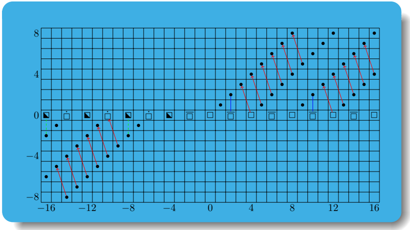
The slice filtration revisited

2016 CMS Winter Meeting
Session on Equivariant geometry and topology
Niagara Falls, ON

December 3, 2016



Mike Hill
UCLA
Mike Hopkins
Harvard University
Doug Ravenel
University of Rochester

The slice spectral sequence and the slice filtration

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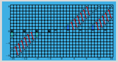

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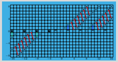

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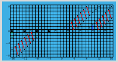

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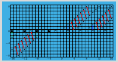

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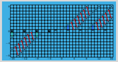

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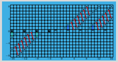

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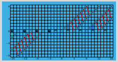

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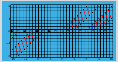
It makes use of equivariant constructions



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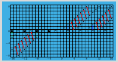

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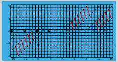
It makes use of equivariant constructions such as isotropy separation and geometric fixed points, which we will describe in due course.



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Localizing subcategories

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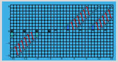

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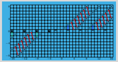

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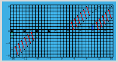

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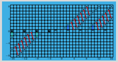

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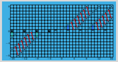

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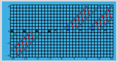

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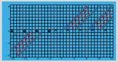

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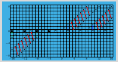

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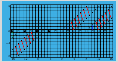

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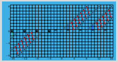
Example

Let $\mathcal{M}$ be either $\mathcal{T}$ (pointed spaces)



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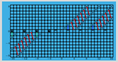
Let $\mathcal{M}$ be either $\mathcal{T}$ (pointed spaces) or $\mathcal{S}p$ (spectra)



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Example

Let $\mathcal{M}$ be either $\mathcal{T}$ (pointed spaces) or $\mathcal{S}p$ (spectra) and let $\tau_n \subset \mathcal{M}$ be the subcategory of $(n-1)$-connected spaces or spectra.

Localizing subcategories (continued)

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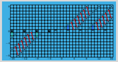

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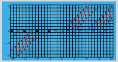
The complement $\tau^{\perp}$ of τ is the subcategory of objects Y such that the space $\mathcal{M}(X, Y)$ is contractible for all X in τ.



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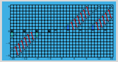

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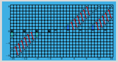

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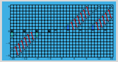
For $\tau_n \subseteq \mathcal{T}$ or $\tau_n \subseteq \mathcal{S}p$ as above, $\tau_n^{\perp}$ is the subcategory n-coconnected spaces or spectra, meaning ones with no homotopy in dimensions n and above.



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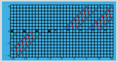

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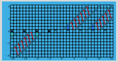
Let $T = \{T_\alpha\}$ be a set of objects in $\mathcal{M}$. The localizing subcategory generated by T is smallest subcategory of $\mathcal{M}$ containing the objects of T



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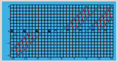

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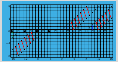
The localizing subcategory τ_n above of $(n-1)$-connected spaces or spectra



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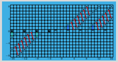

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Let $T = \{T_\alpha\}$ be a set of objects in $\mathcal{M}$. The localizing subcategory generated by T is smallest subcategory of $\mathcal{M}$ containing the objects of T and closed under weak equivalence, cofibers, extensions and arbitrary wedges.

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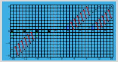
The localizing subcategory τ_n above of $(n-1)$-connected spaces or spectra is the one generated by the object S^n. In the stable case we can define a spectrum S^n for $n < 0$.



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Localizing subcategories (continued)

A localizing subcategory τ of pointed topological model category $\mathcal{M}$ is a full subcategory with three properties:

(i) Any object weakly equivalent to one in τ is also in τ.

(ii) If $W \to X \to Y$ is a cofiber sequence with W in τ, then X is in τ iff Y is in τ. We are not requiring W to be in τ if X and Y are.

