Modelling Catastrophe Bond Pricing in the Primary Market - A Loglinear Approach

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Introduction

The 2017 hurricane season caused more economic damage than any other hurricane season in US history, with total losses above 200 billion USD. Hurricane Harvey flooded a third of the fourth largest city in the US, Houston, and more than 200,000 homes were damaged or destroyed. Hurricane Maria was the worst storm to hit Puerto Rico in over 80 years, leaving 75% of the impoverished US territory without power for at least six weeks [19]. Every year, natural catastrophes threaten communities and their respective economies around the globe.

Despite a growing awareness of the toll that catastrophes can take in high-risk regions, people are migrating to beachfront properties and cities in Florida, Texas, and California. While people flock towards the catastrophic risks, insurance companies exit. After Hurricane Katrina in 2005, All State, one of the largest insurers in the US, stopped writing homeowners' insurance policies in states along the Gulf Coast [4]. Catastrophic loss, by nature, is difficult to insure. Unlike automobile insurance, where claims generally are independent of each other and the annual aggregate of claims does not deviate substantially from expectations, insurance covering catastrophic loss face lump risks. One week, there could be near zero claims filed, and the next over 100,000. In order to limit exposure to catastrophic risk, insurance companies rely on reinsurers to bear part of the risk. However, reinsurers have a limited supply of capital, and are unable to bear the burden of the highest layers of insurance losses. Thus, insurers have remained exposed to events they seemingly cannot afford to cover, like Hurricane Katrina [6].

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As a result, the catastrophe bond market has emerged as a means for insurers and reinsurers to hedge against rare catastrophic events. A catastrophe bond ("cat" bond) is essentially a reinsurance contract sold in pieces to institutional investors. The benefits of a cat bond are two-fold. Insurers, or reinsurers, gain access to the capital markets as a means of financing their catastrophe exposures, and investors gain access to a previously unaccessible market with high risk-to-return and low beta. This "win-win" has helped establish the cat bond market as a major component of transferring catastrophic risk for insurance and reinsurance companies globally [2].

As shown by the Fama-French three factor model, an econometrically derived model for risk pricing can give insights to both scholarly and private sector pursuits. Since the inception of the cat bond market, a collective effort among private and academic researchers have laid the groundwork for empirical analysis of cat bond pricing. The earliest empirical study of cat bond prices by Lane (2000) analyzes cat bonds issued in 1999. He views the expected excess return, or the spread less the expected loss, as a power function of the conditional expected loss and the probability of first loss. However, after years of observing the cat bond market, Lane (2008), along with Mahul, use a multiple linear regression approach with the spread premium of cat bonds as the dependent variable and expected loss as the main explanatory variable. They abandon conditional expected loss as a determinant of spreads, since the variable does not vary greatly between cat bonds. They are also the first to establish cyclicality as a significant determinant of cat bond spreads, similar to the reinsurance cycle. The linear approach to cat bond pricing has been continued in most empirical works thereafter. Dieckmann (2009) examines a cross-section of secondary market cat bond prices around the occurrence of Hurricane Katrina via multiple linear regression to evaluate how prices are affected by large catastrophic events. Braun (2016) studies a complete data set of cat bond issuances from 1997 through 2012 to offer the most accurate linear model of cat bond pricing at issue to date. He finds the corporate junk bond cycle to be a determinant of cat bond prices, as well as the prominence of the sponsor of the cat bond. Galeotti, Guertler, and Winkelvos (2012) evaluate the different pricing models proposed in the literature using issuance data from 1999 through 2009 and determine a linear approach with expected loss as the main determinant of spread premium to be most appropriate.

While a linear approach has been the model of choice to examine cat bond pricing, there have been alternative methods brought forth in the literature that have merit. Major and Kreps (2003), analyzing traditional reinsurance prices, suggests a loglinear relationship between spread and expected loss. Wang (2004) proposes an entirely different method of relating expected loss to spread. Wang transforms the decumulative distribution function of loss using a Student's t-distribution, essentially relating a transformed version of expected loss to spread [8].

The empirical work to date has provided a great breadth of information on the pricing of cat bonds. Expected loss is solidified as the main determinant of cat bond spread premium, and variables for perils covered, regions covered, and cyclicality have shown to have significant impact on spread premiums in the majority of empirical studies. However, as with any young market, the research thus far has used a very limited amount of data. Thus, it is important to reevaluate which determinants are significant drivers of spreads with new data.

My contribution to the literature is as follows. I construct and analyze a data set of 519 cat bond tranches issued from 1999 through 2017. I establish that the loglinear approach to evaluating catastrophic risk pricing introduced by Major and Kreps (2003), rather than the linear approach that most empirical research has chosen, is more appropriate. Building upon the simple loglinear model, I develop a multivariate loglinear model that includes cyclical as well as cat bond-specific variables that have been established in the literature and are highly statistically significant. After developing the model using roughly the first-half of the data set, I use the remaining half for out-of-sample tests and comparison to previously introduced cat bond pricing models.

In the next section, I briefly review the history of reinsurance that led to the introduction of cat bonds. This is followed by an overview of the structure of cat bonds. Afterwards, the data is introduced, along with descriptive statistics to understand what I have collected. Given an understanding of the data I have collected, I discuss the construction of my econometric model. Penultimately, my model is compared to other models proposed in the literature through in-sample and out-of-sample tests. Finally, I summarize results and the paper is concluded.

A Brief History of Catastrophe Reinsurance

Reinsurance is, essentially, insurance for an insurance company. The first known reinsurance contract dates back to 1370 between two individual underwriters to reinsure a ship's cargo through part of its journey to Bruges [12]. According to Gerathewhol, "The treaty, written in Latin, concerned the cargo of a ship sailing from Genoa to Sluis (near Bruges in Flanders) for which the direct 'insurer' transferred the more hazardous part of the voyage from Cadiz (in Andalusia) to Sluis to another 'insurer' who thus provided 'reinsurance coverage.'" [10, qtd in.]

In 1842, one of the largest city fires in history engulfed a quarter of the city of Hamburg. The fire bankrupted the City Fire Fund, which was established after the London fire of 1666, and left many German insurance companies in ruins. Insurance companies became aware of overbearing fire risks throughout the industrial revolution but had not realized the inadequacy of their funds until the Hamburg fire. In response, the first independent reinsurance company, Cologne

Reinsurance, was established in 1846. The idea was good, but still needed to be fleshed out. During the next few decades, most reinsurers had trouble staying in business. "In the years 1871 to 1873, no less than twelve independent reinsurance institutions were founded in Germany, of which very few survive today (1929). The pressure of competition led to unwholesome practices, and soon many of these newly formed companies found themselves in dire straits." [12]

Fast-forward to the early 1990s, reinsurance and insurance evolved from the business of taking on the unknown risks of the high seas to one of calculated risk management. Insurance covers everything from automobiles to Egon Ronay's taste buds, and most companies (at least the ones that stay in business) are hyperaware of the risks they are taking. From the lessons of Hamburg, reinsurance became a vital parasite that protects insurers from catastrophic risks they simply cannot afford. In order to protect their own solvency, reinsurers diversify risks globally in what is known as risk pooling. Florida may get hit by a major hurricane, but it's very unlikely that major California earthquakes and a European pandemic occur in the same year. It is by no means guaranteed, but reinsurance is not a riskless venture.

Despite reinsurance capital providing somewhat of a safety net, insurance companies remained highly exposed to large catastrophic events. In fact, insurance companies were practically self-insured for events that cause industry losses above $5 billion. On top of that, the capital base for reinsurance companies worldwide made it impossible for reinsurance to cover a single large catastrophic event [6]. This was not a concern for insurance companies, however, since no large storms had occurred over the last 20 years. One of the few people to make note of insurance companies exposures to large catastrophic events was Karen Clark. Insurance companies assumed the largest event loss possible to be in the low billions, while her model was frequently generating losses greater than $30 billion. She preached to the "experts" to no avail, other than one insurance company that took note and funded her research [17]. The ship fared well in calm waters, so why question its structure.

The next few years upended the way the insurance industry treated catastrophe risk. In 1992, Hurricane Andrew struck South Florida, causing $15.5 billion in insured damages. In 1994, the Northridge Earthquake shook Los Angeles, causing $15.3 billion in insured damages. Suffice to say, Karen Clark and her research unit, Applied Insurance Research, started to get taken seriously. The industry bought her models and those of competitors, while facing the reality that they could not afford to self-insure the largest catastrophe risks in the world.

