### MFO Arbeitsgemeinschaft Algebraic K-Theory and the Telescope Conjecture



# Why the Telescope Conjecture?



Doug Ravenel University of Rochester

14 October 2024

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Early 70s Morava K-theory Morava's vision Smith-Toda complexes

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1. Morava K-theory





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### 1. Morava K-theory

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### K(0) is rational cohomology.



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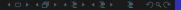
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K(0) is rational cohomology. For each n > 0 and each prime p, there is a nonconnective complex oriented p-local spectrum K(n) with

$$\pi_*K(n) = \mathbb{Z}/p[v_n^{\pm 1}] \qquad \text{where } |v_n| = 2(p^n - 1).$$

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In more detail, a formal group law over a ring *R* is a power series  $F(x, y) = \sum_{i,j} a_{i,j} x^i y^j \in R[[x, y]]$  satisfying

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Commutativity:  $F(y, x) = F(x, y)$ 

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**2** Commutativity: F(y, x) = F(x, y). This means  $a_{j,i} = a_{i,j}$ .

**3** Associativity: F(F(x, y), z) = F(x, F(y, z)).

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Associativity: F(F(x, y), z) = F(x, F(y, z)). This implies complicated relations among the a<sub>i,j</sub>.

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Every complex oriented spectrum *E* has a formal group law over  $\pi_*E$  associated with it.





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E = MU, the complex cobordism spectrum.





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E = MU, the complex cobordism spectrum. In 1969 Daniel Quillen showed that its formal group law has a universal property first studied by Michel Lazard in 1955, defined over

$$MU = \mathbb{Z}[x_i : i > 0] \text{ with } |x_i| = 2i.$$

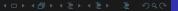
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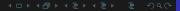


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**2** E = K(n), the *n*th Morava K-theory. The formal group law is characterized by its *p*-fold formal sum,  $[p](x) = v_n x^{p^n}$ .



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⊘ *E* = *K*(*n*), the *n*th Morava K-theory. The formal group law is characterized by its *p*-fold formal sum,  $[p](x) = v_n x^{p^n}$ . This means that its height is *n*. Height is known to be a complete isomorphism invariant for formal group laws over the algebraic closure of  $\mathbb{F}_p$ .



1

Formal group laws with  $[p](x) = x^{p^n}$ were constructed for all *n* and *p* in 1970 by Taira Honda.





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Formal group laws with  $[p](x) = x^{p^n}$ were constructed for all *n* and *p* in 1970 by Taira Honda. Hence all heights occur.

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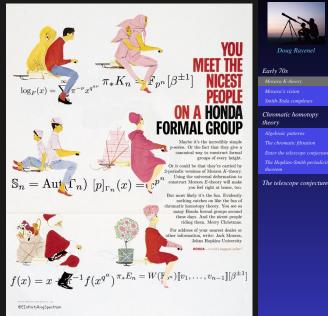
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Poster by Yuri Sulyma



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• Each point  $\theta \in V$  induces a formal group law over  $\overline{\mathbb{F}}_{\rho}$ .





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- Each point  $\theta \in V$  induces a formal group law over  $\overline{\mathbb{F}}_{\rho}$ .
- *V* has an action of  $\mathbb{G}$ . For  $\gamma(x) \in \mathbb{G}$ ,

$$F(x,y) \mapsto \gamma^{-1} \left( F(\gamma(x), \gamma(y)) \right),$$

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For each θ ∈ V, the isotropy or stabilizer group

 *G*<sub>θ</sub> = {γ ∈ *G* : γ(θ) = θ}

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Let *V* denote the "vector space" of ring homomorphisms  $\theta: L \to \overline{\mathbb{F}}_{\rho}$ , and let  $\mathbb{G}$  be the group of functionally invertible power series in 1 variable over  $\overline{\mathbb{F}}_{\rho}$ .





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- There are G-invariant finite codimensional linear subspaces

$$V = V_1 \supset V_2 \supset V_3 \supset \cdots$$

where  $V_n = \{ \theta \in V : \theta(v_1) = \cdots = \theta(v_{n-1}) = 0 \}.$ 

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• The height *n* orbit is  $V_n - V_{n+1}$ .

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- The height  $\infty$  orbit is the linear subspace

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#### 3. Smith-Toda complexes



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Cell diagram for V(2) at p = 5, where  $|v_1| = 8$  and  $|v_2| = 48$ :



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The first 2 cells comprise V(0), the mod p Moore spectrum.

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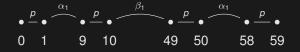
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The first 2 cells comprise V(0), the mod *p* Moore spectrum. The first 4 cells comprise V(1), MFO Workshop Why the Telescope Conjecture?



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Cell diagram for V(2) at p = 5, where  $|v_1| = 8$  and  $|v_2| = 48$ :



The first 2 cells comprise V(0), the mod *p* Moore spectrum. The first 4 cells comprise V(1), and  $V(2)/V(1) \simeq \Sigma^{49}V(1)$ . MFO Workshop Why the Telescope Conjecture?



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There is a cofiber sequence

$$\Sigma^{|v_n|}V(n-1) \xrightarrow{w_n} V(n-1) \longrightarrow V(n).$$

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for  $0 \le n \le 3$  and  $p \ge 2n + 1$ . We know that  $K(n)_* V(n-1) \ne 0$ and that  $w_n$  is a K(n)-equivalence.