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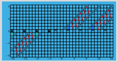

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Localizing subcategories (continued)

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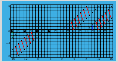

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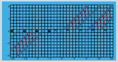

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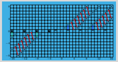

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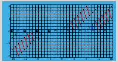
Emmanuel Dror Farjoun with grandkids



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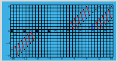
For τ_{n+1} as above (n-connected objects),



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For τ_{n+1} as above (n-connected objects), we denote these two functors by P^n and P_{n+1}.

Localizing subcategories (continued)

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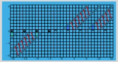

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For τ_{n+1} as above (n-connected objects), we denote these two functors by P^n and P_{n+1}. P^nX is the nth Postnikov section of X, the space or spectrum obtained by killing all homotopy groups above dimension n.

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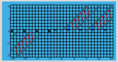

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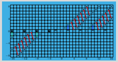

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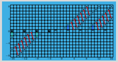

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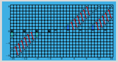

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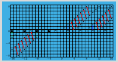

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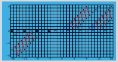

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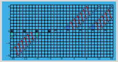

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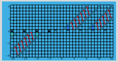
The original slice filtration

We now work in the category of G-spectra $\mathcal{S}p^G$ with a suitable model category structure.



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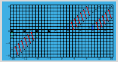

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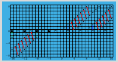

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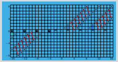

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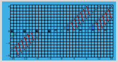

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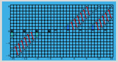

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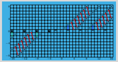
- We call $\widehat{S}(m, H)$ a slice sphere.



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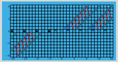

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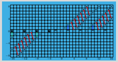

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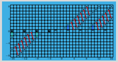
The original slice filtration (continued)

$$\widehat{S}(m,H) := G_+ \underset{H}{\wedge} S^{m\rho_H} \quad \text{for } m \in \mathbf{Z} \text{ and } H \subseteq G.$$



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For each integer n, let

$$T_n^G = \left\{ \widehat{S}(m,H) \colon \ m|H| \geq n \right\} \cup \left\{ \Sigma^{-1}\widehat{S}(m,H) \colon \ m|H| - 1 \geq n \right\}.$$

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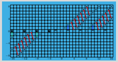

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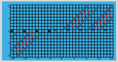

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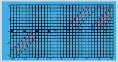

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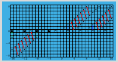

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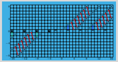

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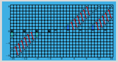

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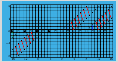

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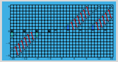

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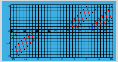

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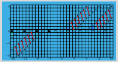

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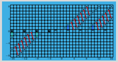

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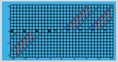

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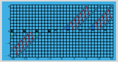

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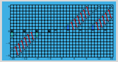

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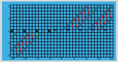

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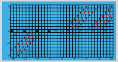

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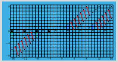

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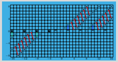

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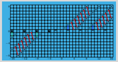

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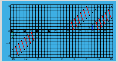
It requires the use of geometric fixed points.



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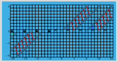
Isotropy separation and geometric fixed points

For a G-spectrum X and a subgroup $H \subseteq G$,



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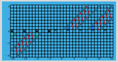

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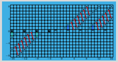

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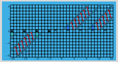

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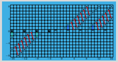

Mike Hill
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Isotropy separation and geometric fixed points

For a G-spectrum X and a subgroup $H \subseteq G$, one can define an ordinary (meaning not equivariant) spectrum X^H, the H-fixed point spectrum of X. These are not fun to work with for two reasons.

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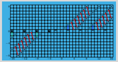

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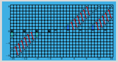

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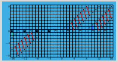

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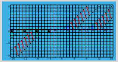

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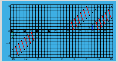

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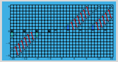

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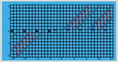

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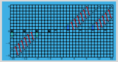

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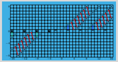

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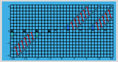

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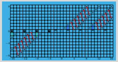

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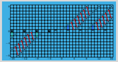

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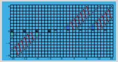

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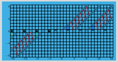

Mike Hill
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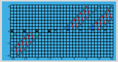
Isotropy separation and geometric fixed points (continued)

How do we construct ΦG?