In California, earthquake insurance took a completely different form following 1994. Earthquakes no longer were covered by homeowners' insurance, and insurance companies fled the market. In 1996 (my birth year!), the California Earthquake Authority took on the role of providing earthquake insurance in California. Understanding the new norm for catastrophic risk, the CEA sought to pool its risk by using a new form of financing that makes use of capital markets, cat bonds. These securitizations of insurance provide capital to the insurance company following a triggering event, like the Northridge Earthquake. Unlike previous small cat bond issuances, the CEA sought over $1 billion in earthquake cover, a deal that would surely show that capital markets are more than willing to bear catastrophe risk. Before the deal could be signed, Berkshire Hathaway, one of the largest reinsurers in the world, stepped in and undercut the offer. This was unforeseen, as reinsurers typically do not expose themselves to this much risk from one insurer. They must have been aware that competition with capital markets could result in lower premiums for the whole reinsurance industry.[1]

Another company, USAA, had plans of utilizing the capital markets as a means of reinsurance. USAA is highly exposed to catastrophe risk, as there are many military personnel in coastal areas like Florida and California. In 1996, USAA formulated a plan with a few investment banks to construct a cat bond to cover the 1996 hurricane season. Due to regulatory hurdles and complications with the novelty and size of the deal, USAA completed its first cat bond sale in 1997 [7]. The deal, named Residential Re after the special purpose vehicle that sold the bonds, provided USAA with $500 million in additional US hurricane cover for one year. USAA's willingness to be at the forefront of the cat bond market would prove incredibly important to the continued growth and evolution of cat bonds. Since 1997, the cat bond market has grown exponentially to over $31 billion in total amount outstanding, and USAA has sponsored at least one cat bond each year.

Structure of Cat Bonds

Cat bonds are bonds structured to provide additional reinsurance coverage to insurance and reinsurance companies for catastrophic events like hurricanes and tsunamis. Unlike typical reinsurance, cat bonds tend to provide coverage for "tail" risk, or events that have less than a 1 in a 100 chance of happening. Due the nature of cat bonds, they were nicknamed "Act of God" bonds. While the cat bonds I intend to analyze only cover "acts of God," there are other cat bonds that cover life insurance risks for example.

Cat bonds are fairly complex. The sponsor, typically an insurance or reinsurance company, does not sell directly to the financial markets. In order to sell cat bonds, the sponsor needs to consult an investment bank(s) to create a special purpose vehicle (SPV). SPVs are fairly easy and cheap to form, and the structure of them benefits the sponsor. They are typically domiciled in the Cayman Islands or Bermuda to decrease regulatory supervision. There are also

[1] Berkshire Hathaway proceeded to buy General Re, the largest reinsurer in the US, in 1998 for $23.5 billion. General Re had merged with Cologne Re, the first reinsurance company, in 1994.

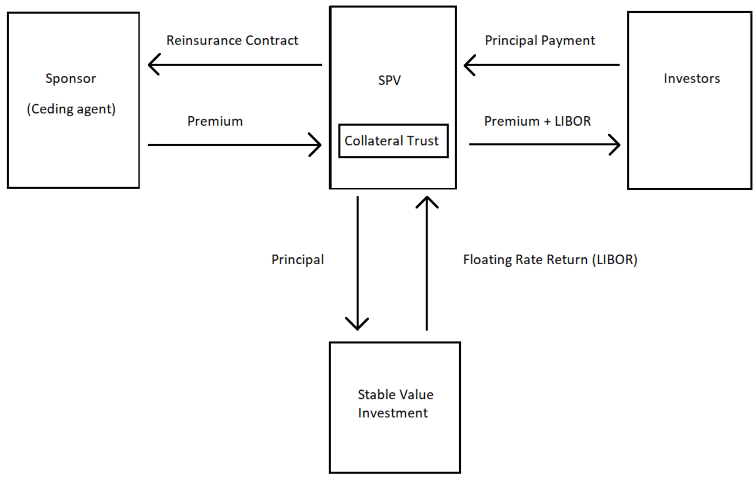


Figure 1: Cat Bond Structure

tax benefits, as assets of SPVs in these countries are exempt from direct taxation. Other benefits include the segregation of the sponsor from the SPV, so the credit quality of the sponsor is separate from that of the SPV [20].

After setting up the SPV, the sponsor enters into a reinsurance contract with the SPV. The sponsor agrees to send the spread premium plus a fixed rate to the SPV in exchange for a layer of reinsurance. This layer is defined by a few values. The "attachment point" is a specific amount or value, usually a dollar amount of losses, such that the contract is "triggered" and the SPV begins paying out to the sponsor. The "limit" is the size of the layer of reinsurance such that any amount exceeding the sum of the attachment point and the limit is no longer covered by the SPV. In cases where the principal can be completely lost, the sum of the attachment point and the limit is termed the "exhaustion point."

The SPV, in turn, sells a cat bond with the exact features of the reinsurance contract to capital market participants. The principal received from investors is invested in AAA-rated short-term securities and placed into a trust. Thus, unlike reinsurance, cat bonds are fully collateralized. The returns from the short-term investment are swapped for a LIBOR floating rate, and LIBOR plus the spread premium is paid to the investors. If the contract is triggered, the principal is at risk and the amount set forth by the contract depending on the event is taken from the trust and given to the sponsor. For most cat bonds, the entire principal is at risk.

While cat bonds are not standardized, there are many features that are common amongst them. A cat bond trigger defines how severe a catastrophic event must be in order for there to be potential losses of the principal. There are different ways to measure the severity of a catastrophic event. The most common triggers are indemnity, parametric, industry loss, and modelled loss. Indemnity triggers are linked directly to the sponsor's losses, so a contract would get triggered if the sponsor pays out claims above a certain amount. The benefit of using an indemnity trigger is mostly for the sponsor, as losses are truly covered as desired with no basis risk. Basis risk is risk that the trigger is not perfectly correlated with losses. A problem with the indemnity trigger is the wait required after an event that may breach the trigger. Claims are not received overnight, so investors sometimes must wait beyond the maturity date to learn what percentage of the principle will be returned to them. This is known as the loss adjustment period. Parametric triggers are linked to a measurable that is related to the catastrophe being covered. For example, a parametric trigger for hurricane coverage might be wind speeds at specified stations. Parametric triggers expose sponsors to basis risk, but are much more transparent to investors. Industry loss triggers are linked to industry losses due to the catastrophe in the region covered. Like indemnity triggered bonds, industry loss triggered bonds are susceptible to loss adjustment. Modelled loss triggers are linked to a model of industry losses or the sponsor's losses based on its exposures. The benefit of modelled loss is that there should, ideally, be little basis risk and the model is controlled by a third party (like AIR). The downside of a modelled loss trigger is model risk is introduced. Many cat bonds include hybrid triggers, or combinations of the common triggers [2].

Insurance companies are exposed to two major risks, exceptionally large catastrophes and an exceptional frequency of catastrophe. There are two basic types of cover to control these exposures, per-occurrence cover and aggregate cover. Per-occurrence cover provides the sponsor coverage for a single catastrophic event. For example, suppose a cat bond offers \$500 million in Florida Hurricane cover from a \$2 billion attachment point to a \$2.5 billion exhaustion point for Company X. If Hurricane Wonka causes \$2.4 billion in losses for Company X, investors will lose \$400 million of their principal investment. Notice that per-occurrence cover is pro-rated, such that if the bond is triggered the principal is lost in a linear fashion such that default occurs at the exhaustion point. This is the most common type of per-occurrence bond, however there are others such that the whole principal is not at risk or the bond only attaches after a second triggering event (or even a third or fourth!). Cat bonds that provide per-occurrence coverage often cover the most extreme tail risk for the sponsor.

Aggregate cover provides the sponsor cover after an accumulation of losses. For example, suppose a cat bond named "Ted Danson Re" provides aggregate cover for earthquakes near Ted Danson's house with a deductible of \$40 million, an attachment point of \$200 million, and an exhaustion point of \$400 million.

For every earthquake near Ted Danson's house that causes losses greater than $40 million, that amount is counted towards Ted Danson Re's running total of losses. Once this running total reaches the attachment point, the bond is triggered and investors can lose their principal. The most common periods for aggregation are annual aggregation and term aggregation. Annual aggregation simply means the running total gets reset to zero after each year, and term aggregation means the running total continues until the bond's maturity. Aggregate cover is popular for multi-peril cat bonds, that is bonds that cover more than one type of catastrophic event.

The maturity term of cat bonds is unlike that of a reinsurance contract. While reinsurance contracts are almost exclusively annual contracts, the typical cat bond is a three-year contract, with maturity terms ranging from one to five years. The benefit of selling cat bonds in multi-year layers is clear when looking at companies that release cat bonds regularly, like USAA. By selling multi-year bonds, USAA locks in rates early in a market of rising rates and smooths premium payments. Another possible explanation for multi-year cat bonds is investment banking costs are high, so it is a way to squeeze as much coverage as possible out of one deal [14].

Cat bonds are typically a complement to reinsurance. Reinsurers do not usually cover high layers of insurance losses since insurers cannot afford the cost of reinsurance at high layers and for events that triggers these high layers, solvency of the reinsurer comes into question. Since cat bonds are fully collateralized, there is essentially no credit risk involved (except during financial crises). Through the use of capital markets, insurers and reinsurers have found a way around the capital constraints of traditional markets.

