Monads as a Solution for Generalized Opacity

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Denotations vs. senses

$1 + 1 = 2$

- The two sides of the equality have the same *denotation*
- But they have different *senses*:
  - 2 is the *name* of the number
  - $1 + 1$ is a *process* whose result is the number

Denotations vs. senses

Similarly in natural language

(1) "**Hesperus is Phosphorus**"

is not necessarily a tautology:

(2) "**Reza doesn't believe Hesperus is Phosphorus**"

Denotations vs. senses

Standard analysis:

- "**Hesperus**" and "**Phosphorus**" refer to the same entity (the planet Venus)
- Their senses are different:
  - "**Hesperus**" → *the evening star*
  - "**Phosphorus**" → *the morning star*
- Two different *descriptions* (processes) that lead to the same entity

But. . .

In certain contexts an expression like (3)

(3) "**Sandy is Sandy**"

is also not a tautology. . .

Capgras syndrome

From Wikipedia:

> "[Capgras syndrome] is a disorder in which a person holds a delusion that a friend, spouse, parent, or other close family member has been replaced by an identical-looking impostor."

- Assume Kim suffers from Capgras syndrome
- The following sentence is not necessarily unsatisfiable:

(4) "**Kim doesn't believe Sandy is Sandy**"

- Saying "**Sandy**" is ambiguous seems wrong

Capgras syndrome

- Kim's model: two "*Sandy*" entities
  - $Sandy_\sigma$
  - $ImpostorSandy_{Kim}$
- Speaker's model: one "*Sandy*" entity
  - $Sandy_\sigma$
- $ImpostorSandy_{Kim} \neq Sandy_\sigma$

Capgras syndrome

(5) "**Kim doesn't believe Sandy is Sandy**"

- At the time of the utterance, the name "Sandy" refers to $ImpostorSandy_{Kim}$ for Kim
- For the speaker the name refers to $Sandy_{\sigma}$
- One instance of "Sandy" is interpreted from Kim's *persepective*
- The other from the speaker's *perspective*

(6) "**Kim doesn't believe** $ImpostorSandy_{Kim} = Sandy_{\sigma}$"

Indiana Pi Bill

In 1897 Dr. Edwin J. Goodwin presented a bill to the Indiana General Assembly for

> "[...] introducing a new mathematical truth and offered as a contribution to education to be used only by the State of Indiana free of cost"

He had copyrighted that

(7) $\pi = 3.2$

Thus

(8) "**Edwin doesn't believe** $\pi = \pi$"

Our approach in a nutshell

- Contentious expressions are given denotations that depend on an additional *interpretation index*
- *Perspectives* rather than *senses*
- Default interpretation index = speaker's index
- Verbs like "believe" can change the default index to another index (e.g. the subject's index)
- Ontology: real entities + mental entities (relative to interpretation indices)

Other opaque contexts

(9) "**Reza doesn't believe Hesperus is Phosphorus**"

(10) "**Mary Jane loves Peter Parker but she doesn't love Spiderman**"

but not in cases like (11)

(11) "**Dr. Octopus killed Spiderman but he didn't kill Peter Parker**" (compare with "**murder**")

- Overlap with *intensionality* but not the same point of view

Outline of the talk

- *Opacity*
- Naive implementation of the idea
- 3 problems for simple type extension
- Monads as a solution

Naive implementation:New types

- A single new type i for interpretation indices
- Simplistically $i = e$

Naive implementation:Contentious expressions

- Given an expression of type τ, we signal it is contentious by assigning it the type $i \rightarrow \tau$
- Example: "**Kim doesn't believe Sandy is Sandy**"
- "**Sandy**" contentious, type assigned $i \rightarrow e$
- Example: "**Reza doesn't believe Jesus is Jesus**"
- "**Jesus**" contentious, type assigned $i \rightarrow e$

