Worker Heterogeneity and Endogenous Separations in a Matching Model of Unemployment Fluctuations*

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Abstract

We model worker heterogeneity in the rents from being employed in a Diamond-Mortensen-Pissarides model of matching and unemployment. We show that heterogeneity, reflecting differences in match quality and worker assets, reduces the extent of fluctuations in separations and unemployment. We find that the model faces a trade-off–it cannot produce both realistic dispersion in wage growth across workers and realistic cyclical fluctuations in unemployment.

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1. Introduction

Shimer (2005), Hall (2005), Costain and Reiter (2008), and Gertler and Trigari (2009) each argue that matching models with flexible wages fail to explain business cycle fluctuations–the models generate much more procyclical wages and much less cyclical unemployment and job finding rates than observed. But, as discussed by Costain and Reiter, Mortensen and Nagypal (2007), and Hagedorn and Manovskii (2008), this negative conclusion rests on employment having substantial economic rents relative to the monetary, home production, and leisure benefits to not being employed. For example, Hagedorn and Manovskii, by allowing benefits to unemployment to replace 95% of the payout toemployment, are able to rationalize the cyclical volatility of unemployment under the matching model with flexible wages and exogenous separations. So establishing the rents from employment is key to judging how well the matching model captures cyclical fluctuations. Judging the size of these rents a priori is problematic as they reflect, not only direct payments, but also individuals' valuations of leisure and home production.

We shed light on this question by considering endogenous separations. We introduce heterogeneity in reservation wages into a business cycle model of separations, matching, and unemployment. As in Mortensen and Pissarides (1994), we allow workers to face shocks to their employment matches, with bad draws possibly leading to endogenous separations. We depart from Mortensen and Pissarides by allowing for diminishing marginal utility in consumption—necessary for wealth to affect labor supply—and for imperfect insurance as in Aiyagari (1994)—which affects workers' reservation wages. As a result, willingness to trade work for search depends on the worker's wealth. Workers with lower savings, reflecting bad past earnings shocks, are less willing to separate. The heterogeneity in match quality and assets jointly determine the distribution of rents to being employed. In turn, this distribution drives both the level and cyclicality of unemployment.

We find a trade-off between generating realistic dispersion in wage growth across workers and generating realistic cyclical fluctuations in unemployment. As stated above, one resolution of the Shimer puzzle is to allow for only a small wedge between the productivity and wages of employment and the benefits of unemployment. This directly implies that differences between workers' wages and reservation wages (for not separating) exhibit a distribution compressed near zero. In turn, the model must yield very high separation rates, much higher than in the data, unless shocks to match quality and wages are implausibly small. For instance, with the replacement rate suggested by Hagedorn and Manovskii, the model can generate reasonable rates of separation and unemployment only if shocks to match quality are so small that wages changes within jobs are an order of magnitude smaller than suggested by empirical studies. With Shimer's calibrated replacement rate of 40%, by contrast, substantial shocks to match quality are required to match separation and unemployment rates, shocks much more consistent with the dispersion of wage growth found in micro data. But with these reasonable match quality shocks, selection through endogenous separations yields few matches near the threshold for destruction. In turn, this reduces responses in separations and unemployment to aggregate shocks; consequently, the model to fails to capture the cyclicality of unemployment.

The model is presented in the next section then calibrated in Section 3. In Section 4 we examine the model's steady-state features. We show that both a high replacement rate and little heterogeneity, in match quality and assets, are key for producing an economy with many workers with low rents from employment—the scenario that generates a large response of unemployment to aggregate shocks. We require our benchmark economy to exhibit realistic separation and unemployment rates and reasonable dispersions in wage rates of wage growth. In turn, this requires a relatively low replacement rate and significant match quality shocks. We consider an alternative economy that matches the average unemployment with a high replacement rate, but it requires extremely small shocks to match quality.

The model's cyclical predictions are presented in Section 5. The model can generate a very cyclical unemployment rate, but only if there is little dispersion in match quality. With little cross-sectional dispersion there is an important spike up in separations at the onset of a downturn. Secondly, again for low dispersion, the rents to vacancy creation are highly procyclical. Thirdly, the model generates a new avenue for cyclicality in unemployment—in response to higher expected unemployment duration, separations become skewed toward workers with higher assets and higher reservation match qualities. Because these workers generate smaller expected surplus to employers, this acts to further depress vacancy creation in a recession. However, for our benchmark model that displays reasonable dispersions in match wages and wage growth, we find that separations, vacancies, and unemployment all exhibit much less cyclicality than seen in the data.

Besides Mortensen and Pissarides (1994), an antecedent to our model is Chang and Kim (2006, 2007). They show that the cross-sectional distributions of wealth and worker productivity play a critical role in determining the elasticity of aggregate labor supply in a competitive equilibrium. Nakajima (2007), Shao and Silos (2007), and Krusell, Mukoyama, and Sahin (2008) have also recently adopted diminishing marginal utility in consumption and imperfect risk sharing into the Mortensen-Pissarides model.[1] However, only Shao and Silos allow for heterogeneous productivity; and none of these authors allows for endogenous separations. Our message is that allowing reasonable heterogeneity, reflecting differences in match quality and in worker assets and consumption, reduces cyclical fluctuations in separations and unemployment. Under linear utility this heterogeneity would be reduced to that from match quality. Our calibrated model generates a positive correlation between match quality and consumption. As a result, with risk aversion it generates a a tighter distribution of match surplus near zero and, for this reason, somewhat more cyclical separations and unemployment than under linear utility.