Data Source

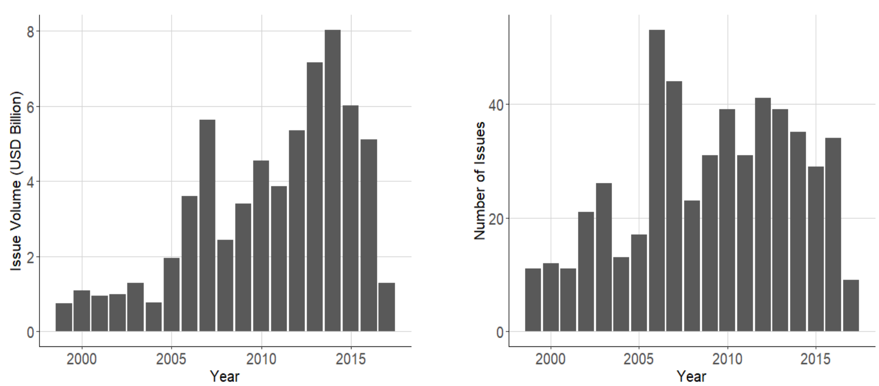
While empirical research on cat bond pricing is becoming more popular, it is still quite cumbersome to gather the data necessary to conduct such research. I must express gratitude towards Lane Financial LLC, as the project would be impossible without use of their annual publication. I combine information from their publications with the Artemis Deal Directory (artemis.bm), and cross-reference tables within these sources to create a consistent data set. In total, my data set comprises 576 cat bond tranches issued between January 1999 and March 2017. Of those 576 cat bonds, 57 are removed due to missing or erroneous fields.[2] Beyond cat bond specific data[3], other information is merged with the data set. The Lane Financial LLC Synthetic Rate on Line Index is used to capture the cyclical movements in the cat bond market. This index is calculated by evaluating cat bond and ILW (Industry Loss Warranty) prices in the secondary market with somewhat constant expected loss, thus trying to

[2]A description of the removal process can be found in the Appendix.
[3]A summary of cat bond-specific information collected can be found in the Appendix.

capture variations in the pricing of catastrophe risk in capital markets. Likewise, the Guy Carpenter Rate on Line Index is included as a reinsurance price index. The Guy Carpenter Index is published on an annual basis, and is intended to evaluate changes in pricing of global catastrophe risk in the reinsurance market. In Braun (2016), it is shown that corporate bond spreads of similar rating have an effect on cat bond spread premiums. Thus, like Braun, I include the quarterly average of the Bank of America Merrill Lynch U.S. High Yield BB Option-Adjusted Spread as obtained from FRED.

Descriptive Statistics

The cat bond market is a young and growing market. As of 2017, there is slightly more than 31 billion USD in cat bonds outstanding. This represents a compounded annual growth rate of 20.2% since 1997 (not adjusting for inflation), a little over double the growth rate of the high yield corporate bond market.[4] Figure 2 shows the annual issue volume in our data set of cat bond issues. In Figure 2(b), the spike in issue volume in 2006 is due to the issue of the Successor series, which is composed of 28 cat bond tranches. In Figure 2(a), there is a clear deviation from the upwards trend in total issuance amount in 2008, which is due to the global financial crisis. Credit risk is limited in cat bond investment since the principal is fully collateralized. However, four cat bonds suffered losses due to the Lehman Brothers default [16]. This represented a minor dip in investor confidence, as the market recovered in the following years.



(a) USD Volume

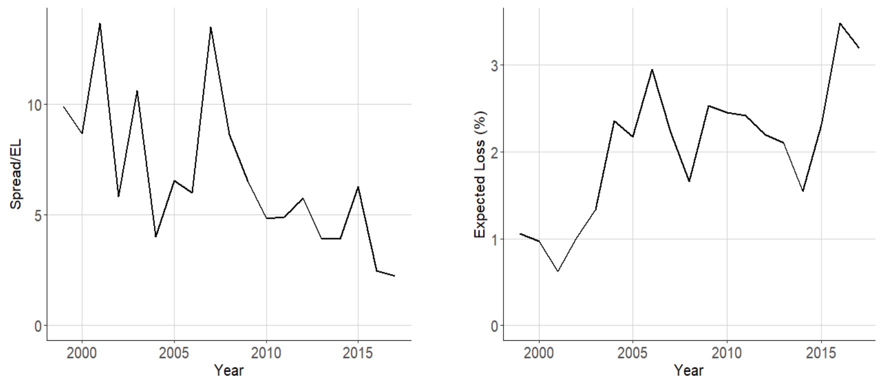
(b) Issue Volume

Figure 2: Cat Bond Issuance

Figure 3(a) and Figure 3(b) show the average ratio of spread premium to expected loss (Spread/EL) and average expected loss for each year, respectively. Average Spread/EL is on a downward trend, with a spike following Hurricane

[4]However, the high yield corporate bond market has over two trillion USD outstanding.

Katrina. High prices following large catastrophic events is consistent with reinsurance cylicality. The downward trend of Spread/EL is due to the increasing trend in expected loss as well as the maturation of the cat bond market. As the market matures and prices fall, insurance and reinsurance companies are more willing to supplement reinsurance of non-tail catastrophe risk with cat bonds. Capital markets are more able to handle the asymmetry of catastrophe risk, and are willing since high-return catastrophe risk offers diversification benefits [11]. Thus, the decreasing trend in cat bond pricing could continue, albeit at a decreasing rate.



(a) Average Spread/Expected Loss

(b) Average Expected Loss

Figure 3: Trends in Cat Bond Pricing

Table 1 shows summary statistics of important cat bond characteristics across the entirety of the data set of cat bond issues. The average spread is 788.2 basis points, and the average expected loss is 2.187%. Both spread and expected loss vary greatly across bonds. The lowest spread observed is 65 basis points while the highest is 4920 basis points. Expected loss varies between 0.007% and 13.06%. Most cat bonds are closer to average, with spread premiums typically between 400 and 1000 basis points, and expected loss between 0.5% and 3%. While the average ratio of spread to expected loss (Spread/EL) is 6.543, this is skewed by the extreme. Most cat bonds in the data set have a spread between 2.5 and 7.0 times expected loss. On average, the size of a cat bond tranche is 123.717 million USD, however cat bond tranches range in size from 1.8 to 1500 million USD. Most cat bonds range from two to five years of risk exposure, but there are some that mimic typical reinsurance exposure terms of one year or less.

While spread and expected loss clearly vary across the sample of cat bonds, these characteristics are in part dependent upon the risk factors of the cat bond. Table 2 breaks down the sample of cat bonds by certain relevant exposures. The majority of cat bonds, roughly 81%, are exposed to US catastrophic events. The average spread of cat bonds exposed to US risks is 846.525 basis points, significantly higher than the average spread of cat bonds not exposed to US risks,

Table 1: Summary Statistics for Cat bond Data Set

	Mean	St. Dev.	Min	$25^{th} Pctl.$	Median	$75^{th} Pctl.$	Max
Spread (bp)	788.207	537.55	65.00	450.00	635.00	1,000.00	4,920.00
EL (%)	2.187	2.106	0.007	0.855	1.370	2.840	13.06
Spread/EL	6.543	13.061	1.396	2.926	4.162	6.408	175.00
Size (USD MM)	123.717	117.955	1.80	50.00	100.00	160.00	1,500.00
Exposure Term (months)	35.888	11.88	5.00	35.50	36.00	44.50	61.00

540.798 basis points. Likewise, the average ratio of spread to expected loss is 6.889 for cat bonds exposed to US risks and 5.077 for cat bonds not exposed to US risks.

Two major catastrophic perils that are covered by insurance companies are US hurricane and Japanese earthquake. As such, the majority of cat bonds, about 65%, cover US hurricane risks. About 14% of cat bonds cover Japanese earthquake, and 27.4% cover neither US hurricane nor Japanese earthquake. Cat bonds that cover both US hurricane and Japanese earthquake have issue sizes substantially lower than the average cat bond. Cat bonds that cover US hurricane and Japanese earthquake have an average spread of 1384.576 basis points, while cat bonds that cover neither risk have an average spread of 512.458 basis points. Likewise, spreads of cat bonds that cover both risks are about 11 times expected loss, while around 7.5 times expected loss for cat bonds that cover neither risk. Cat bonds that only cover Japanese earthquake have a similar average expected loss to cat bonds that cover neither US hurricane nor Japanese earthquake, yet have a higher ratio of spread to expected loss. Cat bond spreads seem to be dependent upon the type of perils the bond covers.

Trigger type is important to both the investors and the sponsor. An indemnity trigger is the most popular trigger of cat bonds in the data set. Roughly 35% of cat bonds have an indemnity trigger, 29% have an industry loss trigger, 23% have a parametric-based trigger, and 13% have either multiple trigger types or another trigger type. Cat bonds with parametric triggers have substantially lower issue sizes on average than cat bonds with industry loss or indemnity triggers. This is due to indemnity triggers becoming more popular in the later years of the data set, as well as indemnity triggers being the trigger of choice for one of the larger annual cat bond issues, Residential Re. The average expected loss of cat bonds with parametric-based and indemnity triggers are similar, 1.975% and 1.837% respectively. However, the ratio of spread to expected loss is 6.55 for cat bonds with parametric-based triggers and about 8.03 for cat bonds with indemnity triggers. This is consistent with the idea that investors care about the transparency of risks in the cat bond market. Industry loss triggers and "other" triggers exhibit the highest spread and expected loss on average. Cat bonds with industry loss triggers have an average spread of 897.71 basis points and an average expected loss of 2.394%, while cat bonds with "other" triggers have an average spread of 1072.955 basis points and an expected loss of 5.739%.

