sq^1 and Sq^2.

Classically we have

$$\begin{aligned}\mathscr{A}(1)_* &= \mathscr{A}_*/(\tau_0^4, \tau_1^2, \tau_2, \dots; \xi_1^2, \xi_2, \dots)\\ &= \mathbb{Z}/2[\tau_0, \tau_1, \xi_1]/(\tau_0^2 + \xi_1, \tau_1^2, \xi_1^2).\end{aligned}$$

Equivariantly we have

$$\mathscr{A}^{C_2}(1)_\star = \mathbf{M}_\star[\tau_0, \tau_1, \xi_1]/(\tau_0^2 + \overline{u}\xi_1 + a\tau_1, \tau_1^2, \xi_1^2).$$

1.21

5.1 Inverting a

Inverting the element a

$$\mathscr{A}^{C_2}(1)_\star = \mathbf{M}_\star[\tau_0, \tau_1, \xi_1]/(\tau_0^2 + \overline{u}\xi_1 + a\tau_1, \tau_1^2, \xi_1^2).$$

Recall that $\mathbf{M}_\star = \mathbf{M}_\star^{\mathbb{R}} \oplus NC$ and $\mathbf{M}_\star^{\mathbb{R}} = \mathbb{Z}/2[a, u]$. Thus $\mathbf{M}_\star^{\mathbb{R}}$ is $\mathbf{M}_\star$ without the negative cone.

Suppose we formally invert a, which is the algebraic counterpart to passing to geometric fixed points. This will kill NC because each element in it is a-torsion. Thus we get a 4-term exact sequence

$$0 \to NC \to \mathbf{M} \to a^{-1}\mathbf{M} = a^{-1}\mathbf{M}^{\mathbb{R}} \to \mathbf{M}^{\mathbb{R}}/(a^{\infty}) \to 0.$$

The multiplicative relation $\tau_0^2 + \overline{u}\xi_1 + a\tau_1 = 0$ can be rewritten as

$$\tau_1 = a^{-1}(\tau_0^2 + \overline{u}\xi_1).$$

It follows that

$$\begin{aligned} a^{-1}\mathscr{A}^{C_2}(1)_\star &= a^{-1}\mathbf{M}_\star[\tau_0, \tau_1, \xi_1]/(\tau_0^2 + \overline{u}\xi_1 + a\tau_1, \tau_1^2, \xi_1^2). \\ &= a^{-1}\mathbf{M}_\star^{\mathbb{R}}[\tau_0, \xi_1]/(\tau_0^4, \xi_1^2). \end{aligned}$$

1.22

Inverting the element a (continued)

We have

$$\begin{aligned} \mathscr{A}^{C_2}(1)_\star &= \mathbf{M}_\star[\tau_0, \tau_1, \xi_1]/(\tau_0^2 + \overline{u}\xi_1 + a\tau_1, \tau_1^2, \xi_1^2) \\ a^{-1}\mathscr{A}^{C_2}(1)_\star &= a^{-1}\mathbf{M}_\star^{\mathbb{R}}[\tau_0, \xi_1]/(\tau_0^4, \xi_1^2) \\ &= \mathbb{Z}/2[a^{\pm 1}, u][\tau_0, \xi_1]/(\tau_0^4, \xi_1^2), \end{aligned}$$

and the right unit is

$$u \mapsto u + a\tau_0 \quad \text{and} \quad u^2 \mapsto u^2 + a^2\tau_0^2 = u^2 + a^2(\overline{u}\xi_1 + a\tau_1).$$

The resulting Ext group is easily seen to be

$$a^{-1}\mathrm{Ext}^{*,\star}_{\mathscr{A}_\star^{C_2}(1)}(\mathbf{M}_\star, \mathbf{M}_\star) = \mathbb{Z}/2[a^{\pm 1}, u^4][h_1],$$

where $h_1 = [\xi_1] \in \mathrm{Ext}^{1,1+\sigma}$. This element is related to the equivariant Hopf map η mentioned at the start of the talk. The nonnilpotence of h_1 in this Ext group is related to that of η in the equivariant stable homotopy category. 1.23

5.2 Killing a

Killing the element a

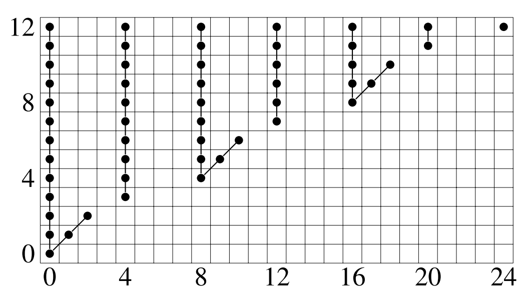
Again we have

$$\mathscr{A}^{C_2}(1)_\star = \mathbf{M}_\star[\tau_0, \tau_1, \xi_1]/(\tau_0^2 + \overline{u}\xi_1 + a\tau_1, \tau_1^2, \xi_1^2).$$

Now we consider the effect of formally killing $a \in \mathbf{M}_\star$. Like inverting a, this will kill the negative cone since each element in it is divisible by a. Thus we have

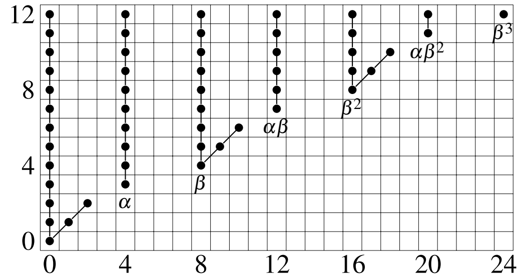
$$\mathbf{M}_\star/(a) = \mathbf{M}_\star^{\mathbb{R}}/(a) = \mathbf{M}_\star^{\mathbb{C}} = \mathbb{Z}/2[u].$$

Recall that if we also set $u \mapsto 1$, we get the classical quotient $\mathscr{A}(1)_*$. Its Ext group is well known and is shown below.



