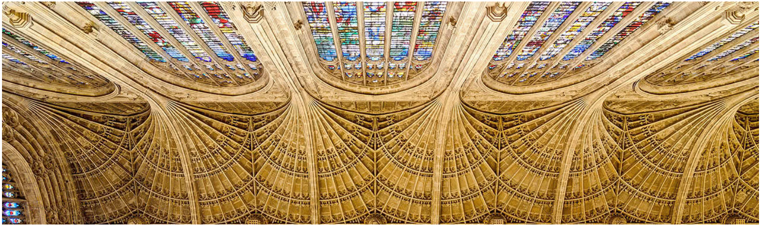

Model category structures for equivariant spectra

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Mike Hopkins Harvard University
Doug Ravenel University of Rochester

Conference on Equivariant and Motivic Homotopy Theory
Isaac Newton Institute, Cambridge, UK
August 15, 2018

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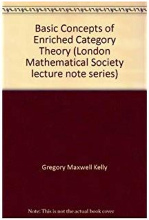
Enriched category theory



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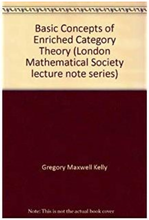
Enriched category theory



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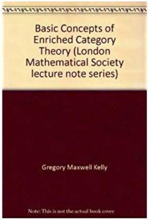

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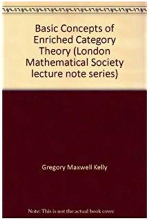

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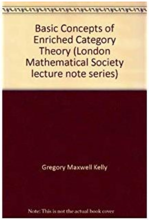

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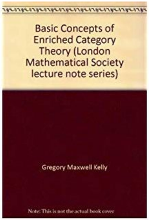

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$$c_{X,Y,Z} : \mathcal{C}(Y, Z) \times \mathcal{C}(X, Y) \to \mathcal{C}(X, Z),$$

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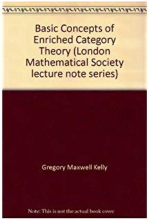

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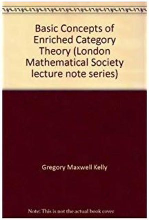

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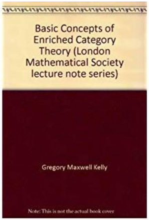

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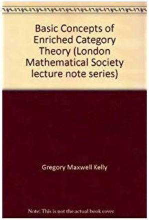

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$$c_{X,Y,Z} : \mathcal{C}(Y, Z) \otimes \mathcal{C}(X, Y) \to \mathcal{C}(X, Z).$$

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Enriched category theory (continued)

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Let $(\mathscr{J}, \oplus, 0)$ *be a small symmetric monoidal category enriched over a cocomplete closed symmetric monoidal category* $\mathcal{V}$*.*

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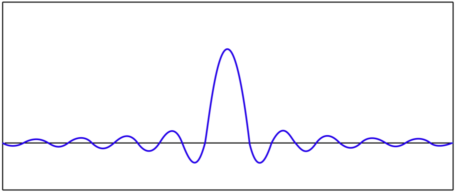

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$$\begin{tikzcd}
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& \mathscr{J} &
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The functor $X \otimes Y$ is the left Kan extension of the composite $\otimes(X \times Y)$ along $\oplus$.

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Some equivariant homotopy theory

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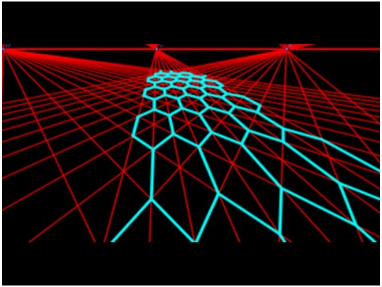
Three ways to construct new model categories from old ones



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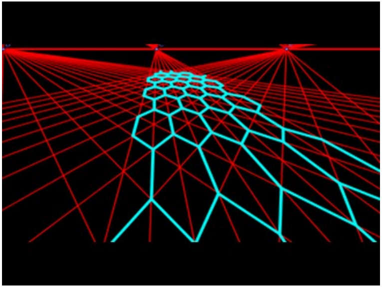

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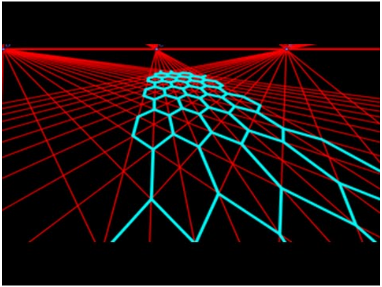

1. Given a model category $\mathcal{M}$ and a small category J, we define the projective model structure on the functor category $\mathcal{M}^J$ as follows.