Table 2: Descriptive Statistics for Cat Bonds

	No.	Percent (%)	Spread (bp)	Median Spread	EL (%)	Spread/EL	Size (USD MM)
Exposure							
US	420	80.925	846.525	697.5	2.294	6.889	122.746
Other	99	19.075	540.798	450	1.735	5.077	127.833
Peril							
US Hurricane	304	58.574	891.916	750	2.373	5.177	126.659
Japan EQ	40	7.707	486.925	400	1.260	9.553	136.257
Both	33	6.358	1,384.576	1,350	4.413	11.064	54.476
Neither	142	27.360	512.458	475	1.533	7.570	129.977
Trigger							
Parametric	119	22.929	666.861	550	1.975	6.550	78.097
Industry	150	28.902	897.710	775	2.394	5.070	134.602
Indemnity	184	35.453	675.280	600	1.837	8.029	159.642
Other	66	12.717	1,072.955	955	3.074	5.739	81.074

Modelling Cat Bond Spreads

The inverse relationship between the ratio of spread premium to expected loss (Spread/EL) and expected loss has been noted in early empirical research on reinsurance pricing. In 2001, Froot analyzes 25 years of reinsurance contract data from Guy Carpenter and broke down Spread/EL by probability of exhaustion, noticing higher Spread/EL for lower probabilities of exhaustion. To model the relationship, Major and Kreps (2003) chooses to use a power function between spread premium and expected loss.

$$Spread_i = \alpha EL_i^{\beta} + \epsilon_i \qquad \text{or}$$
$$ln(Spread_i) = \alpha + \beta ln(EL_i) + \epsilon_i \qquad (1)$$

Lane (2000) takes a different approach, in one of the first empirical studies on cat bonds, by separating expected loss into a product of the probability of first loss (PFL) and the conditional expected loss (CEL). Probability of first loss can be considered the frequency of loss while conditional expected loss is the severity of loss. He considers the excess expected return (EER), or spread premium less the expected loss, to be related to probability of first loss and conditional expected loss by a power function.[5]

$$EER_i = \alpha PFL_i^{\beta_1} CEL_i^{\beta_2} + \epsilon_i \qquad \text{or}$$
$$ln(EER_i) = \alpha + \beta_1 ln(PFL_i) + \beta_2 ln(CEL_i) + \epsilon_i \tag{2}$$

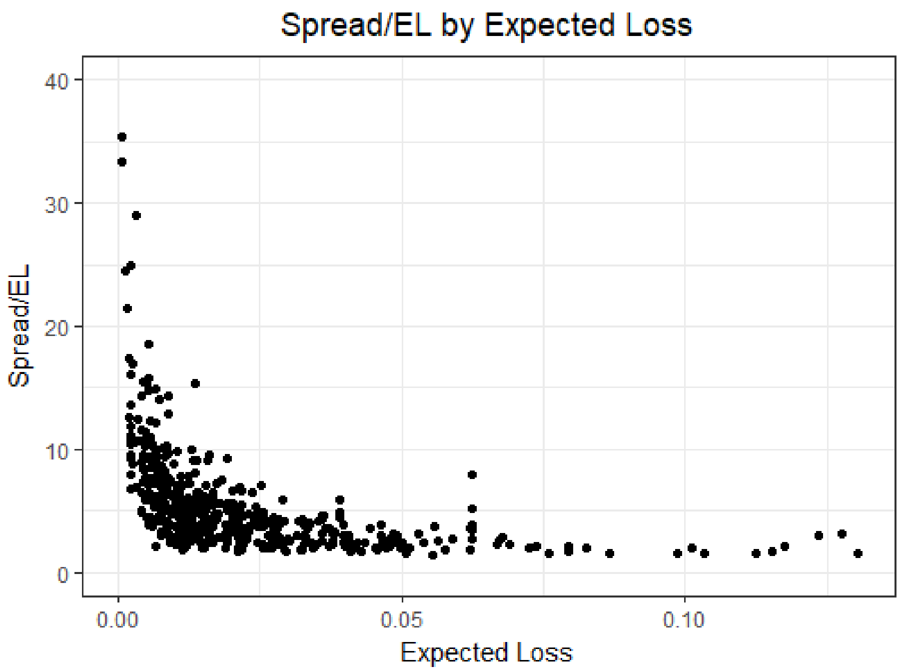


Figure 4

The relationship these pioneers of empirical research in catastrophe risk pricing observed is prevalent in the data. As shown in Figure 4, the Spread/EL increases in an increasing fashion as expected loss decreases. As Galeotti et al (2012) recommends, a simple linear model (3) can fit this data. Supposing the constant term is positive, as expected loss decreases towards zero, Spread/EL increases to infinity. In (1), the simple loglinear model, this same relationship is possible if $\beta < 1$. Thus, further testing of the data and theoretical questioning is required to determine whether a linear or loglinear model is best to represent the data.

$$Spread = \alpha + \beta EL + \epsilon \tag{3}$$

In order to stray from the linear model, there needs to be proof that the loglinear model at least has the potential to better fit the data. I begin by running a simple linear regression and loglinear regression on the full sample of

[5] It is important to note that Lane abandons this idea in 2008, deciding rather to price cat bonds at issue via a linear model.

Table 3: Estimates of Linear and Loglinear Beta by Expected Loss Range

	N	Average Linear Beta	Average Loglinear Beta
0% to 0.5%	46	-1.63	0.57
0.5% to 0.75%	51	2.28	0.57
0.75% to 1%	49	1.85	0.61
1% to 1.2%	39	1.50	0.63
1.2% to 1.4%	73	2.11	0.61
1.4% to 1.8%	47	2.22	0.61
1.8% to 2.5%	52	2.31	0.61
2.5% to 3.5%	53	1.90	0.64
3.5% to 5%	58	2.05	0.62
≥5%	42	2.02	0.61

Notes: Nine observations were removed due to $\frac{Spread}{EL} \geq 30$, as to prevent the argument that the negative β estimate is skewed by a few outliers.

cat bonds. Intuitively, if the data is described by the simple linear model, we would expect that $\frac{Spread-\alpha\hat{Lin}}{EL}$ is a constant independent of the expected loss. Likewise, the loglinear model suggests that $\frac{ln(Spread)-\alpha\hat{Log}}{ln(EL)}$ is a constant independent of expected loss. Table 3 shows that while the loglinear estimate for β is fairly constant for all values of expected loss, the linear estimate is severely off for expected loss lower than 0.5%. Thus, it seems like the loglinear model creates a better overall fit to the data.

The benefit of a linear model is transparency. The coefficient, "λ", of a variable is easily understood as a one unit increase in the variable results in an increase in the spread premium of "λ". This implies that outside of the main driver of spread premiums, expected loss, there are some variables that simply increase or decrease spread premiums independent of expected loss. Let's look at a linear model that includes expected loss and a dummy variable for exposure to Florida hurricane as explanatory variables.

$$Spread_i = \alpha + \beta_1 EL_i + \beta_2 FL\_Hurr_i + \epsilon_i \tag{4}$$

Similar to the logic before, we would expect β_2 in the above model to be constant and independent of expected loss. However, as shown in Table 4, the estimate of the coefficient of the FL_Hurr dummy variable increases with expected loss, and is most evident in the outer 20th percentiles. This relationship between expected loss and another variable can only be captured in a linear model by including further variables and interaction terms, which decreases the transparency of the linear model and leads to over-fitting. While a linear model struggles with this relationship, a loglinear model (5) can handle it. As expected loss increases, so does the effect of the dummy variable FL_Hurr. Dummy variables in a loglinear model have a "multiplier" effect, which seems appropriate for explaining the effect of variables like FL_Hurr on the spread

Table 4: Estimating β_2 by Expected Loss Range

	FL_Hurr	$\neg$ FL_Hurr	β_2 Estimate
0% to 0.75%	36	61	154.42
0.75% to 1.2%	41	47	216.69
1.2% to 1.8%	65	55	203.62
1.8% to 3.5%	73	32	156.93
>3.5%	76	24	322.88

Notes: 13 observations were removed as outliers as in Table 1.

premium of cat bonds.

$$Spread = \alpha EL^{\beta_1} e^{\beta_2 FL\_Hurr} + \epsilon \tag{5}$$

Developing the Model

While I believe the appropriate model to explain the spread premium of cat bonds is an extension of the Kreps and Major (2003) model, empirical research in more recent years has shed light on possibly good explanatory variables. My goal is not to find every variable that can be added to my model with statistical significance, but rather to create a parsimonious model that is stable in out-of-sample tests and has higher explanatory power out-of-sample than a simple log-linear approach.[6]

In beginning to extend the simple loglinear model, I first look to macro factors that should theoretically have an effect on cat bond pricing. One obvious macro variable that should affect cat bond prices is reinsurance market prices. Cat bonds are essentially a reinsurance vehicle for sponsors, so it would make sense for prices to correspond with those in the reinsurance market. In Froot (2001), he discusses the possible explanations for the reinsurance cycle which include restrictions of the supply of capital to catastrophe risk due to market imperfections. Ideally, the effect of reinsurance cycles on the cat bond market will decline (along with reinsurance cycles themselves), but that is a long-run expectation. Thus, it makes sense to include some sort of variable to reflect the cyclicality of the catastrophic risk market.