Our trade-off between realistic dispersion in wage growth and realistic cyclicality of unemployment intersects with arguments in Hornstein, Krusell, and Violante (2007)–both papers express a difficulty for the DMP model that connects to the cross-sectional dispersion of wages. Hornstein, et al., show that, with substantial dispersion in initial wage offers, the Diamond-Pissarides-Mortensen model implies unemployment durations that are far higher than seen in the data. Hornstein, et al., do not consider the implications for business cycles. This is not surprising, as they see the model as inconsistent with first moments of the data. If we apply their reasoning to our models, it would say we should rule out both the high-volatility and benchmark models–neither generates notable dispersion in initial match wages, which they believe is considerable. We do not see it this starkly because we believe it is difficult to measure dispersion in initial wages due solely to match quality. Whereas Hornstein, et al., focus on the dispersion in quality of new matches, we focus entirely on the shocks to productivity and wages within matches, which is central for separations. We assume new matches begin from the same match quality, as in Mortensen and Pissarides. An advantage of our focus, we would argue, is that it is much easier to see that wage movements within matches (jobs) are empirically important than to identify the importance of dispersion in initial match offerings, separate from individual effects. Our conclusion is that the the dispersion in wage growth within jobs rules out the low-rents, high-volatility economy, but not the benchmark economy that fails to generate cyclical volatility.

[1] Other papers that entertain wealth effects in modeling search include Pissarides (1987), Gomes, Greenwood, and Rebelo (2001), and Hall (2006). Haefke and Reiter (2006) generate dispersion in reservation wages, while maintaining linear utility and no match-specific productivity, by assuming heterogeneity in individuals' value of home production. Several papers (Darby, Haltiwanger, and Plant, 1985, Baker, 1992, and Pries, 2007) have argued that lower job-finding rates during recessions may reflect a compositional shift toward workers who display lower job-finding rates. But these papers impose this shift exogenously, whereas our model, by allowing for wealth effects, predicts such a shift in recessions toward unemployed workers with high reservation match qualities.

2. Model

We build on the model of cyclical unemployment in Mortensen and Pissarides (1994). We depart from Mortensen and Pissarides by letting workers be risk averse–a key feature of wealth effect in labor supply, face a borrowing constraint, and value leisure, distinct from goods consumption, from being unemployed.

2.1. Environment

There is a continuum of infinitely-lived workers with total mass equal to one. Each worker has preferences defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + B \cdot l_t \right\},$$

where $0 < \beta < 1$ is the discount factor, and $c_t(> 0)$ is consumption. The parameter B denotes the utility from leisure when unemployed. l_t is 1 when unemployed and otherwise zero. In Mortensen and Pissarides (1994), and many extensions, there is no valuation of leisure; so a marginal rate of substitution between leisure and consumption is not defined. Here the marginal rate of substitution ($c^{-\gamma}/B$) is decreasing in c. This provides the basis for a worker's reservation match quality to be increasing in consumption and thereby savings.

Each period a worker either works (employed) or searches for a job (unemployed). A worker, when working, earns wage w. If unemployed, a worker receives an unemployment benefit b. Each can borrow or lend at a given real interest rate r by trading the asset a. But there is a limit, $\underline{a}$, that one can borrow; that is $a_t > \underline{a}$. Real interest rate r is determined exogenously to fluctuations in this particular economy (small open economy).

There is also a continuum of identical agents we refer to as entrepreneurs (or firms). Entrepreneurs have the ability to create job vacancies with a cost κ per vacancy. Entrepreneurs are risk neutral, diversifying ownership of their investments across many vacancies and across economies, and maximize the discounted present value of profits

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \pi_t \, .$$

There are two technologies in this economy, one that describes the production of output by a matched worker-entrepreneur pair and another that describes the process by which workers and entrepreneurs become matched. A matched pair produces output

$$y_t = z_t x_t \,,$$

where z_t is aggregate productivity and x_t is idiosyncratic match-specific productivity, i.e., match quality. Both aggregate productivity and idiosyncratic productivity evolve over time according to Markov processes, respectively $Pr[z_{t+1} < z'|z_t = z] = D(z'|z)$ and $Pr[x_{t+1} < x'|x_t = x] = F(x'|x)$. For newly formed matches, idiosyncratic productivity starts at the mean value of the unconditional distribution, which is denoted by $\bar{x}$.

The number of new meetings between the unemployed and vacancies is determined by a matching function

$$m(v, u) = \eta u^{1-\alpha} v^{\alpha} \,,$$

where v is the number of vacancies and u is the number of unemployed workers. The matching rate for an unemployed worker is $p(\theta) = m(v, u)/u = \eta\theta^{\alpha}$, where $\theta = v/u$ is the vacancy-unemployment ratio, the labor market tightness. The probability that a vacant job matches with a worker is $q(\theta) = m(v, u)/v = \eta\theta^{\alpha-1}$.

A matched worker-firm constitutes a bilateral monopoly. We assume the wage is set by bargaining between the worker and firm over the match surplus. This is discussed in the next subsection. The match surplus reflects the value of the match relative to the summed worker's value of being unemployed and the entrepreneur's value of an unmatched vacancy, which is zero in equilibrium. There are no bargaining rigidities; separations are efficient for the worker-firm pair, occurring if and only if match surplus falls below zero.

The timing of events can be summarized as follows.

1. At the beginning of each period matches from the previous period's search and matching are realized. Also, aggregate productivity z and each match's idiosyncratic productivity x are realized.

2. Upon observing x and z, matched workers and entrepreneurs decide whether to continue as an employed match. Workers breaking up with an entrepreneur become unemployed. There is no later recall of matches.

3. For employed matches, production takes place with the wage reflecting worker-firm bargaining. Also at this time, unemployed workers and vacancies engage in the search/matching process.