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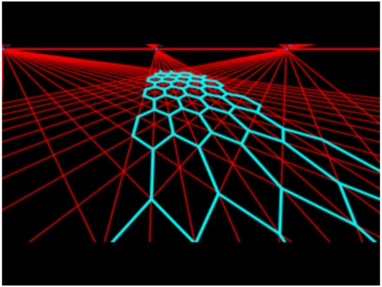

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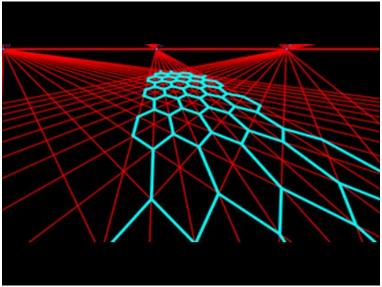

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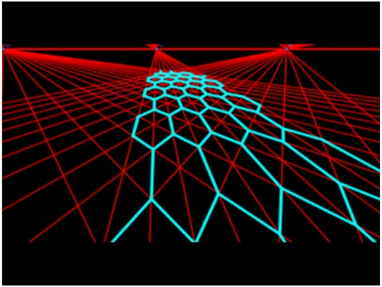
Three ways to construct new model categories from old ones



1. Given a model category $\mathcal{M}$ and a small category J, we define the projective model structure on the functor category $\mathcal{M}^J$ as follows. A map (aka natural transformation) $f : X \to Y$ between functors is a weak equivalence or a fibration if $f_j : X_j \to Y_j$ is one for each object j in J. Cofibrations are defined in terms of left lifting properties.

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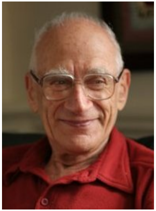

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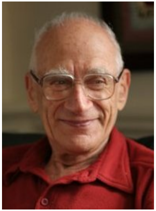

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2. Given a model category $\mathcal{M}$ and a pair of adjoint functors

$$F : \mathcal{M} \leftrightarrows \mathcal{N} : U,$$

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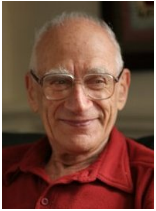

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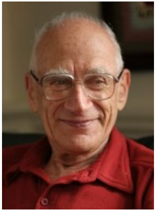

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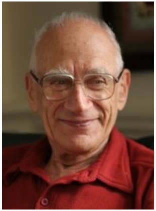

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$$F : \mathcal{M} \leftrightarrows \mathcal{N} : U,$$

the Kan transfer theorem says that under certain conditions there is model structure on $\mathcal{N}$ that makes the above a Quillen adjunction. A morphism in $\mathcal{N}$ is a weak equivalence or a fibration iff its image under U is one.

Three ways to construct new model categories from old ones (continued)

3. Bousfield localization.

Pete Bousfield

Three ways to construct new model categories from old ones (continued)

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Three ways to construct new model categories from old ones (continued)

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Three ways to construct new model categories from old ones (continued)

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Three ways to construct new model categories from old ones (continued)

3. Bousfield localization. Given a model category $\mathcal{M}$ satisfying certain conditions, we can define a new model structure $\mathcal{M}'$ with the same underlying category as follows. $\mathcal{M}'$ has the same cofibrations as $\mathcal{M}$, but more weak equivalences and hence more trivial cofibrations.