A couple of indexes have been used in empirical research for this purpose. One is an annual index created by the reinsurance company Guy Carpenter. It measures the change in rate-on-line, the annual premium received by the reinsurance company divided by the limit of catastrophe coverage, to measure the change in global catastrophe reinsurance prices. Gürtler et al (2016) make use of the Guy Carpenter Index to test the effect of the reinsurance cycle on cat bond spread premiums in a panel-data model. The other index is a quarterly index

[6]Galeotti et al (2012) find loglinear models with multiple regressors perform significantly worse than the simple loglinear model (1) in out-of-sample tests.

created by Lane Financial LLC, called the Synthetic Rate on Line Index, which is based on the average spread premiums for cat bonds and ILWs (Industry Loss Warrants) given a constant expected loss. Lane and Mahul (2008) introduced the use of this index in modelling cat bond spread premiums to control for cyclic effects in the cat bond market. Since this study, it has been used in multiple papers for the same purpose, including Braun (2016) which further refines the linear model. The problem with using the Synthetic Rate On Line Index is that it will not help determine how cat bond pricing is related to the reinsurance cycle as the index represents capital market financed catastrophe risk pricing, of which a significant portion are cat bonds. Thus, it more so measures the cat bond pricing cycle. However, while it is not useful in determining the impact of the reinsurance cycle, it can help determine the impact of capital market financed catastrophe risk pricing cycles on cat bonds, which may or may not correspond with the reinsurance cycle.

As Lane (2008) suggests, it makes sense to use the Lane Synthetic Index in the model to control for time-variant cyclical effects. Thus, the lagged index value of the Lane Financial Synthetic Rate on Line Index is included in the pricing model. Using the lagged index value rather than the in-quarter index value is important, as the index value is reported at the end of the quarter and is thus not known during the quarter reported. Simply put, investors use the latest information available to them, not future information. Table 5 shows how the loglinear model's explanatory power improves with each iteration.

$$ln(Spread_i) = \alpha + \beta_1 ln(EL_i) + \beta_2 ln(Lane\_Index_i) + \epsilon_i \tag{6}$$

After accounting for the undisputed main driver of spread premiums, expected loss, and any cyclical effects, further bond-specific information needs to be considered. More specifically, it seems appropriate that a spread premium is partly driven by what it covers. In Lane and Mahul (2008), a linear model is applied that accounts for both cyclic effects and peril-specific expected loss. They note a high Spread/EL ratio for US hurricane and a low Spread/EL ratio for off-peak perils, like Japanese typhoon, after accounting for cyclic effects. Likewise, Braun (2016) finds off-peak perils are priced significantly lower than peak perils. Peak perils that have caused the most catastrophic damage and are most prevalent in the cat bond market are US hurricane, US earthquake, and Japanese earthquake. To account for this, I create a dummy variable for exposure to US perils, one for exposure to US hurricane, and another for exposure to Japanese earthquake. These exposures account for a significant percentage of the spread.

$$\begin{aligned} ln(Spread_i) =& \alpha + \beta_1 ln(EL_i) + \beta_2 ln(Lane\_Index_i) + \\ & \beta_3 US\_Exposure_i + \beta_4 Hurr_i + \beta_5 JPN\_EQ_i + \epsilon_i \end{aligned} \tag{7}$$

In order for a cat bond to begin paying out to the sponsor following a catastrophic event, the cat bond needs to be triggered. All triggers are not made equal, both for the investors and the sponsors. Sponsors prefer cat bonds with

Table 5: Results of Model Improvements

	Dependent variable:			
	ln(Spread)			
	(1)	(6)	(7)	(8)
	LogEL		Mev2	Mev
Constant	9.226***	8.899***	8.03***	8.272***
	(0.076)	(0.094)	(0.106)	(0.112)
ln(EL)	0.624***	0.586***	0.467***	0.496***
	(0.022)	(0.022)	(0.019)	(0.019)
ln(Lane_Index)		0.411***	0.564***	0.451***
		(0.081)	(0.065)	(0.066)
US_Exposed			0.269***	0.205***
			(0.055)	(0.055)
Hurr			0.199***	0.191***
			(0.044)	(0.043)
JPN_EQ			0.162***	0.148***
			(0.034)	(0.033)
Parametric				−0.146***
				(0.032)
Observations	249	249	249	249
R^2	0.756	0.779	0.863	0.875
Adjusted R^2	0.754	0.776	0.860	0.871
Residual Std. Error	278.6	265.8	210.1	201.5
Note:				*p<0.1; **p<0.05; ***p<0.01

less basis risk, like indemnity triggers. Investors prefer cat bonds with more transparency. Cat bonds with indemnity triggers are less transparent since investors require knowledge of the sponsor's portfolio of risks, which is highly complex [2]. Cat bonds with parametric triggers, be that a parametric index or a pure parametric trigger, offer the most transparency to investors compared with other triggers. Investors need only know the probability distribution of wind speeds at a few wind stations, for example. Therefore, it seems to follow that investors would require a lower return for cat bonds with a parametric trigger, as the probability distribution of losses is more certain.[7]

$$\begin{aligned} ln(Spread_i) =& \alpha + \beta_1 ln(EL_i) + \beta_2 ln(Lane\_Index_i) + \beta_3 US\_Exposure_i + \\ & \beta_4 Hurr_i + \beta_5 JPN\_EQ_i + \beta_6 Parametric_i + \epsilon_i \end{aligned} \tag{8}$$

The results of the loglinear models on the sample of cat bonds with issue dates prior to April 2010 and with outliers removed is shown in Table 5. Parameters are calculated using a nonlinear least squares approach. This is necessary since using ordinary least squares (OLS) on a loglinear model with $ln(Spread)$ as the dependent variable minimizes square residuals relative to $ln(Spread)$ rather than *Spread*, which results in model bias.

To summarize this section, while a linear model has been the choice of most empirical research in the last decade, I believe a loglinear model better fits the data. Despite diverting from the aforementioned linear model, there are many insights into the common drivers of cat bond spreads in the literature. Of these possible variables, I have chosen expected loss, the Lane Synthetic Rate-on-Line Index, a dummy variable for exposure to US catastrophes, a dummy variable for exposure to US hurricanes, a dummy variable for exposure to Japanese earthquakes, and a dummy variable for a parametric trigger as explanatory variables in my loglinear model.

Model Comparison

Fitting a pricing model to the entirety of the sample is tempting since the sample size is not large. Including the entirety of the sample allows for parameters to overcome the effects of time-variation, which is a factor in modelling cat bond spreads [1]. Considering the cat bond market is young, there are certainly a variety of variables that have at one point or another been statistically significant predictors of spread premiums. For example, prior to 2008, it was considered that cat bonds with indemnity triggers had significantly higher spread premiums. Dieckmann (2008) made note of this discretion, and had no explanation for it beyond that the investors are faced with a problem that mirrors the moral hazard problem of reinsurers. Perhaps as a result of the disfavor, cat bond issues with indemnity triggers were low each and every year

[7] Galeotti et al. (2012) also found an inverse relationship between the parametric trigger and spread premiums.

prior to 2010. However, since 2011, cat bonds with indemnity triggers account for more than half the cat bond issues. Interestingly enough, the indemnity trigger premium has become insignificant. There is merit in trying to explain the economic significance of all variables with statistically significant relation to spreads. However, this is not the goal of this paper. Rather, the goal of my model is to offer stable and consistent explanation of spread premiums. This is why out-of-sample tests are highly important to this paper.

As mentioned previously in regards to Table 5, my model is developed using data before April 2010. This is to allow roughly half of the sample to be used for out-of-sample tests. Despite desire to use the entirety of the sample up to April 2010 for in-sample tests, some observations are removed due to their potential for heavy influence on the model. These observations are identified prior to in-sample testing by plotting studentized residuals of a full model against the Cook's D value as described in Braun (2016).[8]

For comparison, I include models that have proven successful in the literature. These include the simple linear model (3), the simple loglinear model (1), Braun (2016) linear model (8), and the Wang (2004) transformation model (9). For comparison, I include a linear version of my model, which will be called L_Mevorach.

- L_Mevorach

$$\begin{aligned} Spread_i =& \alpha + \beta_1 EL_i + \beta_2 Lane\_Index_i + \beta_3 US\_Exposure_i + \\ & \beta_4 Hurr_i + \beta_5 JPN\_EQ_i + \beta_6 Parametric_i + \epsilon_i \end{aligned} \tag{9}$$

- Braun (2016) model is a linear model that includes a dummy variable for exposure to peak territory, a dummy variable for being sponsored by Swiss Re, and a dummy variable for being rated investment grade. The model contains no intercept because theoretically if these variables proxied for cat bond risk, then all variables equalling zero would correspond to the risk-free rate. It is important to note that while the variables Swiss_Re and Inv_Grade seemed relevant within Braun's sample, since 2013 there have only been three cat bonds sponsored by Swiss Re and only one investment grade cat bond.[9]

$$\begin{aligned} Spread_i =& \beta_1 EL_i + \beta_2 Peak_i + \beta_2 Swiss\_Re_i + \beta_3 Inv\_Grade_i + \\ & \beta_4 Lane\_Index_i + \beta_5 BB\_Spread_i + \epsilon_i \end{aligned} \tag{10}$$

[8]For further information regarding this method, see the Appendix.

[9]Upon attempting to fit the best linear model to the in-sample data, I arrive at a result very similar to the Braun model. Thus, the Braun model can be thought of as the linear model of best fit, while my model is the loglinear model of best fit.