2.2. Value functions

Consider a recursive representation, where W, U, J, and V denote respectively the values for the employed, unemployed, a matched entrepreneur, and a vacancy. All value functions depend on the measures of workers. Two measures capture the distribution of workers: $\mu(a,x)$ and $\psi(a)$, respectively, represent the measures of employed workers and unemployed workers during the period.[2] The evolution of these measures is given by $\mathbf{T}$, i.e., $(\mu',\psi') = \mathbf{T}(\mu,\psi,z)$. For notational convenience, let $\mathbf{s} = (z,\mu,\psi)$.

From the model discussion, it follows that the worker's value of being employed is

$$W(a,x,\mathbf{s}) = \max_{a'_e}\left\{ u(c_e) + \beta E\left[\max\{W(a'_e,x',\mathbf{s}'),U(a'_e,\mathbf{s}')\}|\, x,\mathbf{s}\right] \right\}, \tag{2.1}$$

subject to

$$c_e = (1+r)a + w - a'_e\,,$$

$$a'_e \geq \underline{a}\,.$$

The value of being unemployed, recalling that $p(\theta)$ is the probability that an unemployed worker matches, is

$$U(a,\mathbf{s}) = \max_{a'_u}\left\{ u(c_u) + \beta(1-p(\theta(\mathbf{s})))E\left[U(a'_u,\mathbf{s}')|\,\mathbf{s}\right] + \beta p(\theta(\mathbf{s}))E\left[W(a'_u,\bar{x},\mathbf{s}')|\,\mathbf{s}\right] \right\}, \tag{2.2}$$

subject to

$$c_u = (1+r)a + b - a'_u\,,$$

$$a'_u \geq \underline{a}\,,$$

where $u(c_u)$ includes the leisure value B from being unemployed.

For an entrepreneur the value of a matched job is:

$$J(a,x,\mathbf{s}) = zx - w(a,x,\mathbf{s}) + \beta E\left[\max\{J(a'_e,x',\mathbf{s}'),V(\mathbf{s}')\}|\, x,\mathbf{s}\right]\,. \tag{2.3}$$

[2] Let $\mathcal{A}$ and $\mathcal{X}$ denote sets of all possible realizations of a and x, respectively. Then $\mu(a,x)$ is defined over σ-algebra of $\mathcal{A}\times\mathcal{X}$ while $\psi(a)$ is defined over σ-algebra of $\mathcal{A}$.

The value of a vacancy is:

$$V(\mathbf{s}) = -\kappa + \frac{1}{1+r} q(\theta(\mathbf{s})) \int E\left[J(a'_u, \bar{x}, \mathbf{s}')|\mathbf{s}\right] d\widetilde{\psi}(a'_u) + \frac{1}{1+r}(1 - q(\theta(\mathbf{s})))V(\mathbf{s}')\,, \tag{2.4}$$

where recall that κ is the vacancy posting cost and $q(\theta)$ is the probability that a vacancy is filled. $\widetilde{\psi}(a'_u)$ denotes the measure of unemployed workers at the end of a period after decisions on asset accumulation are made.

2.3. Wage Bargaining

The setting allow for bilateral bargaining between a matched vacancy and worker. We follow much of the literature in assuming that wages reflect a Nash bargaining solution, such that

$$\underset{w}{\text{argmax}} \ \Big(W(a, x, \mathbf{s}; w) - U(a, \mathbf{s}; w)\Big)^{\frac{1}{2}} \Big(J(a, x, \mathbf{s}; w) - V(\mathbf{s}; w)\Big)^{\frac{1}{2}} \tag{2.5}$$

for all $(a, x, \mathbf{s})$.[3]

The Nash solution generates a wage that is increasing in a worker's assets, reflecting that being unemployed is less painful for a worker with greater assets. (Below see Figure 1.) In turn, this makes the vacancy creation decision depend on the assets of the unemployed. We believe these features potentially generalize to settings with wage posting by firms and directed search by workers. For instance, Acemoglu and Shimer (1999) model directed search by risk averse workers. They show that the distribution of posted wages exhibits a higher mean, with longer queues, if workers are less risk averse, as then workers are less willing to take lower wages in order to raise the probability of employment. We would expect increased assets for the unemployed, for given risk aversion, to exhibit comparative statics in this same direction in their setting.

[3] Rubinstein (1982) demonstrates in a stationary environment that the Nash solution can be interpreted as the outcome of a noncooperative game with sequential offers. In our stochastic setting without linear utility this interpretation does not literally hold (Coles and Wright, 1998.) We adopt the Nash solution, however, partly for comparability with the related literature.

Because we allow for workers to display risk aversion, there is a motive for employers to insure workers' incomes. With perfect commitment, by both firms and workers, this implies we should not wage dispersion in response to idiosyncratic shocks or wage responses to aggregate shocks. We should also observe severance-type payments that insure workers in the event of separations. We do not allow such insurance, implicitly assuming that commitment fails. To the extent such insurance is important, we anticipate it would have the following two effects on interpreteting our model results. Such insurance would reduce aggregate cyclicality in consumption, causing separations and unemployment to exhibit greater cyclicality. Secondly, it would reduce the dispersion in wages and wage growth. (In the extreme we should see none of the dispersion in rates of wage growth reported by Topel and Ward.) Thus, to rationalize the same degree of dispersion in wage growth that we calibrate to would require substantially greater shocks to match quality. We believe this would require a model calibration, to be consistent with average separation rates, that would yield much less cyclicality.