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Three ways to construct new model categories from old ones (continued)

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Three ways to construct new model categories from old ones (continued)

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The main construction

Suppose we have a diagram of small categories enriched over $\mathcal{T}^G$ (to be named later),

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$$\begin{tikzcd} \mathscr{J}_G^+ \arrow[r, hook, "k_+"] \arrow[d, hook, "i"'] & \widetilde{\mathscr{J}}_G^+ \arrow[d, hook, "\widetilde{i}"] \\ \mathscr{J}_G \arrow[r, hook, "k"] & \widetilde{\mathscr{J}}_G \end{tikzcd}$$

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$$\begin{tikzcd} & {[\mathscr{J}_G^+, \mathcal{T}^G]} \arrow[d, "i_!"'] & {[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]} \arrow[l, "k_+^*"'] \arrow[d, "\widetilde{i}_!"] \\ \mathcal{S}p^G \arrow[r, equal] & {[\mathscr{J}_G, \mathcal{T}^G]} & {[\widetilde{\mathscr{J}}_G, \mathcal{T}^G]} \arrow[l, "k^*"'] \end{tikzcd}$$

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where k^* and k_+^* are induced by precomposition, and $i_!$ and $\widetilde{i}_!$ are induced by left Kan extension. The category $\mathscr{J}_G$ is chosen so that the functor category $[\mathscr{J}_G, \mathcal{T}^G]$ is that of orthogonal G-spectra and equivariant maps.

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The main construction (continued)

$$\begin{tikzcd}
{[\mathscr{J}_G^+, \mathcal{T}^G]} \arrow[ddd, "i_!"'] & & & {[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]} \arrow[lll, "k_+^*"'] \arrow[ddd, "\widetilde{i}_!"] \\
& \mathscr{J}_G^+ \arrow[r, "k_+"] \arrow[d, "i"'] & \widetilde{\mathscr{J}}_G^+ \arrow[d, "\widetilde{i}"] & \\
& \mathscr{J}_G \arrow[r, "k"] & \widetilde{\mathscr{J}}_G & \\
\mathcal{S}p^G = [\mathscr{J}_G, \mathcal{T}^G] & & & {[\widetilde{\mathscr{J}}_G, \mathcal{T}^G]} \arrow[lll, "k^*"']
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Now we proceed as follows.

The main construction (continued)

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Now we proceed as follows.

(i) Start with the projective model structure on $[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]$.

The main construction (continued)

$$
\begin{tikzcd}
{[\mathscr{J}_G^+, \mathcal{T}^G]} \arrow[ddd, "i_!"'] & & & {[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]} \arrow[lll, "k_+^*"'] \arrow[ddd, "\widetilde{i}_!"] \\
& \mathscr{J}_G^+ \arrow[r, "k_+"] \arrow[d, "i"'] & \widetilde{\mathscr{J}}_G^+ \arrow[d, "\widetilde{i}"] & \\
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Now we proceed as follows.

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The main construction (continued)

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Now we proceed as follows.

(i) Start with the projective model structure on $[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]$. It is equifibrant, while the projective model structure on $[\mathscr{J}_G, \mathcal{T}^G]$ is not.

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(ii) The composite functor $i_! k_+^* = k^* \widetilde{i}_!$ is a left adjoint, so we can use the Kan transfer theorem to get a model structure on $\mathcal{S}p^G$. This transferred model structure is also equifibrant.

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{[\mathscr{J}_G^+, \mathcal{T}^G]} \arrow[ddd, "i_!"'] & & & {[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]} \arrow[lll, "k_+^*"'] \arrow[ddd, "\widetilde{i}_!"] \\
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(ii) The composite functor $i_! k_+^* = k^* \widetilde{i}_!$ is a left adjoint, so we can use the Kan transfer theorem to get a model structure on $\mathcal{S}p^G$. This transferred model structure is also equifibrant.

(iii) Expand the transferred class of weak equivalences on $\mathcal{S}p^G$ to that of stable equivalences

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The main construction (continued)

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(iii) Expand the transferred class of weak equivalences on $\mathcal{S}p^G$ to that of stable equivalences and apply Bousfield localization.

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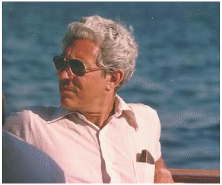
Defining the four small categories

Mike Mandell



Peter May

$\mathscr{J}_G$ is the Mandell-May category.

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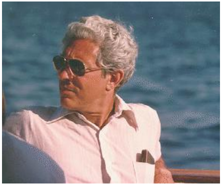

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$\mathscr{J}_G$ is the Mandell-May category. Its objects are finite dimensional orthogonal representations V of G.

