

# Multidimensional Semantics with Unidimensional Glue Logic\*

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## 1 Introduction

- (1) A: Most fucking neighbourhood dogs pee on a damn hydrant on this street.  
B: No, that's not true.  
⇒ No, the neighbourhood dogs don't pee on a hydrant on this street.  
⊄ No, there's nothing wrong with dogs and/or their peeing on hydrants on this street.
- (2) A: John Lee Hooker, the bluesman from Tennessee, appeared in *The Blues Brothers*.  
B: No, that's not true.  
⇒ No, John Lee Hooker did not appear in *The Blues Brothers*.  
⊄ No, John Lee Hooker was not from Tennessee.  
B: True, but actually John Lee Hooker was born in Mississippi
- Potts (2005, 2007) and Arnold and Sadler (2010) have claimed that the analysis of these sentences requires two semantic levels, typically called 'dimensions'.
    1. The 'at-issue' dimension represents the aspect of meaning that is under discussion and is sensitive to logical operators such as negation.
    2. The 'side-issue' dimension (né 'CI dimension') represents an aspect of meaning that contributes information that is speaker-oriented, often peripheral, and not under discussion or up for grabs.
  - In example (1), the fact that the speaker hates dogs and/or their urinary habits cannot be controversial: the speaker communicates this by his/her choice of words.
  - A crucial aspect of Potts's multidimensional type theory is that information can flow from at-issue content to side-issue content, but not vice versa.
  - This intuition is captured by Arnold and Sadler (2010) in Glue Semantics for LFG, following Potts (2005: 87), by treating at-issue and side-issue content as corresponding to elements of a tensor pair in the glue logic. A similar approach was proposed by Nouwen (2007) in a dynamic semantics setting.

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- AnderBois et al. (2010) have claimed that multidimensional semantics for at-issue/side-issue meanings is problematic, because of example like the following.
- (3) John<sub>1</sub>, who by the way almost destroyed his<sub>1</sub> car yesterday, has bought a motor-cycle, too.
  - Here we seem to see full interaction between the two dimensions. An anaphor in the side-issue appositive relative clause is finding its antecedent in the main clause at-issue content. This is to be expected on the standard account. However, we also have unexpected information flow from the side-issue content to the at-issue content, since the presupposition triggered by “too” is satisfied by the information contributed in the side-issue appositive, that John has another vehicle.

## 2 Main Claims

- A multidimensional treatment of conventional implicature (appositives, expressives, etc.) is necessary (in agreement with Potts and Arnold and Sadler).
- The multidimensionality can be restricted to the meaning language, such that the logic of composition is still unidimensional (contra Arnold and Sadler).
- Our intuition is that at-issue and side-issue meaning are generally separate, but there are limited interactions (contra AnderBois et al.).
- Monads are a good language for expressing multidimensionality in the meaning language while capturing limited interaction between dimensions. The following properties of monads are of particular interest in this respect:
  1. Monads can be used as containers for more than one value.
  2. Monads can be used to impose an order of evaluation.
  3. Different types of monads can be layered in a uniform way, but with a common interface. This allows us to use monads both to capture multidimensional effects and dynamic effects.

## 3 Overview

1. *Introduction*
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5. Background: Monads
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## 4 Interdimensional Meaning Interaction