1.24

Killing the element a (continued)



As a ring with generators and relations, we have

$$\mathrm{Ext}_{\mathscr{A}(1)_*} = \mathbb{Z}/2[h_0, h_1, \alpha, \beta]/(h_0h_1, h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

where

$$h_0 = [\tau_0] \in \mathrm{Ext}^{1,1}, \qquad h_1 = [\xi_1] \in \mathrm{Ext}^{1,2},$$
$$\alpha = \langle h_1^2, h_1, h_0 \rangle \in \mathrm{Ext}^{3,7},$$
$$\text{and } \beta = \langle h_1^2, h_1, h_1^2, h_1 \rangle \in \mathrm{Ext}^{4,12}.$$

1.25

Killing the element a (continued)

As a ring with generators and relations, we have

$$\mathrm{Ext}_{\mathscr{A}(1)_*} = \mathbb{Z}/2[h_0, h_1, \alpha, \beta]/(h_0h_1, h_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

where

$$h_0 = [\tau_0] \in \mathrm{Ext}^{1,1}, \qquad h_1 = [\xi_1] \in \mathrm{Ext}^{1,2},$$
$$\alpha = \langle h_1^2, h_1, h_0 \rangle \in \mathrm{Ext}^{3,7},$$
$$\text{and} \quad \beta = \langle h_1^2, h_1, h_1^2, h_1 \rangle \in \mathrm{Ext}^{4,12}.$$

The complex motivic answer is only slightly different.

$$\mathrm{Ext}_{\mathscr{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0h_1, uh_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

where

$$h_0 = [\tau_0] \in \mathrm{Ext}^{1,1}, \qquad h_1 = [\xi_1] \in \mathrm{Ext}^{1,1+\sigma},$$
$$\alpha = \langle uh_1^2, h_1, h_0 \rangle \in \mathrm{Ext}^{3,5+2\sigma},$$
$$\text{and} \quad \beta = \langle uh_1^2, h_1, uh_1^2, h_1 \rangle \in \mathrm{Ext}^{4,8+4\sigma}.$$

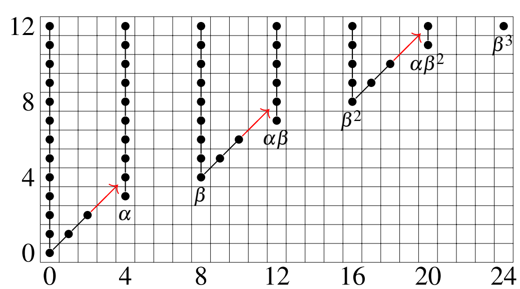
Note that while $uh_1^3 = 0$, all powers of h_1 itself are nontrivial, as was the case when we inverted a.

1.26

Killing the element a (continued)

$$\mathrm{Ext}_{\mathscr{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0, h_1, \alpha, \beta]/(h_0h_1, uh_1^3, h_1\alpha, \alpha^2 + h_0^2\beta),$$

Here is an illustrative chart.



Each red arrow is shorthand for a diagonal tower of elements related by h_1 and killed by u. The elements in black are u-torsion free. As before, an element in $\mathrm{Ext}^{f,x+y\sigma}$ is shown at $(x+y-f,f)$. For example, the elements

$$h_1^4 \in \mathrm{Ext}^{4,4+4\sigma} \qquad \text{and} \qquad h_0\alpha \in \mathrm{Ext}^{4,6+2\sigma}$$

would both appear at $(4,4)$. 1.27

5.3 The polar spectral sequence

The polar spectral sequence

$$\mathrm{Ext}_{\mathscr{A}^{\mathbb{C}}(1)_\star} = \mathbf{M}^{\mathbb{C}}[h_0,h_1,\alpha,\beta]/(h_0h_1,uh_1^3,h_1\alpha,\alpha^2+h_0^2\beta),$$

Filtering $\mathbf{M}_\star^{\mathbb{R}}$ (which is $\mathbf{M}_\star$ without the negative cone) by powers of a, the polar filtration, yields the polar spectral sequence

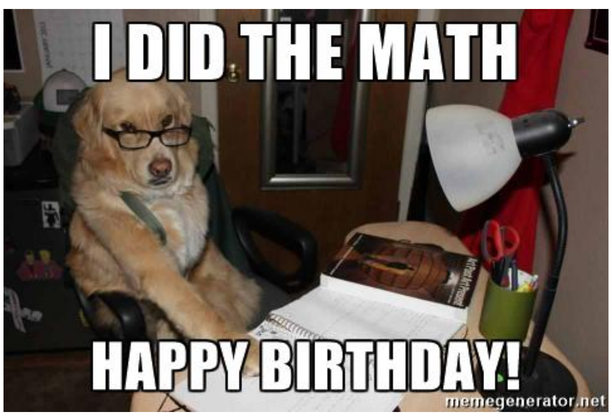
$$\mathbb{Z}/2[a]\otimes \mathrm{Ext}_{\mathscr{A}^{\mathbb{C}}(1)_\star} \Longrightarrow \mathrm{Ext}_{\mathscr{A}^{\mathbb{R}}(1)_\star}.$$

It has three differentials:

$$d_1(u) = ah_0, \qquad d_2(u^2) = a^2uh_1 \quad \text{and} \quad d_3(u^3h_1^2) = a^3\alpha.$$

This leads to a ring with 9 generators and 22 relations.

The answer for the negative cone is even more complicated. 1.28



HAPPY BIRTHDAY,

PAUL! 1.29