SIXTH ANNUAL UPSTATE NEW YORK NUMBER THEORY CONFERENCE

APRIL 30 - MAY 1, 2016

ABSTRACTS

PLENARY TALKS

Alan Adolphson (Oklahoma State University)

Distinguished-root formulas for generalized Calabi-Yau hypersurfaces

Abstract: The theory of A-hypergeometric differential equations has made it possible to give explicit formulas for roots of zeta and L-functions of varieties over finite fields. We review the theory of A-hypergeometric differential equations, discuss when their series solutions have integral coefficients, and explain how those solutions can be used to express the roots of zeta and L-functions.

Julio Andrade (University of Exeter, UK)

Mean values of L-functions and its derivatives in function fields

Abstract: In this talk I will discuss mean values of L-functions over function fields. In particular, I will prove a second moment of quadratic Dirichlet L-functions associated to monic irreducible polynomials over finite fields and if time permits I will also discuss the mean values of derivatives of L-functions in the same setting.

Edray Goins (Purdue University)

Metabelian Galois Representations

Abstract: We are used to working with Galois representations associated to elliptic curves by considering the action of the absolute Galois group on torsion points. However there is a slightly more exotic way to view this construction once we realize that the Tate module of an elliptic curve is just the abelianization of the étale fundamental group of the punctured torus.

In this talk, we discuss how to construct a class of Galois representations by considering covers of elliptic curves which are branched over one point. We discuss how this is related to the question of surjectivity of certain Galois representation, and how to construct representations with image isomorphic to the holomorph of the quaternions. We will not assume extensive knowledge of étale cohomology. This is joint work with Rachel Davis.

Youness Lamzouri (York University)

Large values of class numbers of real quadratic fields

Abstract: We construct an infinite family of real quadratic fields for which the class numbers are as large as possible, improving a result of Montgomery and Weinberger. These values are achieved using a special family of fields, first studied by Chowla. In a subsequent work, joint with A. Dahl, we investigate the distribution of class numbers in Chowlas family, and show a strong similarity between this distribution and that of class numbers of imaginary quadratic fields. As an application of our results, we determine the average order of the number of real quadratic fields in Chowlas family with class number h.

Hugh Montgomery (University of Michigan)

Trigonometric extrema

Abstract: After describing the solutions of three different extremal problems for trigonometric polynomials, we discuss how they may be applied, particularly to Kronecker's approximation theorem, which in turn is useful in discussing values of the zeta function.

Robert Pollack (Boston University)

Slopes of modular forms and the ghost conjecture

Abstract: In this talk, we present a new conjecture on slopes of p-adic modular forms. We write down a relatively simple and explicit power series over weight space and conjecture, in the Buzzard-regular case, that this power series knows the slopes of the U_p operator acting on all spaces of overconvergent modular forms. Precisely, we conjecture that the Newton polygon of our series evaluated at a weight k (classical or not) matches the Newton polygon of the characteristic power series of U_p acting on weight k overconvergent modular forms. We call this power series the "ghost series" as its spectral curve hovers around the true spectral curve.

In this talk, we will explain this ghost conjecture and its connections to other conjectures on slopes (e.g. Buzzard's conjecture, the Gouvea-Mazur conjecture, Coleman's spectral halo) and discuss implications for the shape and structure of the eigencurve.

This is a joint project with John Bergdall.

CONTRIBUTED TALKS

Andrew Bridy (University of Rochester)

Ramification and Galois Theory in Preimage Fields of Rational Maps

Abstract: Let K be a number field or a function field of characteristic 0, and let $f \in K(x)$ and $a \in K$. I study the tower of extensions given by the preimage fields $K_n := K(f^{-n}(a))$. I prove that except for a certain well-known family (postcritically finite maps), an analogue of Zsigmondy's theorem holds for ramified primes in this tower. For number fields, this requires assuming the abc conjecture. This result has applications to the Galois theory of these extensions, and in some cases I show an analogue of Serre's open image theorem holds for Galois representations arising from these towers. This is joint work with Tom Tucker.

Jack Buttcane (SUNY Buffalo)

The Kuznetsov formula on $GL(3)$

Abstract: The $GL(2)$ Kuznetsov formula gives a connection between Kloosterman sums and Fourier coefficients of Maass forms. I will discuss its generalization to $GL(3)$ and applications to the theory of exponential sums and $GL(3)$ L-functions.