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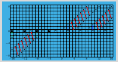

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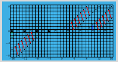

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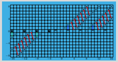

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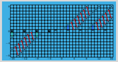

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Example

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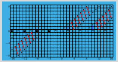

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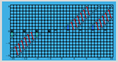

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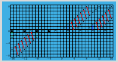

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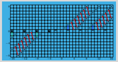
When $\mathcal{F}$ contains all subgroups of G,



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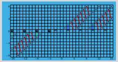
When $\mathcal{F}$ contains just the trivial subgroup, then $E\mathcal{F}$ is the usual contractible free G-space EG, the infinite join of G.

When $\mathcal{F}$ contains all subgroups of G, then $E\mathcal{F}$ is a point.



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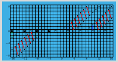

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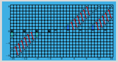

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For any nonempty family $\mathcal{F}$ of subgroups of G closed under inclusion and conjugation, there is a G-space $E\mathcal{F}$ with

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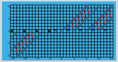

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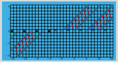

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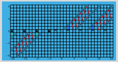

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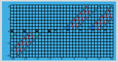

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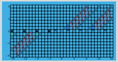

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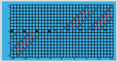

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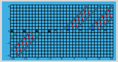

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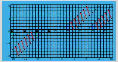

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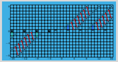

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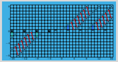

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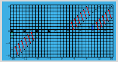

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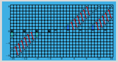

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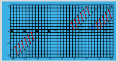

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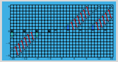

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The new definition of the slice filtration

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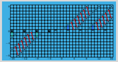

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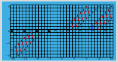

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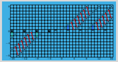

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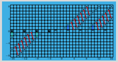

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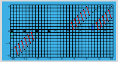

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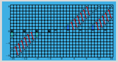

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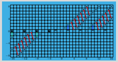

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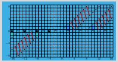
4. *For each integer n there is an equivalence of categories $\tau_n^G \to \tau_{n+|G|}^G$ given by $X \mapsto X \wedge S^{\rho_G}$ with inverse given by $X \mapsto X \wedge S^{-\rho_G}$.*



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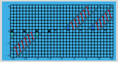
The localizing subcategories $\mathcal{S}p^G_{\geq n}$ (defined in terms of slice spheres) and τ_n^G are equal,



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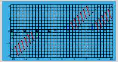

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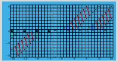

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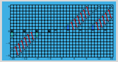

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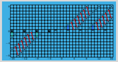

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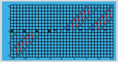
The localizing subcategories $\mathcal{S}p^G_{\geq n}$ (defined in terms of slice spheres) and τ_n^G are equal, so they lead to the same slice towers.

It is easy to show that the slice sphere $\widehat{S}(m, H)$ is in $\tau^G_{m|H|}$, which implies that $\mathcal{S}p^G_{\geq n} \subseteq \tau_n^G$. The converse is more delicate, and requires an argument by induction on the order of G



Mike Hill
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The new definition of the slice filtration (continued)

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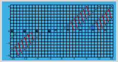
$$E\mathcal{P}_+ \wedge X \to X \to \widetilde{E}\mathcal{P} \wedge X.$$



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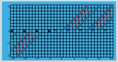

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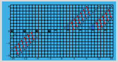

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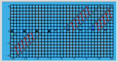

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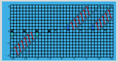

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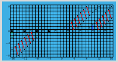

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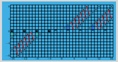

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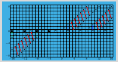

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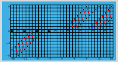

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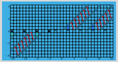

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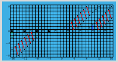

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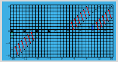

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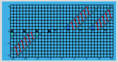

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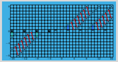

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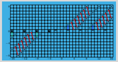

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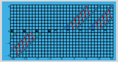

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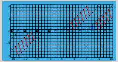

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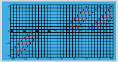

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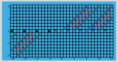
Similar statements hold for $G_+ \underset{K}{\wedge} S^V$ and $G_+ \underset{K}{\wedge} S^{-V}$ for a representation V of a subgroup $K \subseteq G$.