2.4. Evolution of measures

The measures for workers employed and unemployed, $\mu(a,x)$ and $\psi(a)$, evolve as follows.

$$\begin{aligned}\mu'(A^0,X^0) = & \int\limits_{A^0,X^0}\int\limits_{\mathcal{A},\mathcal{X}} \mathbf{1}_{\{x'\geq x^*(a',\mathbf{s}'),a'=a'_e(a,x,\mathbf{s})\}} dF(x'|x)d\mu(a,x)da'dx' \\ & + p\left(\theta(\mathbf{s})\right)\int\limits_{A^0}\int\limits_{\mathcal{A}} \mathbf{1}_{\{x'=\bar{x},a'=a'_u(a,\mathbf{s})\}} d\psi(a)da'dx'\end{aligned}, \tag{2.6}$$

$$\begin{aligned}\psi'(A^0) = & \int\limits_{A^0}\int\limits_{\mathcal{A},\mathcal{X}} \mathbf{1}_{\{x'< x^*(a',\mathbf{s}'),a'=a'_e(a,x,\mathbf{s})\}} dF(x'|x)d\mu(a,x)da' \\ & + \left(1-p\left(\theta(\mathbf{s})\right)\right)\int\limits_{A^0}\int\limits_{\mathcal{A}} \mathbf{1}_{\{a'=a'_u(a,\mathbf{s})\}} d\psi(a)da'\end{aligned} \tag{2.7}$$

for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

2.5. Equilibrium

The equilibrium consists of a set of value functions, $W(a,x,\mathbf{s})$, $U(a,\mathbf{s})$, $J(a,x,\mathbf{s})$, a set of decision rules for consumption $c_e(a,x,\mathbf{s})$, $c_u(a,\mathbf{s})$, asset holdings $a'_e(a,x,\mathbf{s})$, $a'_u(a,\mathbf{s})$, and separating $x^*(a,x,\mathbf{s})$, the wage schedule $w(a,x,\mathbf{s})$, the labor-market tightness $\theta(\mathbf{s})$, and a law of motion for the distribution, $(\mu',\psi') = \mathbf{T}(\mu,\psi,z)$. Equilibrium is defined by the following.

1. (Optimal Savings): Given θ, w, μ , ψ, and $\mathbf{T}$, a' solves the Bellman equations for W, U, J and V in (2.1), (2.2), (2.3), and (2.4).

2. (Optimal Separation): Given W, U, J, V, μ, ψ, and $\mathbf{T}$, x^* satisfies $J(a,x^*,\mathbf{s}) = 0$.

3. (Nash Bargaining): Given W, U, J and V, w **satisfies** $J(a,x,\mathbf{s}) = (W(a,x,\mathbf{s}) - U(a,\mathbf{s}))\times u'(c_e(a,x,\mathbf{s}))^{-1}$.

4. (Free Entry): Given w, x^*, J, μ, ψ, and $\mathbf{T}$, the vacancies are posted until $V = 0$.

5. (Rational Expectations): Given a'_e, a'_u and x^*, the law of motion for distribution $(\mu',\psi') = \mathbf{T}(\mu,\psi)$ is described in (2.6) and (2.7).

3. Model Calibration

We calibrate our model in order to present its predictions for business cycle fluctuations. But, prior to considering business cycles, in Section 4 we display the model's steady-state features, in particular showing how the heterogeneity of worker's match quality and assets determine the distribution of rents to employment.

3.1. The benchmark economy

We consider two calibrated models that yield the same steady-state rates of separations and unemployment, but differ sharply in their predictions for the average level, and dispersion, in match rents. Our benchmark calibration reflects nontrivial rents to employment that reflect dispersion in wages due to differences in match quality. These rents are roughly consistent with the dispersion observed for wage growth within matches (e.g., Topel and Ward, 1992) and with the dispersion for wage levels that has been attributed to match effects estimated on matched employer, employee data (Woodcock, 2008). We also describe an alternative calibration that is designed to generate sizable cyclical fluctuations—large enough to match the observed volatility of aggregate unemployment in the data. But this calibration hinges on having almost no dispersion in match quality, which requires extremely small dispersion in wage growth within matches.

Starting with preferences, we assume a relative risk aversion parameter γ equal to one. We choose a monthly discount factor β of 0.995 and an annualized real interest rate of 6%. These together generate average assets equal to 18 months of labor earnings, which is about the median ratio of net worth to family earnings we calculate from the Survey of Income and Program Participation (SIPP) data. (See Bils, Chang, and Kim, 2007, for more details on statistics derived from the SIPP.) We set the borrowing constraint to six, so approximately six month labor income, as we see few households in the SIPP with unsecured debt exceeding this amount.

The key outcomes we target are the average rates of unemployment and separations. We target an average unemployment rate of 6% and a monthly separation rate of 2%. A separation rate of 2% is consistent with the rate of monthly separations in the SIPP data, based on separations that are not job-to-job and that do not result in a return to the same employer within four months. The SIPP associates a distinct employer code for each job–so it is possible to observe worker recalls to an employer. We see that about half of separations out of work exhibit a return to the original employer. We view these short separations with recall like a reduction in hours; they do not correspond to separations to engage in search/matching. Based on the CPS data, Fallick and Fleishman (2004) and Shimer (2005) construct monthly rates of separation out of employment of, respectively, 4.0% and 3.4%. But if half these, like those in the SIPP result in recall, then this would correspond to separation rates without recall close to our 2% assumed rate. Both in the CPS (e.g., Fallick and Fleishman) and in the SIPP (e.g., Nagypal, 2006) job-to-job separations are nearly as sizable as separations out of employment (including those with recall). We do not count these job-to-job flows in calibrating the model. Key to our calibration is the rents to employment relative to search unemployment–observing high rates of job-to-job mobility does not inform us that unemployment is a good substiutute for employment.