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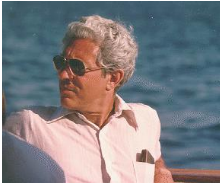

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$\mathscr{J}_G$ is the Mandell-May category. Its objects are finite dimensional orthogonal representations V of G. The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of the following vector bundle.

Defining the four small categories

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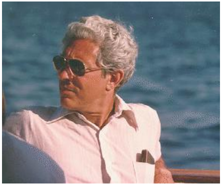

Peter May

$\mathscr{J}_G$ is the Mandell-May category. Its objects are finite dimensional orthogonal representations V of G. The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of the following vector bundle.

Let $O(V, W)$ be the (possibly empty) Stiefel manifold of isometric embeddings (which need not be equivariant) of V into W.

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Defining the four small categories

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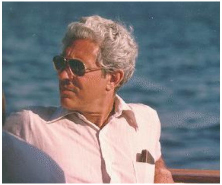

Peter May

$\mathscr{J}_G$ is the Mandell-May category. Its objects are finite dimensional orthogonal representations V of G. The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of the following vector bundle.

Let $O(V, W)$ be the (possibly empty) Stiefel manifold of isometric embeddings (which need not be equivariant) of V into W. For each such embedding $f : V \hookrightarrow W$ one has the orthogonal compliment $V^{\perp}$ of $f(V)$ in W,

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Defining the four small categories

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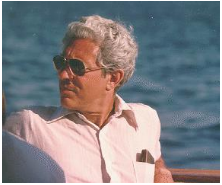

Peter May

$\mathscr{J}_G$ is the Mandell-May category. Its objects are finite dimensional orthogonal representations V of G. The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of the following vector bundle.

Let $O(V, W)$ be the (possibly empty) Stiefel manifold of isometric embeddings (which need not be equivariant) of V into W. For each such embedding $f : V \hookrightarrow W$ one has the orthogonal compliment $V^\perp$ of $f(V)$ in W, which is the fiber of our vector bundle over $O(V, W)$.

Defining the four small categories (continued)

The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of a certain vector bundle over the embedding space $O(V, W)$.

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Defining the four small categories (continued)

The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of a certain vector bundle over the embedding space $O(V, W)$.

The Mandell-May category is symmetric monoidal under direct sum.

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Defining the four small categories (continued)

The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of a certain vector bundle over the embedding space $O(V, W)$.

The Mandell-May category is symmetric monoidal under direct sum. This means that the functor category $\mathcal{S}p^G = [\mathscr{J}_G, \mathcal{T}^G]$,

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Defining the four small categories (continued)

The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of a certain vector bundle over the embedding space $O(V, W)$.

The Mandell-May category is symmetric monoidal under direct sum. This means that the functor category $\mathcal{S}p^G = [\mathscr{J}_G, \mathcal{T}^G]$, our category of equivariant spectra,

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Mike Hopkins
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Defining the four small categories (continued)

The morphism space $\mathscr{J}_G(V, W)$ is the Thom space of a certain vector bundle over the embedding space $O(V, W)$.

The Mandell-May category is symmetric monoidal under direct sum. This means that the functor category $\mathcal{S}p^G = [\mathscr{J}_G, \mathcal{T}^G]$, our category of equivariant spectra, is closed symmetric monoidal by the Day Convolution Theorem.

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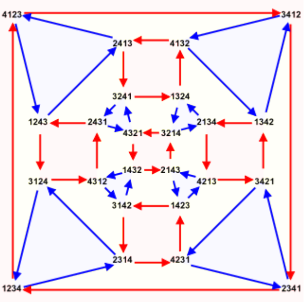
Defining the four small categories (continued)



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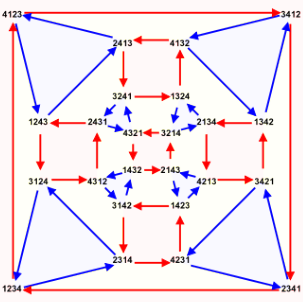
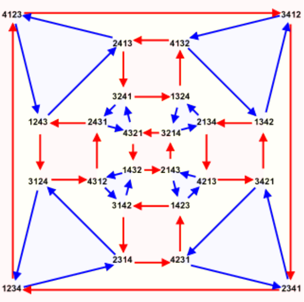
Defining the four small categories (continued)



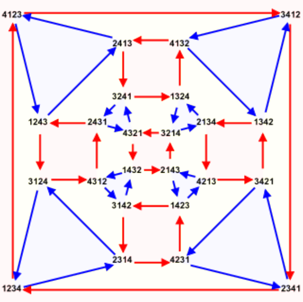
$\widetilde{\mathscr{J}}_G$ is the equifibrant Mandell-May category.