Michael Chou (University of Conneticutt)

Torsion of rational elliptic curves over the maximal abelian extension of $\mathbb{Q}$

Abstract: A theorem of Ribet states that as we range over all elliptic curves $E/\mathbb{Q}$, the size of the torsion subgroup over $\mathbb{Q}^{ab}$, the maximal abelian extension of $\mathbb{Q}$, is bounded. We will present techniques in order to determine precisely $E(\mathbb{Q}^{ab})_{\text{tors}}$ for a given curve $E/\mathbb{Q}$. We further present a list containing all possible torsion subgroups appearing in this way.

Dan Collins (Cornell)

Heegner points for $x^3 + y^3 = p$

Abstract: The question "which primes are the sum of two rational cubes" is a classical Diophantine problem. Results that primes in certain residue classes are not sums of two cubes go back to Sylvester around 1880. Systematic results in the positive direction are much more recent: proving that primes in certain residue classes are sums of two cubes can be done by constructing a nonzero Heegner point on the associated elliptic curve. I'll give an overview of this problem, and outline my approach for getting nonzero Heegner points by working with a 3-adic L-function.

Rachel Davis (Purdue)

Origami Galois representations

Abstract: Let $(E, \mathcal{O})$ be an elliptic curve defined over $\mathbb{Q}$. An origami is a pair (C, f), where C is a curve and $f : C \to E$ is a map, branched at most above one point. The name comes from pictures that we will show. We define Galois representations associated to specific origami that generalize the Galois representations arising from the Galois action on the Tate module of E. This is joint work with Edray Goins.

John Doyle (University of Rochester)

Preperiodic portraits for unicritical polynomials over $\mathbb{C}(t)$

Abstract: If φ is a polynomial defined over a field K, we say that a point $\alpha \in K$ has preperiodic portrait (M, N) for φ if α enters an N-cycle after M steps under iteration by φ. Even if α is not preperiodic, α may become preperiodic modulo some prime in K; for example, this is necessarily the case if K is a number field, since all of the residue fields are finite. One may therefore ask the following question, considered by Ingram-Silverman and Faber-Granville for number fields and Ghioca-Nguyen-Tucker for function fields: Given a polynomial $\varphi(z) \in K[z]$, a point $\alpha \in K$, and a pair of integers (M, N), does there exist a prime $\mathfrak{p}$ such that α has preperiodic portrait (M, N) for φ modulo $\mathfrak{p}$? I will fully answer this question for the rational function field $K = \mathbb{C}(t)$ and the unicritical polynomials $\varphi(z) = z^d + t$.

Evan Dummit (University of Rochester)

Characterizations of Quadratic, Cubic, and Quartic Residue Matrices

Abstract: A recent paper of D. Dummit, Granville, and Kisilevsky showed the existence of unusually large biases in a number of prime-counting problems. While investigating this phenomenon, the following question arose: given n odd primes $p_1, ..., p_n$ where p^* denotes $(-1)^{(}p - 1)/2$, how many possible configurations are there for the splitting behavior of p_i in $\mathbb{Q}(\sqrt{p_j^*})$ for the possible pairs (i, j)? A natural way to organize this information is via the "quadratic residue matrix" of Legendre symbols $\left(\frac{p_i}{p_j}\right)$, which is a seemingly natural object that does not appear to have been previously studied. In my talk, I will give a simple characterization of these quadratic residue matrices along with natural generalizations to the cubic and quartic cases. (This is joint work with D. Dummit and Kisilevsky.)

Fan Ge (University of Rochester)

Connections between Zeros of $\zeta(s)$ and $\zeta'(s)$: Old and New results

Abstract: The distribution of zeros of $\zeta'(s)$ is intimately connected to both horizontal and vertical distribution of zeros of $\zeta(s)$, which directly or indirectly related to problems such as the Riemann Hypothesis, Landau-Siegel zeros, and class numbers of quadratic fields. In this talk, we briefly review some known results on the relationship between the zeros of ζ and ζ', and then we discuss some recent progress in this topic. New results include progress on Soundararajan's conjecture, solution to half of Radziwiłł's conjecture, and conditional estimate for the number of zeros of ζ' (which corresponds to Littlewood's result for ζ). One key ingredient in proofs of all the three is a function investigated by Yitang Zhang in his 2001 work solving half of Soundararajan's conjecture.