For our primary results, we follow Mortensen and Pissarides in treating all separations as chosen endogenously, that is all matches have an option to continue, though in some cases this would be at very low productivity. We also explore the implications for our results of allowing for a mixture of endogenous and exogenous separations.

The 6% rate for unemployment and monthly separation rate of 2% imply a steady-state monthly job finding rate of 31%. This rate is consistent with transition hazards reported by Meyer (1990). The vacancy posting cost κ is chosen so that the vacancy-unemployment ratio (θ) is normalized to 1 in the steady state. The matching technology is Cobb-Douglas; $m(v, u) = .31\, v^{\alpha} u^{1-\alpha}$ hits the steady-state finding rate. We set the matching power parameter α to 0.5.

Remaining to calibrate are the payouts to being unemployed, which are unemployment insurance b and leisure utility B, and the magnitude of match-specific shocks. These are key determinants of rates of separations and unemployment. If unemployment is made more attractive, everything else equal, this clearly leads to higher separation and unemployment rates. We calibrate our benchmark economy to generate rents to employment comparable to that in Shimer (2005). To do so, we first considered a special case of our model that, like Shimer's, has linear utility and no match-quality shocks or endogenous separations—separations occur exogenously at a rate of 2% monthly. We follow Shimer by calibrating unemployment insurance to $b = 0.4$, with $B = 0$. That economy generates capitalized value of a matched job (J) of 1.65, that is, a little over one and half months of match output. This in turn directly implies a vacancy creation cost κ of 0.52 (half of a month's output). We calibrate our benchmark economy to exhibit the same size of values, $J = 1.65$ and $\kappa = 0.52$. Keeping $b = 0.4$, we find this requires a value for leisure of $B = 0.15$. That is, a consumer views this leisure comparably, in terms of flow utility, to 15% higher consumption.

Greater match-quality shocks, like higher replacement rates, create more separations and higher average unemployment. We set the persistence of the match-specific shock to be quite high, $\rho_x = 0.97$, to accord with the high persistence typically estimated for individual wage earnings. At an annual frequency, the persistence of wage ranges across estimates ranges from 0.75 to .95 depending on how one treats measurement error and other matters of specification (e.g., see Chang and Kim, 2007). At a monthly frequency these numbers imply a high auto-correlation.[4] We particularly stress Topel and Ward's statistics on dispersion in wage growth based on administrative data. They show an annual autocorrelation in the *growth rate* of wages of -0.33. When we produce the same statistic, based on wage growth within matches, our calibrated models (all versions) generate a value of -0.27. So we believe the persistence we employ is empirically sensible.

Finally, we set the standard deviation of these match-quality shocks in order achieve the target separation and unemployment rates of 2 and 6%. This dictates $\sigma_x = 0.13$. These match-quality shocks produce a plausible match to individual earnings data. In particular, they are consistent with the dispersion in the growth rate of wages within job matches reported by Topel and Ward (1992). Topel and Ward examine quarterly wages for full-time workers based on earnings reported to Social Security for the primary job. We highlight the Topel and Ward study because of its use of administrative data, which should minimize the impact of measurement error. They report a cross-sectional standard deviation of wage-growth relative to four quarters prior, *within job matches*, of 19%. We calculate the growth rate in the same fashion, that is quarterly wages relative to four quarters earlier for the same employer match, for our calibrated model economies. For our benchmark economy the standard deviation of this growth rate is 18%, quite close to that reported by Topel and Ward.[5]

3.2. The high-volatility economy

For contrast, we consider a cyclically sensitive economy calibrated so that, in response to aggregate shocks to productivity, it exhibits a standard deviation of quarterly unemployment rate that is 9.5 times that in productivity—where 9.5 reflects the ratio of these standard deviations reported by Shimer (2005).

To achieve this targeted cyclicality, while maintaining an average rate of 6% unemployment,

[4] Our choice for ρ_x is limited distinctly below one by computational concerns–the simulations are sometimes unstable with a stochastic process with persistence very close to 1.

[5] Our benchmark model generates a standard deviation of wage *levels* across workers that also equals 18%. We also examined the distribution of long-term match wages, that is the average wage over each match. The standard deviation of average match wages is 11% for the model. These figures are more difficult to relate to the empirical literature. Woodcock (2008) allows for individual, employer, and match components in explaining dispersion in earnings for a matched employer-employee sample and finds an important match component. Woodcock's estimated standard deviation for the match component in earnings is 28%. This figure is much larger than the dispersion in average match wage of 11% for our benchmark model with important match shocks. But, more to the point, it is far, far greater than the dispersion in wages produced by the high-volatility economy.

we free up the leisure value of unemployment B and the variability of match-quality shocks σ_x, keeping other parameters at their benchmark values.[6] The economic payoffs while unemployed are key, not only to the average rate of unemployment, but also to its cyclical volatility (Hagedorn and Manovskii, 2008, and Mortensen and Nagypal, 2007)—less surplus to employment increases cyclical volatility of vacancies and unemployment. By contrast, greater volatility of match-specific productivity (higher σ_x) has opposite impacts on the level versus cyclical volatility of unemployment. Greater match-quality shocks create more separations and higher average unemployment, but actually reduce the cyclical volatility of separations and unemployment. With greater match-quality shocks, workers become sorted over time into matches with significant match surplus. This makes their separations less responsive to cyclical fluctuations in productivity. Because the level of unemployment is increasing in both B and σ_x, but its cyclicality responds oppositely to the two parameters, we can maintain unemployment's average rate of 6%, while increasing its cyclicality, by appropriately increasing B in conjunction with decreasing σ_x. We find that the combination $B = 0.51$, $\sigma_x = 0.014$ produces a standard deviation of unemployment that is 9.5 times that for productivity. We show that this economy, though generating realistic cyclicality, yields implausibly little cross-sectional dispersion for wage growth within matches.