- AnderBois et al. (2010) review a number of circumstances, initially discussed by Potts (2005: 52ff.), in which at-issue content seems to require access to side-issue content, which would be precluded by Potts's type theory.
  1. Presupposition
    - (4) Mary, a good drummer, is a good singer too.
  2. Anaphora
    - (5) Jake<sub>1</sub>, who almost killed a woman<sub>2</sub> with his<sub>1</sub> car, visited her<sub>2</sub> in the hospital.
  3. VP ellipsis
    - (6) Lucy, who doesn't help her sister, told Jane to.
  4. Nominal ellipsis
    - (7) Melinda, who won three games of tennis, lost because Betty won six.
- AnderBois et al. (2010) conclude from this kind of data that there is only one dimension of meaning, since there seems to be interaction between the at-issue and side-issue meanings and the multidimensional treatment is founded on an intuition of independence.
- AnderBois et al. instead propose that there are two modes of discourse update, one for at-issue material and one for side-issue material. At-issue material is *proposed* and open for correction, questioning, etc. Side-issue material is instead *imposed* and the update eliminates possible interpretations that are inconsistent with the side-issue meaning.
- The job of dimensions in Potts's theory, and also in our version, is to keep track of how information was introduced (as at-issue or side-issue contributions). AnderBois et al.'s proposal hardwires this distinction into discourse update, and therefore effectively fails to eliminate the bipartite nature of the Potts theory.
- Our intuition is instead that at-issue and side-issue content are largely separate, but that at-issue content can access side-issue content in certain limited circumstances.
- This intuition is motivated by the fact that side-issue content always ends up outside the scope of logical operators, such as negation, question-forming operators, etc., which was an important part of the initial motivation for Potts's claim of multidimensionality. On the AnderBois et al. theory, this lack of interaction with logical operators is unexplained.
- Assuming, then, that we wish to keep a multidimensional treatment, the next question is how to capture multidimensionality in a type-logical setting such as Glue Semantics.

- Arnold and Sadler (2010) follow Potts (2005) in capturing multidimensionality in the logic for composition. In the context of Glue Semantics, this means in the glue logic terms on the right side of glue meaning constructors.
- We propose an alternative approach in which multidimensionality is captured in the meaning language, while leaving the glue logic unidimensional, for the following reasons:
  1. In principle, it might be necessary to propose more than two dimension. In such a case, the commutative tensor conjunction in linear logic does not provide enough structure to properly distinguish between dimensions or to refer to information in a particular dimension subsequently.
  2. The lack of structure in the tensor conjunction makes it difficult to control at-issue/side-issue interactions of the kind discussed above.
  3. Tensors in proof goals make it more difficult to state the correct condition on proof termination and therefore potentially lose some of the linguistic leverage provided by linear logic's resource sensitivity (Asudeh 2004, 2012).
- Monads provide a single mechanism for capturing multidimensionality and interaction.

## 5 Background: Monads

- Monads were first used to give a unified analysis of various semantic phenomena by Shan (2001)
- The main intuition behind monads is that they are a way to reproduce the structure of a space of values and functions in a richer setting that carries more information, in the sense that we can specify more things about the values and functions.
- We can move from the information-poor space to the information-rich space as follows:
  - A value or function in the poor space is mapped to an information-enriched counterpart by associating the value or function with some sort of default information. In this way, we get an object of the right information-rich type, without committing to any particular enriched information.
  - For example, in the case of multidimensionality, the values and functions that contribute only to at-issue material can be mapped to a richer space where they have a vacuous side-issue component.
- A more operational way to look at monads is to consider them as computations that yield values.
- $\eta$  is the mapping from the information-poor space to the information-rich space.
- $\star$  is the mechanism for extracting values from computations and creating new computations using these values.  $\star$  also allows ordering for side-effects of computation.
- $M$  is the label for the information-rich counterpart of the original, information-poor types.
- For example, the *Writer* monad maps to an enriched type that pairs a value with a collection of propositions. For *Writer*:
  - $\eta$  maps any value  $x$  to the pair  $\langle x, \{ \} \rangle$
  - $\star$  is a binary function that takes 1) an input pair of a variable and a collection of propositions and 2) a function  $f$  that produces a computation using the first value of the input pair.  $\star$  produces a new computation whose value is the value of the computation produced by  $f$  and a new collection of propositions that is the union of the input collection of propositions with the collection of propositions produced by  $f$ .
  - *Writer* therefore has the effect of logging a collection of propositions.
- In the Glue setting, we want to keep as much as we can of the standard glue logic, but use the mapping facility of monads to obtain the additional side-issue dimension.
- The one augmentation to the glue logic that we require is an additional implication connective that allows some lexical items to be directly specified in terms of the information-rich space.

- This equation defines application for the standard glue logic implication,  $\multimap$ , in the monad-enriched meaning language.

$$A(f)(x) =_{def} f \star \lambda g.x \star \lambda y.\eta (g y) : M (\alpha \rightarrow \beta) \rightarrow M \alpha \rightarrow M \beta \quad (8)$$

- This equation defines abstraction for the standard glue logic implication,  $\multimap$ , in the monad-enriched meaning language.

$$\eta(x) \triangleleft m =_{def} m \star \lambda b.\eta (\lambda x.b) : M \alpha \rightarrow M \beta \rightarrow M (\alpha \rightarrow \beta) \quad (9)$$

$x$  must be a free variable not appearing anywhere else in the proof.