– Wang (2004) builds off of his previous work (1996, 2000) of defining a relationship between the transformed distribution of loss and spread premium. In his latest model, Wang replaces the normal distribution with a Student's t distribution to better represent cat bond risk. In Galeotti et al (2012), this model is approximated using the data available. I replace probability of last loss with probability of exhaust, as they are essentially identical. F_k is the Student's t distribution with k degrees of freedom. Φ is the standard normal distribution ($N(0,1)$). k and λ are the parameters of the model. The approximation of Wang's model is below.

$$\begin{aligned} Spread_i = & \frac{1}{2}[F_k(\Phi^{-1}(Prob\_1st\_Loss_i) + \lambda) + \\ & F_k(\Phi^{-1}(Prob\_Exhaust_i) + \lambda)] + \epsilon_i \end{aligned} \tag{11}$$

In-Sample Test

The in-sample estimations are done using the sample of 249 cat bonds (outliers removed as mentioned previously) with issue dates prior to April 2010. R^2 and Residual Squared Errors are recalculated to be comparable between models. The results are shown in Tables 5 and 6. The constant term is highly significant and positive in the simple linear regression, while less significant and negative for L_Mevorach. This might be due to the constant converging to zero as cat bond risk variability explained by the model increases, thus giving good reason for Braun's omission of the constant term. In the L_Mevorach model, the dummy variable for exposure to Japanese earthquake is insignificant, which is odd considering the same variable is highly significant in the loglinear model. The signs and magnitude of the parameters for all of the models are as expected.

Mevorach and Mevorach2 explain the greatest portion of the variance of cat bond spreads among the models, with Adjusted R^2 of 87.1% and 86.0% respectively. The multivariate linear models, L_Mevorach and Braun, explain slightly less of the variance with Adjusted R^2 of 84.9% and 84.0% respectively. It is interesting that the multivariate linear models explain less of the variance than the multivariate loglinear models, since the simple linear model performs better than the simple loglinear model in the in-sample test. This sheds light on the advantage of the loglinear model over the linear model in multivariate modelling of cat bond spread premiums.

Out-of-Sample Test

While in-sample performance shows marginal differences between four of the seven models, model performance is best tested out-of-sample. If a dynamic is unexplained by a model, the effect of that dynamic can be averaged out over the parameters to decrease the impact of the dynamic. On the other side, if a model over-fits the data, extraneous parameters can be highly variable with the

Table 6: In-Sample Results

	(3) EL	(10) Braun	(11) Wang	(9) L_Mev
Constant	359.085***			−46.174
	(23.544)			(80.542)
EL	2.273***	2.238***		2.099***
	(0.078)	(0.075)		(0.072)
Lane_Index		1.381***		2.605***
		(0.346)		(0.533)
US_Exposed				97.840**
				(41.284)
Hurr				150.051***
				(35.531)
JPN_EQ				49.640
				(37.389)
Parametric				−112.773***
				(32.447)
Peak		175.125***		
		(33.658)		
Swiss_Re		−95.856***		
		(29.795)		
IG		−187.945***		
		(51.653)		
BB_Spread		0.316***		
		(0.068)		
$\{\lambda$,k$\}$			$\{7, 0.277\}$	
Observations	249	249	249	249
R^2	0.773	0.844	0.727	0.853
Adjusted R^2	0.772	0.840	0.725	0.849
RSE	268.8	224.5	294.8	218.8
Note:	*p<0.1; **p<0.05; ***p<0.01			

inclusion or exclusion of observations. Under-fitting, over-fitting, and temporal dependence is more clear in out-of-sample testing. The out-of-sample testing involves fitting a model to a data set of cat bonds prior to a date, and using the model parameters to predict cat bond spread premiums on a subset of cat bonds after the specified date. Due to the function of time, this is the longest out-of-sample testing period in the literature.

A few measures will be used to compare out-of-sample fit of the seven models. Mean absolute error (MAE) is the average absolute difference between the actual spread premium and the model prediction. Mean absolute percentage error (MAPE) is the average absolute percentage difference between the actual spread premium and the model prediction. Median absolute percentage error (MEAPE) is simply the median of this statistic as opposed to the average. Out-of-sample R^2 (R^2_{OS}) compares the variance of model prediction with the variance of spread premium in relation to the in-sample mean. If R^2_{OS} is negative, it implies the in-sample mean is a better predictor of spread premiums than the model.

– Mean absolute error (MAE):

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|Spread_i - Spr\hat{e}ad_i\right|$$

– Mean absolute percentage error (MAPE):

$$MAPE = \frac{100}{N}\sum_{i=1}^{N}\frac{\left|Spread_i - Spr\hat{e}ad_i\right|}{Spread_i}$$

– Median absolute percentage error (MEAPE):

$$MEAPE = median(\{\frac{\left|Spread_i - Spr\hat{e}ad_i\right|}{Spread}\}_{i=1}^{N})$$

– Out-of-Sample R^2 (R^2_{OS}):

$$R^2_{OS} = 1 - \frac{\sum_{i=1}^{N}(Spread_i - Spr\hat{e}ad_i)^2}{\sum_{i=1}^{N}(Spread_i - \overline{Spread_{IS}})^2}$$

Table 7 shows the out-of-sample testing of my model (Mevorach/Mev) and the six alternatives. The models are fit using the sub-sample of 249 cat bonds issued from March 1999 to March 2010. The parameters of the models are then applied to the 253 cat bonds in our sample that were issued between April 2010 and March 2017 to predict cat bonds spread premium. These predictions are compared to actual spreads premiums to calculate the statistics in the table.

Reviewing Table 7, the simple linear model as well as the models of Braun and Wang seem to perform the worst with the three highest values of MAE and MAPE, and lowest R^2_{OS}. While Braun's model only performs slightly better than the simple linear model, this is due to two of its explanatory variables, sponsored by Swiss Re and rated investment grade, becoming irrelevant over the out-of-sample period. Another result to note is the simple loglinear model performs better by all measures than the simple linear model despite worse explanatory power in the in-sample test of Tables 5 and 6. This is counter to previous out-of-sample tests in the literature. Perhaps, previous out-of-sample tests did not reach this result because of out-of sample testing limited to three years or less of primary market cat bond data.

Table 7: Out-of-Sample Performance from April 2010 to March 2017

	MAE	MAPE	MEAPE	R^2OS
LogEL	276.213	46.221	39.061	0.277
Mev	228.524	39.525	34.710	0.568
Mev2	186.875	32.586	26.969	0.705
L_Mev	245.827	40.349	36.584	0.410
EL	268.800	45.943	38.744	0.278
Braun	264.236	44.172	42.071	0.285
Wang	271.784	47.430	39.101	0.327

The most interesting comparison is between Mevorach, Mevorach2, and L_Mevorach. All parameters included in the Mevorach model are highly statistically significant, as well as theoretically justified. However, the Mevorach2 model, which removes the dummy variable for a parametric trigger, performs better in out-of-sample testing, which is consistent with triggers becoming less imporant as the cat bond market matures. The difference between the Mevorach model and the L_Mevorach model highlights the difference between the multivariate linear and loglinear models. These models include the same parameters. However, out-of-sample testing separates these two models. The Mevorach model outperforms the L_Mevorach model by every measure, including MEAPE. MEAPE is important to include since it could be argued that the Mevorach model simply fits outliers better than the L_Mevorach model, which would skew any statistic involving an average upwards for the L_Mevorach model. The median is unresponsive to outliers. Thus, the Mevorach model exhibits closer fit to out-of-sample cat bond data than the L_Mevorach model in all respects.

A possible explanation for the Mevorach model's outperformance in out-of-sample tests is that seven years is simply too long for out-of-sample tests. Perhaps, other models would perform similarly with shorter out-of-sample tests and during different time periods. Figures 2 and 3 intend to address this question. Each model is fit using cat bond data from March 1999 through March of 20xx. The model parameters obtained from in-sample tests are then used to predict cat bond spread premiums for the next three years. The statistics for comparing out-of-sample model performance are reported in the four charts of Figures 6 and 7.[10] These tests are repeated for in-sample data up to 2010, 2011, 2012, 2013, and 2014.

The results from Table 6 hold up in these new out-of-sample tests. The loglinear models of Mevorach and Mevorach2 have a relatively stable R^2_{OS} of approximately 80% for each test while the other models lose their predictive power with time. Based on Figure 6(b), all models have similar average error when using data up to 2010. However, as in-sample data includes later years and the models attempt to predict three years of cat bond issue spreads, the average error of the simple loglinear model, the simple linear model, the Braun model, and the Wang model drift towards 300+ basis points. The L_Mevorach model separates from the Mevorach model as well, with a difference between their average errors of over 50 basis points. The Mevorach2 model proves to be the most accurate model in out-of-sample testing.

It is worth noting that MAPE has an increasing trend for all models, which suggests that there are new factors that the models are not accounting for, or relationships between model explanatory variables and spread is changing over time. The latter possibility has been noted by Braun (2016) and others in the literature.