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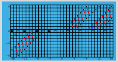

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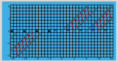

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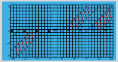

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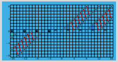

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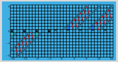

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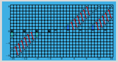
Then $S^V \wedge (-) : \tau_n^G \to \tau_{n+d}^G$ is an equivalence of categories whose inverse is $S^{-V} \wedge (-)$,



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What do the subcategories τ_n^G and $\tau_{=n}^G$ look like? (continued)

Proposition

Let V be a representation of G of degree d.

1. $\dim V^H \geq \lfloor d/|H| \rfloor$ *for all subgroups* $H \subseteq G$ *iff* S^V *is in* τ_d^G.
2. $\dim V^H \leq \lfloor d/|H| \rfloor$ *for all subgroups* $H \subseteq G$ *iff* S^{-V} *is in* τ_{-d}^G.

Recall the ceiling function $\lceil x \rceil$, the smallest integer $\geq x$.

Corollary 1

Smashing with representation spheres. *Suppose there is a degree d representation V of G and an integer n such that*

$$\left\lceil \frac{n}{|H|} \right\rceil + \dim V^H = \left\lceil \frac{n+d}{|H|} \right\rceil \qquad \textit{for all } H \subseteq G.$$

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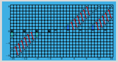

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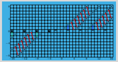

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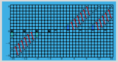
Example

Let G be any finite group,



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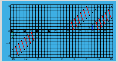
Example

Let G be any finite group, and let V be the trivial representation of degree 1.



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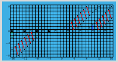
Let G be any finite group, and let V be the trivial representation of degree 1. Then the conditions above are met only when n is divisible by $|G|$.



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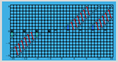
Let G be any finite group, and let V be the trivial representation of degree 1. Then the conditions above are met only when n is divisible by $|G|$. It follows that ordinary suspension $\Sigma : \tau_{m|G|+i}^G \to \tau_{m|G|+i+1}^G$ is an equivalence of categories for $i = 0$,



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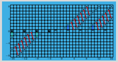

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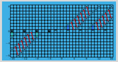

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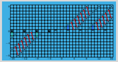
Corollary 2

Smashing layers with representation spheres. *Suppose that for a given V,*



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Suppose there is a degree d representation V of G and an integer n such that

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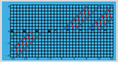
Then $S^V \wedge (-) : \tau_n^G \to \tau_{n+d}^G$ is an equivalence of categories whose inverse is $S^{-V} \wedge (-)$, and conversely.

Corollary 2

Smashing layers with representation spheres. *Suppose that for a given V, the conditions of Corollary 1 are met for both $n = m$ and $n = m + 1$.*



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Suppose there is a degree d representation V of G and an integer n such that

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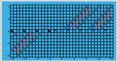
Smashing layers with representation spheres. *Suppose that for a given V, the conditions of Corollary 1 are met for both $n = m$ and $n = m + 1$. Then*

$$S^V \wedge (-) : \tau_{=m}^G \to \tau_{=m+d}^G$$

is an equivalence of layer categories whose inverse is $S^{-V} \wedge (-)$.



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Suppose there is a representation V of degree d and an integer n such that

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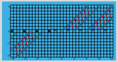

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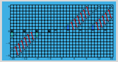
Example

An equivalence among the subcategories τ_n^G and $\tau_{=n}^G$.



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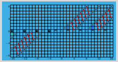
Example

An equivalence among the subcategories τ_n^G and $\tau_{=n}^G$. *Let G be any finite group*



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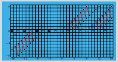
An equivalence among the subcategories τ_n^G and $\tau_{=n}^G$. *Let G be any finite group and $V = \rho_G$.*



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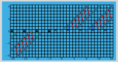

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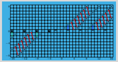
An equivalence among the subcategories τ_n^G and $\tau_{=n}^G$. *Let G be any finite group and $V = \rho_G$. Then the conditions of both corollaries hold for any n. Hence $S^{\rho_G} \wedge (-)$ induces an equivalence between τ_n^G and $\tau_{n+|G|}^G$,*



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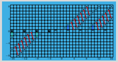

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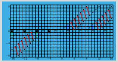

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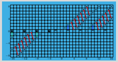

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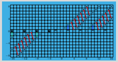

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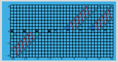
Another equivalence among the subcategories τ_n^G. *Let G be any finite group and $V = \overline{\rho}_G$, the reduced regular representation.*



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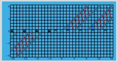

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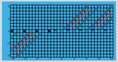

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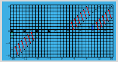
What do the subcategories τ_n^G and $\tau_{=n}^G$ look like? (continued)

Example

More equivalences among the subcategories τ_n^G.