Table 1 summarizes the calibrated parameters with values employed for both the benchmark and high-volatility economies.

4. Steady-state Statistics and the Distribution of Match Rents

We present statistics for the model's steady state to illustrate how a worker's assets and match quality determine his wage, reservation match quality, and the surplus from employment. We focus on the distribution of surplus from employment because this is key in determining cyclicality of separations, vacancy creation, and unemployment for the model. We contrast the distribution of rents to employment from our benchmark model to those for the economy calibrated to generate high cyclical volatility in unemployment.

Starting with the benchmark economy, Figure 1 displays the values of the wage, $W - U$, and J as functions of a worker's assets. These relations are illustrated for three different values for match quality x. Higher values of match quality are directly associated with higher wages and capitalized value of employment W, while irrelevant for U. So both $W - U$ and J correspondingly

[6] It requires a very slightly different discount factor ($\beta = 0.9949$, versus 0.9948 for the benchmark) to hit average asset holding of 18 months earnings.

increase with match quality. Focusing on assets, both W and U increase with assets. But having low assets particularly lowers the value of being unemployed, resulting in a lower bargained wage. Figure 1 displays this positive relation between assets and wages. Both $W - U$ and J (reflecting the higher wage) decrease in worker assets. The sharpest positive relation of the wage to assets, and opposite reaction in J, is concentrated at the very low end of assets, near or below zero.[7] Focusing on firm rents J, we see that high assets lessens the expected rents of hiring a worker. In turn this provides a channel from assets, specifically the assets of the unemployed, to vacancy creation—high assets among the unemployed, everything else equal, reduces desired vacancies. This implies the cyclicality of assets for the unemployed will influence (oppositely) the cyclicality of vacancy creation.

The top panel of Figure 2 presents the distribution of assets separately for employed and unemployed workers. Because the unemployed draw down assets to maintain consumption, they exhibit average assets of 21% less than the employed (14.7 compared to 18.1). The unemployed exhibit lower consumption, by 9%, than the employed. The model succeeds in generating a fairly wide dispersion in assets, given workers differ only in their histories of match qualities and unemployment durations–its Gini coefficient for asset holdings is 0.44. The wealth distribution is highly concentrated in the data. For example, from the PSID's 1984 survey the Gini coefficient for wealth for "primary households"—families with households heads ages 35 to 55 with 12 years of schooling—is 0.70.[8] In particular, the richest 5% of households in the PSID owns 43% of total wealth, whereas in our model this share is 16%. However, the middle and left tail of the wealth distribution for the model differs less from the data. The PSID shows that primary households in the 1st, 2nd, 3rd, 4th, and 5th quintiles own respectively 1.0, 7.1, 13.0, 21.1, and 57.8% of total wealth; for the model these respective shares are 1.5, 9.1, 16.6, 26.2, 46.7%.[9]

[7] The assumptions of Nash bargaining and a coefficient of risk aversion of one imply that J equals $W - U$ times the worker's consumption. For this reason J decreases less than $W - U$ with worker assets. This is more relevant at low asset levels, where consumption responds more to assets. For instance, for $x = 1$, an increase in assets from 0 to 5 yields a drop in J of about two-thirds that in $W - U$.

[8] Family wealth in the PSID reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets.

[9] We should note that the wealth distribution for all households is more skewed than that of primary households. Across all households, from the 1st to 5th quintiles, the shares of total wealth are respectively, -0.5, 0.5, 5.1, 18.7, and 76.2%.

It is also important to judge the dispersion in assets relative to dispersion in earnings. This is much higher in the data than model, presumably because we abstract from differences in human capital. Among the PSID primary households, the Gini coefficient of earnings is 0.42, compared to 0.11 for the model. We could increasing the model's cross-sectional dispersion in earnings and wealth by allowing larger match shocks. But this, in turn, exaccerbates the trade off between cross-sectional dispersion of earnings and cyclical volatility, strengthening our conclusion. For example, for our high-volatility economy the Gini coefficients of wealth and earnings are only 0.24 and 0.01, respectively.

The bottom panel of Figure 2 displays how a worker's critical value for match quality x^* depends positively on assets—the critical match quality increases with assets throughout the range of relevant asset holding. Projecting this policy for x^* on the distribution for assets in the top panel of Figure 2 yields the distribution for x^*. This distribution exhibits a standard deviation of 3.3%.

Statistics for unemployment, turnover, and assets for the benchmark economy are presented in Table 2. The table reports that the cross-sectional standard deviation of (ln)wages is 18%. As discussed under calibrating, the standard deviation of annual wage growth within a match, calculated to parallel the treatment in Topel and Ward, also equals 18%, close to the Topel and Ward's figure of 19%.

Figure 3 presents the distribution of workers' ln(wages) relative to the critical wage, $\ln(w^*)$, at which the worker is indifferent to separating. (w^* is the bargained wage associated with critical match quality x^*.) This difference, $\ln(w) - \ln(w^*)$, reflects the flow rents associated with the employment match. These rents are significant for the benchmark economy, averaging 26%. If we consider a drop in match quality sufficient to reduce the wage by 10%, holding w^* unaffected, this would induce only about 16% of workers to separate. The standard deviation across workers of the differential $\ln(w) - \ln(w^*)$ equals 17.8%. This dispersion is largely driven by dispersion in the wage, not the reservation wage (w^*), and in turn reflects the dispersion in match quality, x. Recall that ln(wages) has a standard deviation of 18.0%. By contrast $\ln(w^*)$ has standard deviation of only 1.5%.