Eva Goedhart (Smith College)

On the Family of Diophantine Equations $(a^2 x^k - 1)(b^2 y^k - 1) = (abz^k - 1)^2$

Abstract: For a, b, $k \in \mathbb{Z}^+$ with $k \geq 7$, the equation $(a^2 x^k - 1)(b^2 y^k - 1) = (abz^k - 1)^2$ has no solutions in integers x, y, $z > 1$ with $a^2 x^k \neq b^2 y^k$. I will present the main ideas of the proof which uses standard results on continued fractions and a Diophantine approximation theorem due to Bennett.

Jamie Juul (Amherst College)

Periodic Points of $x^d + c$ over Finite Fields

Abstract: Let k be a number field and let $f(x)$ be a polynomial or rational function defined over k. For any prime p in the ring of integers $\mathcal{O}_k$ of k, we can view $f(x)$ as a map on the residue field $\mathcal{O}_k/p$ via reduction modulo p. Then we can ask: what proportion of points in $\mathcal{O}_k/p$ are periodic under this map? Heuristics suggest that for most maps the proportion of periodic points should tend to 0 as p increases. In this talk we will describe some

general conditions for when this will hold, focusing on polynomials of the form $f(x) = x^d + c$ as our primary examples.

Tianyi Mao (CUNY Graduate Center)

A Weighted Sum on the Distribution of totally Positive Integers in Totally Real Cubic Fields

Abstract: Hecke studied the distribution of fractional parts of quadratic irrationals with Fourier expansion of Dirichlet series. This method is generalized by Behnke and Ash-Friedberg, to study the distribution of the number of totally positive integers of given trace in a general totally real number field of any degree. I will talk about the asymptotic behavior of a weighted Diophantine sum when the field is cubic, and show how it is related to the structure of the unit group.

Brendan Murphy (Rochester)

Exponential sums via growth in groups

Abstract: An approximate group is a finite subset of group that is nearly closed under multiplication. Recently, Gill and Helfgott classified approximate subgroups of $GL_n(\mathbb{Z}/p\mathbb{Z})$, building on work of many others.

We will see how to derive results on exponential sums over multiplicative subgroups by applying Gill and Helfgott's structure theorem.

If time permits, we will discuss a generalization where exponential sums are replaced by characters on a nilpotent group, and multiplicative subgroups are replaced by orbits of an abelian group acting on the nilpotent group.

This is joint work with Sarah Peluse, Vladislav Petkov, and Lam Pham, completed at the 2016 Arizona Winter School under the direction of Harald Helfgott and Henry Bradford.

Howard Skogman (SUNY Brockport)

Constructing Ramanujan Graphs as Galois Covers of Edge-Weighted Graphs

Abstract: We provide a fairly general construction of sets of Ramanujan graphs as Galois covers of edge-weighted graphs and discuss further generalizations of the construction.

Nicolas Templier (Cornell)

Kloosterman families, Quantum cohomology and geometric Langlands

Abstract: We prove cases of Rietsch mirror conjecture that the Dubrovin quantum connection for projective homogeneous varieties is isomorphic to the pushforward D-module attached to Berenstein-Kazhdan geometric crystals. The idea is to recognize the quantum connection as Galois and the geometric crystal as automorphic. We reveal surprising relations with the works of Frenkel-Gross, Heinloth-Ngo-Yun and Zhu on Kloosterman sheaves. The isomorphism comes from global rigidity results where Hecke eigensheaves are determined by their local ramification. It implies combinatorial identities for the counts of rational curves and the Peterson variety presentation as corollary. Work with Thomas Lam.

Amanda Tucker (University of Rochester)

Multiple zeta values

Abstract: It is widely believed that the numbers $\pi, \zeta(3), \zeta(5), \zeta(7), \ldots \zeta(2k+1)$ are algebraically independent over the rational numbers for any integer $k \geq 1$, but a proof is still elusive. However, in examining this question, one quickly comes (as Euler did in 1775) to a study of the multiple zeta values. Governing these numbers are many algebraic relations whose study is still in its infancy. I will introduce the multiple zeta values and talk about our proof regarding some related symmetries of rational functions. This talk represents joint work with D. Schindler and A. Salerno.

David Zywina (Cornell)

Possible indices for the Galois image of elliptic curves over $\mathbb{Q}$

Abstract: For a non-CM elliptic curve $E/\mathbb{Q}$, its Galois action on all its torsion points can be expressed in terms of a Galois representation. A famous theorem of Serre says that the image of this representation is as "large as possible" up to finite index. We will study what indices are possible assuming that we are willing to exclude a finite number of possible j-invariants from consideration.