Mike Hill
Mike Hopkins
Doug Ravenel

What do the subcategories τ_n^G and $\tau_{=n}^G$ look like? (continued)

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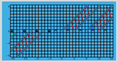
More equivalences among the subcategories τ_n^G.

- *Let $G = C_2$.*



Mike Hill
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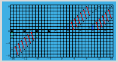
- *Let $G = C_2$. Then the two previous examples show that each τ_n^G is equivalent to τ_0^G,*



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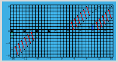
- *Let $G = C_2$. Then the two previous examples show that each τ_n^G is equivalent to τ_0^G, but the layers $\tau_{=0}^G$ and $\tau_{=1}^G$ are distinct.*



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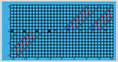

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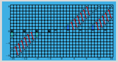

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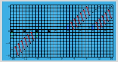

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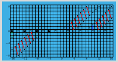

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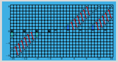

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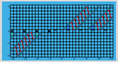

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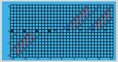

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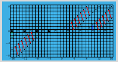

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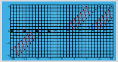

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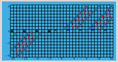

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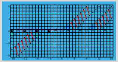

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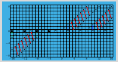

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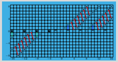

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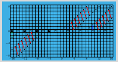
Let $G = C_p$ for p an odd prime, and let $V = \lambda$, a 2-dimensional rotation of order p. Then



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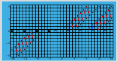
- *The conditions of the Corollary 1 hold provided n is not congruent to 0 or -1 mod p.*



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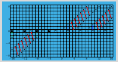
$$\tau_1^G \to \tau_3^G \to \cdots \to \tau_p^G \quad \text{and} \quad \tau_2^G \to \tau_4^G \to \cdots \to \tau_{p-1}^G.$$



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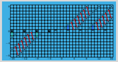

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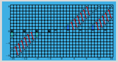
  Hence each τ_n^G is equivalent to τ_1^G or τ_2^G.
- *For n not congruent to 0, -1 or -2 mod p,*



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  Hence each τ_n^G is equivalent to τ_1^G or τ_2^G.
- *For n not congruent to 0, -1 or -2 mod p, X is an n-slice iff $S^\lambda \wedge X$ is an $(n+2)$-slice.*

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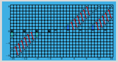
- *For n not congruent to $0, -1$ or -2 mod p, X is an n-slice iff $S^\lambda \wedge X$ is an $(n+2)$-slice. Each n-slice for $0 < n < p$ can be obtained from a from a 1-slice or a 2-slice*



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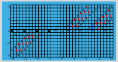

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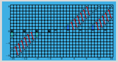
  Hence each τ_n^G is equivalent to τ_1^G or τ_2^G.
- *For n not congruent to 0, -1 or -2 mod p, X is an n-slice iff $S^\lambda \wedge X$ is an $(n+2)$-slice. Each n-slice for $0 < n < p$ can be obtained from a from a 1-slice or a 2-slice by smashing with a power of S^λ. Hence there are three distinct categories of slices,*



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What do the subcategories τ_n^G and $\tau_{=n}^G$ look like? (continued)

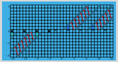
Example

Let $G = C_p$ for p an odd prime, and let $V = \lambda$, a 2-dimensional rotation of order p. Then

- *The conditions of the Corollary 1 hold provided n is not congruent to 0 or -1 mod p. Hence we get equivalences*

  $$\tau_1^G \to \tau_3^G \to \cdots \to \tau_p^G \quad \text{and} \quad \tau_2^G \to \tau_4^G \to \cdots \to \tau_{p-1}^G.$$

  Hence each τ_n^G is equivalent to τ_1^G or τ_2^G.
- *For n not congruent to 0, -1 or -2 mod p, X is an n-slice iff $S^\lambda \wedge X$ is an $(n+2)$-slice. Each n-slice for $0 < n < p$ can be obtained from a from a 1-slice or a 2-slice by smashing with a power of S^λ. Hence there are three distinct categories of slices, those of 0-, 1- and 2-slices.*



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