Defining the four small categories (continued)



$\widetilde{\mathscr{J}}_G$ is the equifibrant Mandell-May category. Its objects are finite dimensional orthogonal representations of finite G-sets.

Defining the four small categories (continued)



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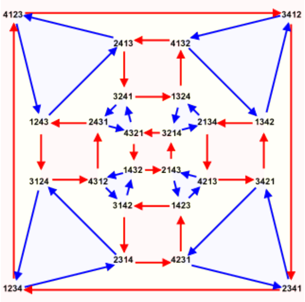
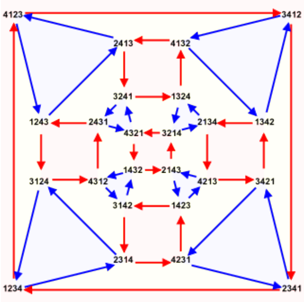
Defining the four small categories (continued)



$\widetilde{\mathscr{J}}_G$ is the equifibrant Mandell-May category. Its objects are finite dimensional orthogonal representations of finite G-sets. For a G-set T there is a category $\mathcal{B}_T G$ whose objects are the elements of T, and for each $(t, \gamma) \in T \times G$ there is a morphism that sends t to γt.

Defining the four small categories (continued)



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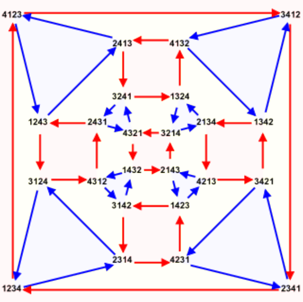
Defining the four small categories (continued)



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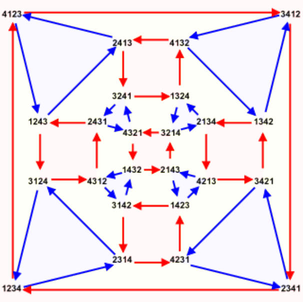
Defining the four small categories (continued)



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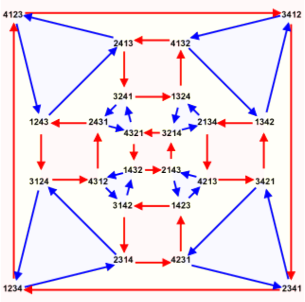

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Defining the four small categories (continued)

Recall that Mandell-May morphism objects involved orthogonal embeddings $V \hookrightarrow W$.

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Defining the four small categories (continued)

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It is a product of ordinary Stiefel manifolds.

Defining the four small categories (continued)

Given an orthogonal embedding

$$(S, V) \xrightarrow{f} (T, W),$$

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Defining the four small categories (continued)

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Defining the four small categories (continued)

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It is a smash product of ordinary Mandell-May morphism spaces.

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the one point union over all possible choices $\bar{f}$.

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Defining the four small categories (continued)

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The positive equifibrant Mandell-May category $\widetilde{\mathscr{J}}_G^+$ is the full subcategory with objects (T, V) in which the representation for each orbit of T has a nontrivial invariant vector.

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The main construction again

$$\begin{tikzcd}
{[\mathscr{J}_G^+, \mathcal{T}^G]} \arrow[ddd, "i_!"'] & & & {[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]} \arrow[lll, "k_+^*"'] \arrow[ddd, "\widetilde{i}_!"] \\
& \mathscr{J}_G^+ \arrow[r, "k_+"] \arrow[d, "i"'] & \widetilde{\mathscr{J}}_G^+ \arrow[d, "\widetilde{i}"] & \\
& \mathscr{J}_G \arrow[r, "k"] & \widetilde{\mathscr{J}}_G & \\
\mathcal{S}p^G = {[\mathscr{J}_G, \mathcal{T}^G]} & & & {[\widetilde{\mathscr{J}}_G, \mathcal{T}^G]} \arrow[lll, "k^*"']
\end{tikzcd}$$

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\arrow[from=1-1, to=4-1, "i_!"']
\end{tikzcd}
$$

(i) Start with the projective model structure on $[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]$.