- These are the elimination and introduction rules, with a kind of Curry-Howard isomorphism to the monad-enriched meaning language, for the standard  $\multimap$  implication.

$$\frac{x : A \quad f : A \multimap B}{A(f)(x) : B} \multimap E \quad \frac{\begin{array}{c} [\eta(x) : A]_i \\ \vdots \\ t : B \end{array}}{\eta(x) \triangleleft t : A \multimap B} \multimap I_i \quad (10)$$

- This equation defines application for the new glue logic implication,  $\multimap_*$ , in the monad-enriched meaning language.

$$A_*(f)(x) =_{def} f x : (M \alpha \rightarrow M \beta) \rightarrow M \alpha \rightarrow M \beta \quad (11)$$

- This equation defines abstraction for the new glue logic implication,  $\multimap_*$ , in the monad-enriched meaning language.

$$x \triangleleft_* m =_{def} \lambda x.m x : M \alpha \rightarrow M \beta \rightarrow (M \alpha \rightarrow M \beta) \quad (12)$$

- These are the elimination and introduction rules, with a kind of Curry-Howard isomorphism to the monad-enriched meaning language, for the new  $\multimap_*$  implication.

$$\frac{x : A \quad f : A \multimap_* B}{A_*(f)(x) : B} \multimap_* E \quad \frac{\begin{array}{c} [x : A]_i \\ \vdots \\ t : B \end{array}}{x \triangleleft_* t : A \multimap_* B} \multimap_* I_i \quad (13)$$

## 6 Analysis

(14) John, who likes cats, likes dogs also.

### Lexicon

<b>comma</b>	$\lambda j \lambda l . j \star \lambda x . l \star \lambda f . \text{write}(f x) \star \lambda \_ . \eta(x) : j \multimap (j \multimap l) \multimap j$
<b>also</b>	$\lambda v . \lambda o . \lambda s . s \star \lambda x . v \star \lambda f . o \star \lambda y . \text{check}(\exists z . f x z \wedge z \neq y) \star \lambda \_ . \eta(f x y) :$ $(d \multimap j \multimap l) \multimap d \multimap j \multimap l$
<b>John</b>	$\eta(j) : j$
<b>who</b>	$\eta(\lambda P . P) : (j \multimap l) \multimap (j \multimap l)$
<b>likes</b>	$\eta(\lambda y \lambda x . \text{like}(x, y)) : c \multimap j \multimap l$
<b>cats</b>	$\eta(\iota x . \text{cat}^*(x)) : c$
<b>likes</b>	$\eta(\lambda y \lambda x . \text{like}(x, y)) : d \multimap j \multimap l$
<b>dogs</b>	$\eta(\iota x . \text{dog}^*(x)) : d$

The lexical entries of **comma** and **also** are dependent on the surface order of their respective arguments. It is possible to reshuffle the argument order without changing the semantic term's interpretation. The correct order can be selected by feeding information about linear order to the semantic derivation as discussed in Asudeh (2009).

### Proof

The proof for example (14) is shown in Figure 1. The result is a pair whose first member represents the at-issue meaning, namely that John likes dogs, while the second member represents the collection of side-issue contributions so far, namely that John likes cats. In the background, the dynamic monad checks, via the function `check`, that presuppositional requirements of **also** are satisfied; namely, that John likes something other than dogs.