To account for this, Table 8 has adjusted the in-sample period to be eight years for each test. In testing April 2014 through March 2017, the in-sample period is April 2006 through March 2014. Thus, stale relationships between spread and the explanatory variables are ignored, so models can update to recent data more easily. Like in Figures 6 and 7, the Mevorach and Mevorach2 models perform best. In fact, the models improve in explanatory power as time moves forward. The R^2_{OS} of the Mevorach and Mevorach2 models for the last out-of-sample test with an in-sample period of April 2006 through March 2014 is 94.2% and 94.1% respectively. The linear models of Braun and L_Mevorach perform similarly to the Mevorach and Mevorach2 models in the first period of testing, but distant themselves with each subsequent out-of-sample test. The clear difference between the Mevorach and L_Mevorach models in out-of-sample testing shows that it is not only the parameters that are driving the model fit, but also the structure of the model.

[10] These figures can be found at the end of the Appendix.

Table 8: Three Years Out-of-Sample Model Comparison

	EL	LogEL	Mev	Mev2	Braun	L_Mev	Wang
In-Sample Period: April 2002 through March 2010							
R^2_{OS}	0.567	0.553	0.710	0.741	0.666	0.722	0.573
MAPE	22.046	23.572	19.389	17.665	21.104	20.180	22.619
MEAPE	17.220	17.317	16.210	14.526	14.826	16.164	16.360
MAE	190.440	195.573	161.528	149.317	168.730	162.175	188.632
In-Sample Period: April 2003 through March 2011							
R^2_{OS}	0.550	0.576	0.767	0.780	0.642	0.694	0.596
MAPE	35.908	34.096	26.136	24.460	30.788	27.902	35.597
MEAPE	23.500	23.000	16.539	15.602	21.382	20.165	23.257
MAE	227.071	223.460	164.303	156.480	196.916	181.871	223.115
In-Sample Period: April 2004 through March 2012							
R^2_{OS}	0.587	0.638	0.838	0.848	0.694	0.734	0.630
MAPE	60.054	46.886	34.306	32.394	45.012	36.802	56.147
MEAPE	52.601	41.798	31.553	28.845	39.942	31.169	44.326
MAE	271.943	243.691	170.613	161.943	224.479	202.247	264.658
In-Sample Period: April 2005 through March 2013							
R^2_{OS}	0.457	0.510	0.855	0.866	0.530	0.679	0.495
MAPE	73.747	57.714	34.778	33.128	56.040	39.928	70.278
MEAPE	62.514	58.020	31.901	30.131	54.855	39.766	65.759
MAE	328.547	293.018	166.075	156.994	286.913	224.307	324.078
In-Sample Period: April 2006 through March 2014							
R^2_{OS}	0.205	0.078	0.942	0.941	0.416	0.806	0.188
MAPE	77.376	71.155	19.975	19.757	57.140	26.747	75.465
MEAPE	62.968	64.397	18.294	17.087	50.185	24.482	66.350
MAE	379.398	389.257	94.305	94.860	311.326	150.657	384.611

Conclusion

Due to data scarcity, empirical research of cat bond pricing has been limited to explaining the variation in spreads of small subsections of cat bonds and extrapolating that analysis to a broader understanding of the market. Perhaps due to these limitations, the loglinear relationship between expected loss and spread first introduced by Major and Kreps (2003) has been put aside as inferior to a linearly modelled relationship. This paper goes against the recent literature and reintroduces the loglinear approach as a viable alternative. Observing the relationship between the ratio of spread over expected loss and expected loss in my data set, a loglinear model appears to provide a better fit than a linear model. This is confirmed in out-of-sample testing, as a simple loglinear model performs slightly better than a simple linear model.

Building off of the simple loglinear model, I include a few variables with practical and theoretical relationship to cat bond spreads, with confirmation of these relationships from previous studies. The variables determined to be highly significant include the expected loss from the simple log linear model, the cat bond cycle, exposure to US catastrophes, exposure to US hurricane, exposure to Japanese earthquake, and a parametric trigger. The multivariate loglinear model developed using these determinants show better in-sample fit than linear alternatives, and more importantly exhibits better out-of-sample fit than well-conceived models from the literature. The most critical finding is that the multivariate loglinear model performs better than its linear counterpart, further emphasizing the practicality of using a loglinear model in the empirical study of cat bond spreads.

The insights of the paper carry implications to further empirical analysis of cat bonds as well as to participants in the cat bond market. Since a loglinear model has been determined to be of better fit to the cat bond market, further empirical research attempting to derive determinants of cat bond spreads should consider this model form. A linear approach and loglinear approach reach different conclusions as to the statistical significance of variables in question, as shown by the insignificance of the dummy variable for Japanese earthquake in the L_Mevorach model. As to the participants in the cat bond market, a greater understanding of the derivation of cat bond spreads can limit barriers to investing in this relatively new security. While investors do not necessarily have problems with the asymmetric risk of catastrophes, further information regarding how cat bonds are priced aids transparency.

The cat bond market has become an effective means for insurance and reinsurance companies to limit their exposure to large catastrophic risk. As an understanding of this market develops, prices should continue to decline as they have since inception. Hopefully, as the price of reinsurance stabilizes and declines due to capital market pressures, so will the price of insurance. Hurricane Harvey caused over 125 billion USD in economic losses, while insured losses were estimated to be less than 30 billion USD. Perhaps, with the proactive involvement of capital markets, this disparity will become a thing of the past.

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Appendix

Summary of Cat Bond-Specific Data Collected

Below is a list of cat bond specific information collected, with descriptions where necessary. This list is exhaustive, and contains variables not contained in the analysis to follow.

Sponsor, SPV, Issue Date, Maturity Date.

Spread Premium (Spread) - The annual yield of the cat bond over 6-month LIBOR as reported at issue.

Expected Loss (EL) - The annual expected loss of the cat bond as reported by a modelling agency (like AIR) in basis points. Expected loss is not consistent between modelling agencies nor across time. However, the estimate reported is the same as the one in the bond's prospectus, and therefore is most relevant.

Trigger - Cat bond trigger as reported by Artemis. Dummy variables for three trigger types are made; parametric, indemnity, and industry loss.

Issue Size - The total principal collected for the cat bond tranche.

Exposure Term - The number of months the cat bond is exposed to catastrophe risk, which is not necessarily until maturity.

Peril - A list of the regions of exposure and their respective catastrophe exposure as reported by Artemis. For example, a cat bond's list of perils could include Florida hurricane and Mediterranean windstorm. Details of the exact regions and catastrophes included in the data set are available below.

Probability of first loss - The probability that the cat bond will be triggered within a year.

Probability of exhaust - The probability that the principal of the cat bond will be exhausted.

Conditional Expected Loss - The expected loss conditional on the cat bond being triggered.

Rating - The rating of the cat bond if assigned a rating by S&P, Moodys, or Fitch.

Coverage type - Annual aggregate, term aggregate, or per-occurrence coverage.

Regions and Perils

Regions:

US regions include Northeast, Southeast, Florida, Texas, Midwest, California, Hawaii, and Northwest.
European regions include United Kingdom, Central Europe, and the Mediterranean.
Various other regions include Japan, Taiwan, Australia, Mexico, Puerto Rico, Caribbean, Canada, and Cayman Islands.

Catastrophes:

Hurricane, earthquake, flood, wildfire, storm Surge, meteorite impact, severe storms, winter storms, wind storms, river flood, worker's compensation from earthquakes, typhoon, temperature fluctuations, cyclone, extreme mortality, and casualty.

Exclusion of Observations

There are 576 cat bond issues in the data set. Below is a table detailing the number of cat bonds excluded in each step to reach the total of 519 cat bonds in the final data set. Expected loss and spread need to be greater than zero since the alternative is not feasible. Probability of first loss and probability of exhaust must exist and be greater than zero since the Wang transformation model makes use of these measurements of risk. If probability of first loss or probability of exhaust equals zero, then the Wang transformation is undefined.

Table 9: Cat Bond Exclusion

Step Description	Number of Cat Bonds
Beginning	576
Expected Loss $>$ 0 and Not Missing	563
Spread Premium $>$ 0 and Not Missing	559
Prob 1st Loss $>$ 0 and Not Missing	555
Prob Exhaust Not Missing	522
Prob Exhaust $>$ 0	519
End	519

Excluding Outliers in In-Sample Testing

In fitting his model to in-sample data, Braun (2016) excludes influential outliers. To identify these influential outliers, he uses Cook's distance and studentized

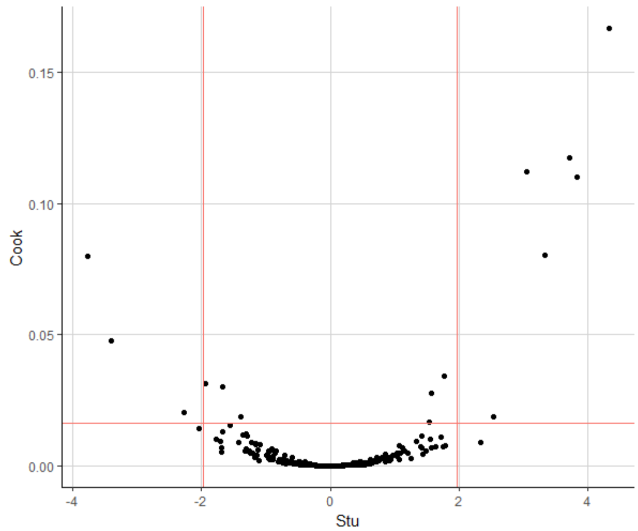


Figure 5: Removing Outliers from In-Sample Data

residuals (also known as jackknifed residuals) of a model that includes all possible explanatory variables. I follow this method.[11]