The magnitude of the differential $\ln(w) - \ln(w^*)$ is key to the economy's cyclical volatility. A negative aggregate shock induces only a small response in separations if few workers display wages close to the reservation wage w^*. Greater dispersion in $\ln(w) - \ln(w^*)$, absent search frictions, implies a less elastic aggregate labor supply response to aggregate shocks—in a search and matching model this is manifested by less response in separations. Secondly, a drop, say of one percent, in aggregate productivity represents a much smaller percentage hit to the expected payout to filling a vacancy if the average rents to employment are large. Therefore, considerable rents, such as depicted for the benchmark economy in Figure 3, will act to reduce the cyclicalilty of both separations and vacancy creation.

We consider an alternative specification that is comparable to our benchmark, but where half of separations are purely exogenous. The key difference for this calibration it that we reduce the size of match specific shocks considerably ($\sigma_x = 0.043$) to cut endogenous separations to half of all separations. We find this reduces the dispersion in match rents by nearly two-thirds. As a result, the model will generate more cyclical separations and somewhat more cyclical unemployment. (We discuss cyclicality in the next section.) But this version of the model generates much less cross-sectional dispersion in wage growth. When we calculate the cross-sectional standard deviation of annual wage-growth within job matches, this dispersion is reduced dramatically from 18% to less than 7%. This is far below the value of 19% reported for this statistic by Topel and Ward.[10]

The high-volatility economy displays much less dispersion in match quality and smaller rents to employment. Results for this model economy are given in Figures 4 and 5. The top panel of Figure 4 presents the distribution of assets separately for employed and unemployed workers; the bottom panel displays how a worker's critical match quality x^* depends on assets. Compared to the benchmark economy, the high-volatility economy generates a smaller dispersion of assets and, as a result, a smaller dispersion of x^*—the standard deviation of x^* is 0.8% for this economy, compared to 3.3% for the benchmark.

Statistics for the high-volatility economy are presented in the right-most column of Table 2. For the high volatility economy assets and consumption differ little between the employed and unemployed. Reflecting the small shocks to match quality, this economy exhibits a cross-sectional standard deviation for (ln)wages of only 1.9%, which we view as unreasonably small. Similarly, the high-volatility economy displays very little dispersion for rates of wage growth within matches. The simulated model data display a cross-sectional standard deviation for wage growth within matches (calculated to parallel Topel and Ward's treatment) also of only 1.9%. That is a full order of magnitude smaller than reported by Topel and Ward.

Figure 5 presents the distribution of workers' ln(wages) relative to reservation wage $\ln(w^*)$. In order to match cyclical volatility of employment, this economy must exhibit a highly elastic aggregate labor supply. This is reflected in a distribution for the differential $\ln(w) - \ln(w^*)$ that is limited to near zero—it averages only 3.0% for workers, with a standard deviation equal to only 1.8%.[11] A drop in match quality sufficient to reduce the wage by 10%, holding w^* unaffected, would induce nearly 100% of workers to separate. Thus, while we are able to generate large cyclical fluctuations with this model, we highlight that there is a severe tradeoff—achieving high cyclical volatility requires implausibly little dispersion in wages from match quality.

[10] An alternative for reducing the rate of endogenous separations would be to reduce the replacement rate, that is reduce parameters b and/or B. But we can anticipate that this will reduce cyclicality for the model, which already falls far short of that observed in the data.

[11] As with the benchmark economy, this dispersion is driven by dispersion in the wage, not w^*. The standard deviation of $\ln(w^*)$ is only 0.6 percent. The correlation between $\ln(w)$ and $\ln(w^*)$ is 0.24. For the benchmark economy that correlation is 0.14.

5. Business cycle predictions

We next characterize the business cycles properties of the model in response to exogenous shifts in aggregate productivity, contrasting results for the benchmark and high-volatility economies. For aggregate monthly productivity shocks we use $\rho_z = 0.95$ and $\sigma_z = 0.0077$. This yields a time series for (logged) TFP, after quarterly averaging and HP filtering, with autocorrelation of 0.84 and standard deviation of 2%. These coincide with the statistics reported by Shimer (2005) for U.S. quarterly labor productivity. We focus on discussing relative volatilities and correlations in describing the model results.

With aggregate fluctuations, productivity z and the measures of workers, μ and ψ, are state variables for agents' optimization problems, as separation decisions depend on subsequent matching probabilities. These, in turn, depend on the next period's measures of workers. Because it is not possible to keep track of the evolution of these measures, we employ Krusell and Smith's (1998) "bounded rationality" method which approximates the distribution of workers by a limited number of its moments. In particular, we assume that agents make use of the average asset holdings of the economy and the fraction of workers who are employed. (The computational appendix gives more detail.). We generate 12,000 monthly periods for a model economy. After dropping the first 3,000 observations, we compute quarterly values, take logs, and apply Hodrick-Prescott filter to produce the business cycle statistics.[12]

Key statistics are highlighted in Table 3. In addition to our benchmark and high-volatility economies, for comparison the table provides results for a model with linear utility, exogenous separations, and no shocks to match quality. We refer to this, in Column 2, as the Shimer model because it is similar to the model calibrated in Shimer (2005). Also for comparison, the first column reports the comparable statistics reported by Shimer for quarterly U.S. data for 1951-2003, where note that all standard deviations are expressed relative to that for labor productivity. Shimer points out that the natural log of the unemployment series exhibits volatility, measured by standard deviation, that is 9.5 times that in labor productivity, whereas for his calibrated model unemployment displays lower volatility by a factor of about one half. Comparing results for our Shimer model in Column 2 to the data essentially replicates this finding—here the relative standard deviation of unemployment to productivity falls short of the data by a factor of 16.