These lead to the construction of the  $v_n$ -periodic families aka Greek letter elements

$\alpha_t \in \pi_{t v_1 -1} \mathbb{S}$	for <i>p</i> ≥ 3
$\beta_t \in \pi_{t v_2 -2p} \mathbb{S}$	for <i>p</i> ≥ 5
$\gamma_t \in \pi_{t v_3 -2p^2-2p+1}\mathbb{S}$	for <i>p</i> ≥ 7





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$$\begin{array}{ll} \alpha_t \in \pi_{t|v_1|-1} \mathbb{S} & \text{for } p \geq 3 \\ \beta_t \in \pi_{t|v_2|-2p} \mathbb{S} & \text{for } p \geq 5 \\ \gamma_t \in \pi_{t|v_3|-2p^2-2p+1} \mathbb{S} & \text{for } p \geq 7 \end{array}$$

 $\alpha_t$  is the composite

$$S^{t|v_1|} \xrightarrow{i} \Sigma^{t|v_1|} V(0) \xrightarrow{w_1^t} V(0) \xrightarrow{j} S^1.$$

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## Algebraic patterns

#### The Adams-Novikov spectral sequence

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### The Adams-Novikov spectral sequence

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These are nicely displayed in the  $E_2$ -term the Adams-Novikov spectral sequence.

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These are nicely displayed in the  $E_2$ -term the Adams-Novikov spectral sequence. In it there are similar families for all *n*.

In 1977 Haynes Miller, Steve Wilson and I constructed the chromatic spectral sequence converging to this  $E_2$ -term.



Annals of Mathematics. 106 (1977), 469-516

Periodic phenomena in the Adams-Novikov spectral sequence

> By HAYNES R. MILLER, DOUGLAS C. RAVENEL, and W. STEPHEN WILSON



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It organizes things into layers so that in the *n*th layer everything is  $v_n$ -periodic. The structure of this *n*th layer is controlled by the cohomology of the *n*th Morava stabilizer group  $\mathbb{G}_n$ .

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## The chromatic filtration

Later we learned that the stable homotopy category itself is similarly organized.





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Let Sp denote the category of spectra.



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Let Sp denote the category of spectra. Given a spectrum *E*, Bousfield constructed an endofunctor  $L_E : Sp \rightarrow Sp$ 



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Let Sp denote the category of spectra. Given a spectrum *E*, Bousfield constructed an endofunctor  $L_E : \text{Sp} \rightarrow \text{Sp}$  whose image category  $L_E\text{Sp}$  is stable homotopy as seen through the eyes of *E*-theory.



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for  $0 \le n \le 3$  and  $p \ge 2n + 1$ .





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Can we generalize this to n > 3?





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Can we generalize this to n > 3? Not exactly.





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Periodicity Theorem

Let X be a p-local type n finite spectrum,

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Periodicity Theorem

Let X be a p-local type n finite spectrum, meaning that  $K(n)_*X \neq 0$  and  $K(m)_*X = 0$  for m < n.

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### Periodicity Theorem

Let X be a p-local type n finite spectrum, meaning that  $K(n)_*X \neq 0$  and  $K(m)_*X = 0$  for m < n. Then for some d > 0 (and divisible by  $|v_n|$ ) there is a map

 $w: \Sigma^d X \to X$  where  $K(n)_* w$  is an isomorphism.

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Let *X* be a p-local type n finite spectrum. Then for some d > 0 (and divisible by  $|v_n|$ ) there is a self-map

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The theorem implies that the cofiber of w has type n + 1.



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When I stated the telescope conjecture in 1984, it was known to be true for n = 0 and n = 1.

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The height one case was proved around 1980 by Mark Mahowald for p = 2 and Haynes Miller for odd primes.





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Thus the statement for n > 1 seemed to be favored by Occam's razor.







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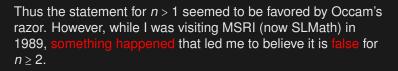
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Thus the statement for n > 1 seemed to be favored by Occam's razor. However, while I was visiting MSRI (now SLMath) in 1989, something happened that led me to believe it is false for  $n \ge 2$ .



San Francisco earthquake of October 17, 1989

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This failure of the telescope conjecture for  $n \ge 2$  is now a theorem of Robert Burklund, Jeremy Hahn, Ishan Levy and Tomer Schlank.

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Jeremy, Tomer, myself, Ishan and Robert at Oxford University, June 9, 2023. Photo by Matteo Barucco. MFO Workshop Why the Telescope Conjecture?



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Jeremy, Tomer, myself, Ishan and Robert at Oxford University, June 9, 2023. Photo by Matteo Barucco.

## THANK YOU!

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### Scratch paper

### MFO Workshop Why the Telescope Conjecture?



Doug Ravenel

Early 70s

Morava K-theory

Morava's vision

Smith-Toda complexes

Chromatic homotopy theory

Algebraic patterns

The chromatic filtration

Enter the telescope conjecture The Hopkins-Smith periodicity

The telescope conjecture

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### *More scratch paper*

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### Even more scratch paper

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