Naive implementation:Index switching expressions

- "**love**" : $\lambda s.\lambda o.love(s)(o(s)) : e \to (i \to e) \to t$
- "**believe**" : $\lambda s.\lambda c.believe(s)(c(s)) : e \to (i \to t) \to t$

Grammatical infrastructure

- Applicable basically to any grammatical formalism
- We use a sort of soft LFG / Categorial grammar approach
- Linear logic as model for semantic composition
- Functional types become linear implications ($\multimap$)
- No generalized lifting in the lexicon

Problem 1:Only contradictory reading

(12) "**Kim doesn't believe Sandy is Sandy**"

- Only reading available:

(13) "**Kim doesn't believe** $ImpostorSandy_{Kim} = ImpostorSandy_{Kim}$"

- "**Sandy**" always interpreted in the scope of "**believe**"

Problem 2:Implication linearity

$\tau \to \delta \to \rho,$	$\tau,$	δ	$\vdash$	ρ
$\tau \to \delta \to \rho,$	$i \to \tau,$	$\delta,$	$\vdash$	$i \to \rho$
$\tau \to \delta \to \rho,$	$i \to \tau,$	$i \to \delta,$	$\vdash$	$i \to i \to \rho$

Problem 3:Linearity again

What if the object of a verb like "**love**" is non contentious?

(14) $e \to (i \to e) \to t, e, e \vdash ?$

$\alpha \vdash i \to \alpha$ not a valid inference in linear logic

Problems

- Problem 1 is a scopal problem
- Problem 2 and 3 seem to stem from *linearity* of implication / function type constructor
- Rejecting linearity seems too strong:

  (15) **John punches Bill**

  (16) $punch(John)(John)$

Solution

- Same pattern emerges (for problem 2 and 3) in unrelated contexts:
  - Multidimensional semantics (e.g. conventional implicatures)
  - Semantics of questions
  - Optional arguments
- Monads as a generalized model for all these phenomena

Monads

- Monad = $\langle M, \eta, \mu \rangle$
- M is a functor, mapping between types: $\tau \mapsto i \to \tau$
- η ("unit") *lifts* object to the monadic type, solves problem 3:

  (17) $\eta = \lambda x.\lambda i.x : \tau \to i \to \tau$

- μ ("join") *compresses* multiple monadic layers into a single one, solves problem 2:

  (18) $\mu = \lambda f.\lambda i.f(i)(i) : (i \to i \to \tau) \to i \to \tau$

$$\frac{}{x : A \vdash x : A}\ id \qquad \frac{\Gamma \vdash B \qquad B, \Delta \vdash C}{\Gamma, \Delta \vdash C}\ Cut$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \multimap B} \multimap R \qquad \frac{\Delta \vdash t : A \qquad \Gamma, x : B \vdash u : C}{\Gamma, \Delta, y : A \multimap B \vdash u[y(t)/x] : C} \multimap L$$

$$\frac{\Gamma \vdash x : A}{\Gamma \vdash \eta(x) : \Diamond A}\ \Diamond R \qquad \frac{\Gamma, x : A \vdash t : \Diamond B}{\Gamma, y : \Diamond A \vdash y \star \lambda x.t : \Diamond B}\ \Diamond L$$

Figure 1: Sequent calculus for a fragment of multiplicative linear logic enriched with a monadic modality, together with a Curry-Howard correspondence between formulae and meaning terms.

Technical detail:$\star$ (“bind”) rather than μ

- μ impractical when writing lexical entries

$$\begin{aligned} \star = \quad & \lambda m.\lambda k.\lambda i.k(m(i))(i) : \\ & (i \to \tau) \to (\tau \to (i \to \delta)) \to (i \to \delta) \end{aligned}$$

- $\star$ can be defined in terms of μ
- $\star$ can be interpreted as sequencing too
- New “scope” mechanism, solves problem 1

Monads in the logic

- New unary connective $\Diamond$, but $\Diamond\alpha \neq i \to \alpha$

Online theorem prover

<http://llilab.carleton.ca/ giorgolo/tp.html>

Monads, intuitively

- Monads as a model of *side effects*: $\Diamond\tau$ computation that results in value of type τ possibly producing side effects