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\end{tikzcd}$$

(i) Start with the projective model structure on $[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]$.

(ii) Use Kan's theorem to transfer it to a model structure on $\mathcal{S}p^G$. This is the positive equifibrant model structure.

The main construction again

$$\begin{tikzcd}
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(i) Start with the projective model structure on $[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]$.

(ii) Use Kan's theorem to transfer it to a model structure on $\mathcal{S}p^G$. This is the positive equifibrant model structure.

(iii) Expand the class of weak equivalences on $\mathcal{S}p^G$ to that of stable equivalences

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The main construction again

$$
\begin{tikzcd}
{[\mathscr{J}_G^+, \mathcal{T}^G]} \arrow[ddd, "i_!"'] & & & {[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]} \arrow[lll, "k_+^*"'] \arrow[ddd, "\widetilde{i}_!"] \\
& \mathscr{J}_G^+ \arrow[r, "k_+"] \arrow[d, "i"'] & \widetilde{\mathscr{J}}_G^+ \arrow[d, "\widetilde{i}"] & \\
& \mathscr{J}_G \arrow[r, "k"] & \widetilde{\mathscr{J}}_G & \\
\mathcal{S}p^G = {[\mathscr{J}_G, \mathcal{T}^G]} & & & {[\widetilde{\mathscr{J}}_G, \mathcal{T}^G]} \arrow[lll, "k^*"]
\end{tikzcd}
$$

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The main construction again

$$\begin{tikzcd}
{[\mathscr{J}_G^+, \mathcal{T}^G]} \arrow[ddd, "i_!"'] & & & {[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]} \arrow[lll, "k_+^*"'] \arrow[ddd, "\widetilde{i}_!"] \\
& \mathscr{J}_G^+ \arrow[r, "k_+"] \arrow[d, "i"'] & \widetilde{\mathscr{J}}_G^+ \arrow[d, "\widetilde{i}"] & \\
& \mathscr{J}_G \arrow[r, "k"] & \widetilde{\mathscr{J}}_G & \\
\mathcal{S}p^G = [\mathscr{J}_G, \mathcal{T}^G] & & & {[\widetilde{\mathscr{J}}_G, \mathcal{T}^G]} \arrow[lll, "k^*"']
\end{tikzcd}$$

(i) Start with the projective model structure on $[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]$.

(ii) Use Kan's theorem to transfer it to a model structure on $\mathcal{S}p^G$. This is the positive equifibrant model structure.

(iii) Expand the class of weak equivalences on $\mathcal{S}p^G$ to that of stable equivalences and apply Bousfield localization. The result is the positive stable equifibrant model structure.

The main construction again

$$\begin{tikzcd}
{[\mathscr{J}_G^+, \mathcal{T}^G]} \arrow[ddd, "i_!"'] & & & {[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]} \arrow[lll, "k_+^*"'] \arrow[ddd, "\widetilde{i}_!"] \\
& \mathscr{J}_G^+ \arrow[r, "k_+"] \arrow[d, "i"'] & \widetilde{\mathscr{J}}_G^+ \arrow[d, "\widetilde{i}"] & \\
& \mathscr{J}_G \arrow[r, "k"] & \widetilde{\mathscr{J}}_G & \\
\mathcal{S}p^G = [\mathscr{J}_G, \mathcal{T}^G] & & & {[\widetilde{\mathscr{J}}_G, \mathcal{T}^G]} \arrow[lll, "k^*"']
\end{tikzcd}$$

(i) Start with the projective model structure on $[\widetilde{\mathscr{J}}_G^+, \mathcal{T}^G]$.

(ii) Use Kan's theorem to transfer it to a model structure on $\mathcal{S}p^G$. This is the positive equifibrant model structure.

(iii) Expand the class of weak equivalences on $\mathcal{S}p^G$ to that of stable equivalences and apply Bousfield localization. The result is the positive stable equifibrant model structure. The positivity condition enables us to define a model structure on the category of equivariant commutative ring spectra.

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THANK YOU!