$$\begin{array}{c}
\frac{\frac{\frac{\frac{\frac{[\![\text{likes}]\!] : c \multimap j \multimap l}{[\![\text{cats}]\!] : c}}{A([\![\text{likes}]\!])([\![\text{cats}]\!] : j \multimap l)}}{[\![\text{who}]\!] : (j \multimap l) \multimap (j \multimap l)}}{A([\![\text{who}]\!])(A([\![\text{likes}]\!])([\![\text{cats}]\!] : j \multimap l))} \\
\boxed{1} \frac{A_*(A_*([\![\text{comma}]\!]([\![\text{john}]\!] \multimap_{o_*} j \multimap_{o_l} j \multimap_{o_*} j)))(A([\![\text{who}]\!])(A([\![\text{likes}]\!])([\![\text{cats}]\!] \multimap_{o_*} j)))}{A_*([\![\text{comma}]\!])([\![\text{john}]\!] : (j \multimap_{o_l} j \multimap_{o_*} j \multimap_{o_*} j))} \\
\frac{[\![\text{john}]\!] : j \quad [\![\text{comma}]\!] : j \multimap_{o_*} (j \multimap_{o_l} j \multimap_{o_*} j \multimap_{o_*} j)}{A_*([\![\text{comma}]\!])([\![\text{john}]\!] : (j \multimap_{o_l} j \multimap_{o_*} j \multimap_{o_*} j))}
\end{array}$$

$$\begin{array}{c}
\frac{[\![\text{also}]\!] : (d \multimap j \multimap l) \multimap_{o_*} d \multimap_{o_*} j \multimap_{o_*} l \quad [\![\text{likes}]\!] : d \multimap j \multimap l}{\frac{A_*([\![\text{also}]\!])([\![\text{likes}]\!] : d \multimap_{o_*} j \multimap_{o_*} l)}{A_*(A_*([\![\text{also}]\!])([\![\text{likes}]\!])([\![\text{dogs}]\!] : j \multimap_{o_*} l))} \quad \boxed{1} \frac{[\![\text{dogs}]\!] : d}{1}}{A_*([\![\text{also}]\!])([\![\text{likes}]\!])([\![\text{dogs}]\!])(A_*(A_*([\![\text{comma}]\!]([\![\text{john}]\!] \multimap_{o_l} j \multimap_{o_*} j \multimap_{o_*} j)))(A([\![\text{who}]\!])(A([\![\text{likes}]\!])([\![\text{cats}]\!] \multimap_{o_*} j))))} : l} \\
\frac{\langle \text{likes } j \text{ ix. dog}^*(x), \{\text{likes } j \text{ ix. cat}^*(x)\} \rangle : l}{\text{check} = 1}
\end{array}$$

Figure 1: Proof for John, who likes cats, likes dogs also



## 7 Conclusion

- The take home messages are as follows:
  1. Multidimensionality is necessary to capture the at-issue/side-issue distinction. Hiding multidimensionality in updates does not help.
  2. The interactions between the two dimensions are restricted — free interaction is not the solution.
  3. This means that we need enough structure to distinguish the different forms of interaction and to limit them.
  4. This additional structure cannot be effectively captured by conjoined terms in the logic of composition.
  5. We have proposed to use monads to simultaneously capture the multidimensionality and to provide enough structure to control interactions between dimensions.

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## A Monad Transformers

- We need a way to combine the monadic characterizations of multidimensionality and of the various phenomena that require interaction between the meaning dimensions.
- In general it is not possible to combine two monads  $(M_1, \eta_1, \star_1)$  and  $(M_2, \eta_2, \star_2)$  to get a third monad  $(M_1 \circ M_2, \eta_1 \circ \eta_2, \star_1 \circ \star_2)$ .
- The solution is to “lift” the monadic mappings to operate directly on informationally rich meaning spaces.
- From each monad we (mechanically) generate a *monad transformer*:
  - The monad transformer encapsulates the same type of computation performed by the original monad (writing/reading from a global state, generating a value in a non deterministic way, etc.).
  - However, rather than mapping from the value space (the informationally poor meaning space) to the monadic space, we create a mapping from another monadic (rich) space to the one representing the computation we are interested in.
  - Effectively, each monad transformer can be seen as a collection of monads distinguished by the monadic space from which they map.
- Monad transformers are monads; therefore their definition is given in terms of the standard operations  $\eta$  and  $\star$ .
- However, we also need an additional operation, usually called `lift` and with type  $M\ x \rightarrow MT\ M\ x$ , where  $M$  is the monad indexing the specific instance of the monad transformer  $MT$ . The function `lift` maps a specific instance of a monadic rich value to an even richer one in the space defined by the monad transformer.