- η creates a trivial computation *without side effects*
- $\star$ *combines* computations and side effects, enforces *order of evaluation*
- In our case, side effect = *dependency on context / environment*

Capgras example

(19) "**Kim doesn't believe Sandy is Sandy**"

Kim	Kim	e
not	$\lambda p.\neg p$	$t \to t$
believe	$\lambda s.\lambda c.\lambda i.believe(s)(c(s))$	$e \to \Diamond t \to \Diamond t$
Sandy	$\{Kim \mapsto Impostor_{Kim}, \sigma \mapsto Sandy_\sigma\}$	$\Diamond e$
is	$\lambda x.\lambda y.x = y$	$e \to e \to t$

Readings

Satisfiable reading $[\![\text{Sandy}]\!] \star \lambda x.[\![\text{believe}]\!](Kim)([\![\text{Sandy}]\!] \star \lambda y.\eta(x = y)) \star \lambda z.\eta(\neg z)$

$$\neg believe(Kim)(Impostor_{Kim} = Sandy_\sigma)$$

Unsatisfiable readings $[\![\text{believe}]\!](Kim)([\![\text{Sandy}]\!] \star \lambda x.[\![\text{Sandy}]\!] \star \lambda y.\eta(x = y)) \star \lambda z.\eta(\neg z)$

$$\neg believe(Kim)(Impostor_{Kim} = Impostor_{Kim})$$

$[\![\text{Sandy}]\!] \star \lambda x.[\![\text{Sandy}]\!] \star \lambda y.[\![\text{believe}]\!](Kim)(\eta(x = y)) \star \lambda z.\eta(\neg z)$

$$\neg believe(Kim)(Sandy_\sigma = Sandy_\sigma)$$

"love" example

(20) "*Mary Jane loves Peter Parker but she doesn't love Spiderman*"

Mary Jane	MJ	e
love	$\lambda s.\lambda o.\lambda i.love(s)(o(s))$	$e \rightarrow \Diamond e \rightarrow \Diamond t$
Peter Parker	PP	e
not	$\lambda p.\neg p$	$t \rightarrow t$
Spiderman	$\{MJ \mapsto SM_{MJ}, \sigma \mapsto PP\}$	$\Diamond e$

Readings

Satisfiable reading $[\![\text{love}]\!](MJ)(\eta(PP)) \star \lambda p.[\![\text{love}]\!](MJ)([\![\text{Spiderman}]\!]) \star \lambda q.\eta(p \wedge \neg q)$

$$love(MJ)(PP) \wedge \neg love(MJ)(SM_{MJ})$$

Unsatisfiable reading $[\![\text{love}]\!](MJ)(\eta(PP)) \star \lambda p.[\![\text{Spiderman}]\!] \star \lambda x.[\![\text{love}]\!](MJ)(\eta(x)) \star \lambda q.\eta(p \wedge \neg q)$

$$love(MJ)(PP) \wedge \neg love(MJ)(PP)$$

Beyond names

- Use the same approach with any possibly contentious expression

(21) "**Tina believes Bob is a woodchuck but she doesn't believe he is a groundhog**"

(22) "**Tina thinks Flipper is a dolphin but she doesn't think he is a marine mammal**"

Beyond names

dolphin	$\{Flipper, Oscar, ...\}$	$e \rightarrow t$
marinemammal	$\{\sigma \mapsto \{Flipper, Oscar, MobyDick, ...\}, Tina \mapsto \{MobyDick, ...\}\}$	$\Diamond(e \rightarrow t)$

To sum up

- Generalized approach to *opacity* based on interpretation indices, *perspectives*
- Not only traditional cases but also *Capgras* examples
- Monads: *weaken linearity* of compositionality, additional *scope* mechanism
- Not limited to synonyms but also applicable to more general entailments