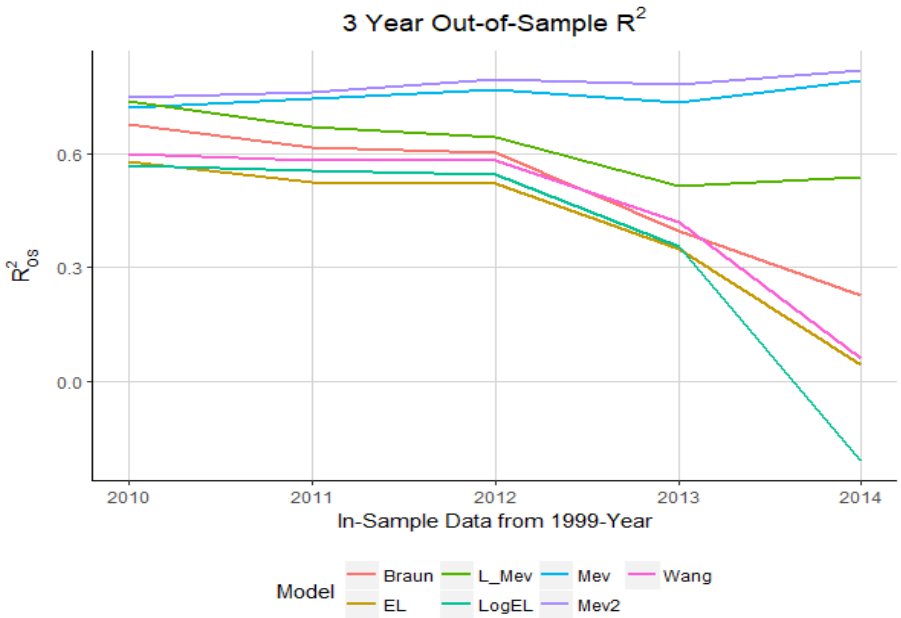
There are 266 cat bonds issued prior to March 31, 2010. The model used has 18 regressors, so the degrees of freedom equals 247. Thus, using the studentized t-distribution, a 5% confidence interval for the studentized residuals is +/- 1.97. Values of Cook's distance greater than $4/247 = 0.016$ are considered extreme. Cat bonds outside of the bottom-center box in the chart below are excluded from the in-sample fit of the models. There are 17 cat bonds outside of the bottom-center box, so there are 249 cat bonds included in in-sample fitting of the models.

Favorite Cat Bond Names

- The "Arbor" Series. In 2003, Swiss Re sponsored multiple cat bonds, each named after an indigenous tree of the region it covers. Sequoia Capital for California, Sakura Capital for Japan, Palm Capital for Florida, Oak Capital for Europe, and of course Arbor Capital for multi-region coverage.
- Shackleton Re in 2006. It sounds like Shaq decided to put away his old life and become a butler, but did not have much creativity in the name department.

[11] Cook's Distance is calculated using the base package in R, and studentized residuals are calculated using the MASS package in R

- In 2007, Javelin was sponsored by Arrow Re. They like pointy things.
- In 2014, Kilimanjaro Re sponsored by Everest Re. They get it.
- Obviously, the winning name is Bonanza Re, sponsored by American Strategic Insurance in 2016. If the theme song is not already stuck in your head, then this paper has been unsuccessful.



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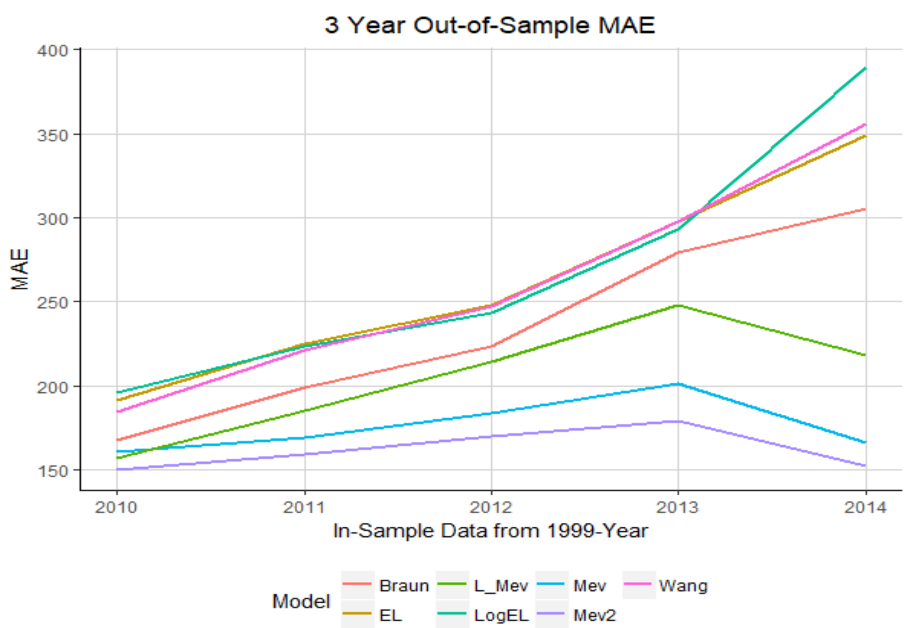

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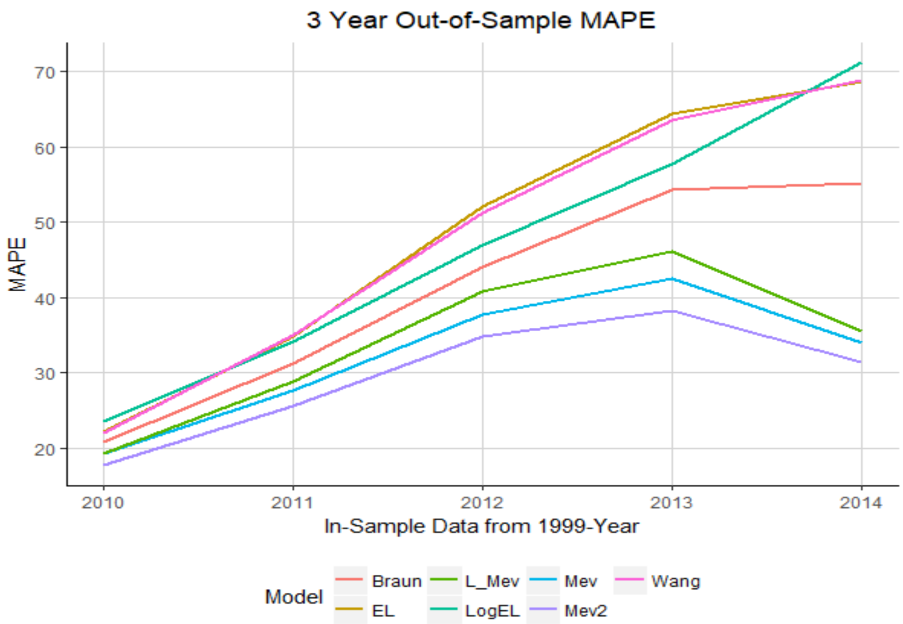
Figure 6: Out-of-Sample Tests



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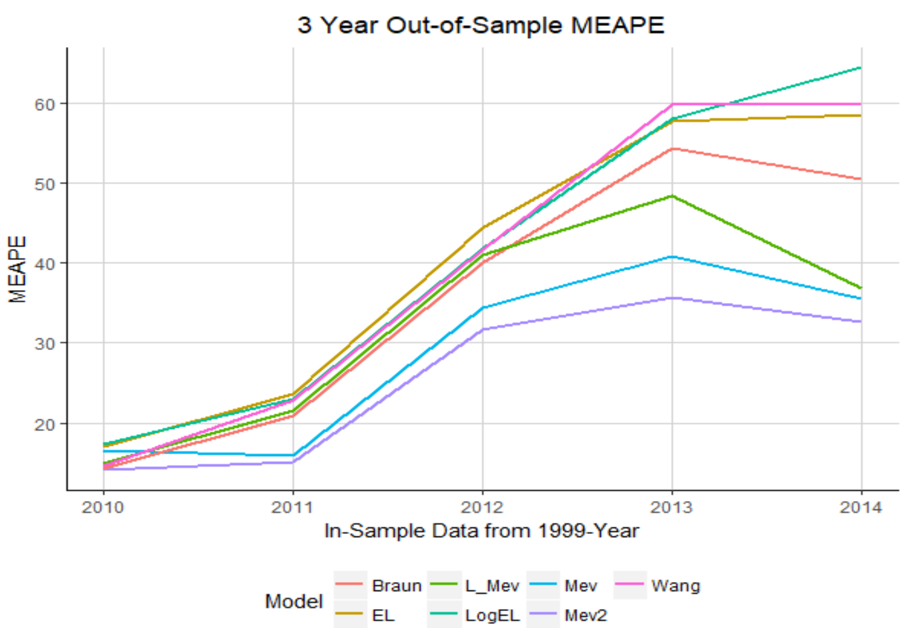

(b)

Figure 7: Out-of-Sample Tests