The cyclical results for our benchmark economy are given in Column 3. The volatility of unemployment falls very far short of that in the data; its relative standard deviation is only

[12] We use H-P smoothing parameter of 10^5 to be comparable to Shimer's treatment.

one-eighth that observed for the data.[13] Although unemployment is twice as volatile as for the Shimer economy, this increased volatility largely reflects the impact of fluctuations in separations. Volatility of the finding rate, as with the Shimer economy, falls far short of that for the data.

Separations are notably countercyclical for the model: the standard deviation for separations is nearly equal that for unemployment, while the correlation between the rates of separations and unemployment is 0.54. (Separations lead the cycle for the model economy, and so are more highly correlated, 0.85, with the change in unemployment rate.) The correlation between Shimer's data measure of separations and unemployment is even higher at 0.71; but separations for the data show a considerably lower standard deviation than that for unemployment.

Vacancies are actually less volatile for our model than for the Shimer economy. This reflects the model's predicted increase in separations during contractions which, in turn, encourages vacancy creation. The relative standard deviation of vacancies is only 0.6 for the model, compared to the data's 10.1. The model's correlation between the unemployment rate and vacancies is only -0.39, compared to -0.89 for the data. Thus the model generates only a weak Beveridge curve relative to the Shimer model, and especially relative to the data. It is common for models with volatility in separations to generate a weaker Beveridge Curve, reflecting the endogenous response of vacancies to separations. For instance, when Shimer (2005) allows for both labor productivity and separation shocks the correlation between unemployment and vacancies drops from -0.93 to -0.43. (Of course, while models with constant separation rates succeed in generating a more negative correlation between unemployment and vacancies, they do so by predicting counterfactually no volatility in separations.) We follow Mortensen and Pissarides in having endogenous separations. Their correlation between u and v is -0.47. This correlation is -0.39 in our benchmark economy, and -0.53 in the linear-utility version of our benchmark.

If we reduce the number of endogenous separations in half by making match shocks smaller, and label half of separations as purely exogenous, we generate somewhat more cyclical volatility because there are a greater number of workers with low rents from matches. But for this case the relative standard deviation of unemployment, relative to productivity, remains only one-third that in the data, compared to one-eighth for the benchmark. Furthermore, this alternative is more counterfactual than the benchmark in important respects. It differs from the benchmark primarily by generating more cyclical separations, not more cyclical findings. As a result it

[13] Because separations are endogenous, fluctuations in aggregate labor productivity do not equal the fluctuations in exogenous productivity, however their cyclical statistics are very similar for our calibrated models. For the benchmark model the standard deviation of labor productivity is 0.201% compared to 0.208% for productivity (both series quarterly and HP filtered). Both series exhibit a quarterly autocorrelation of 0.83.

generates a standard deviation for the separation rate that is more than five times that for the finding rate, wheras in the data the standard deviation for the finding rate is as large or larger. Reflecting that most of the cyclical action is in separations, it generates a perverse Beveridge curve–unemployment and vacancies are highly positively correlated. Finally, we repeat that this model fails not only cyclically, too little action in unemployment and especially in finding rates, but it also generates far too little cross-sectional dispersion in wage growth in matches, generating only a third of that reported by Topel and Ward.

We turn now to our high-volatility model, with results given in the last column of Table 3. The model by construction generates observed volatility in unemployment. Its standard deviation for unemployment is eight times that produced by our benchmark model. Because it exhibits many workers with little employment surplus, separations are much more volatile than for the benchmark model—the standard deviation of separations is 9 times higher. This model also generates much more cyclical vacancies. This primarily reflects that expected surplus of matches is only about one-tenth that for the benchmark economy. In other words, workers are highly concentrated at the margin. Therefore, a shock to aggregate productivity wields a much bigger percentage impact on expected surplus of matching. The high-volatility economy also generates a considerable skewing of separations during downturns toward workers with higher assets. This shift toward workers with higher assets and higher reservation wages in recessions further drives down the value of vacancy creation. (This channel for volatility is distinctive to our model having both risk aversion and endogenous separations.) To separately quantify the impact of this channel, we constructed a version of our high-volatility model where separations are exogenous, but display the same time series properties as the economy with endogenous separations.[14] We find that the selection of workers into the unemployment pool by assets increases the volatility of unemployment by about 12%.

Despite matching cyclical volatility of unemployment, the high-volatility economy displays the qualitative shortcomings of our benchmark model. In particular, separations are far too cyclical relative to vacancies. This model generates an even weaker Beveridge curve correlation between unemployment and vacancies, -0.16, than the benchmark economy. Finally, we repeat that this model can achieve its cyclicality for unemployment only by displaying a cross-sectional dispersion for wages of just 1.9%. Related to this, it generates a cross-sectional standard devia-

[14] We first estimate a two-variable VAR for productivity and the separation rate on data simulated from our model with endogenous separations, where the separation rate depends on current and lagged productivity as well as its own lag. We then employ the estimated VAR to generate shocks for separations as well as productivity for the model simulations.

tion in wage growth within matches (calculated as in Topel and Ward) of only 1.9%. We view this as implausible as it is an order of magnitude less than reported by Topel and Ward.[15]

6. Conclusions

We have introduced worker heterogeneity, in worker assets and match quality, into a model of separations, matching, and unemployment. We emphasize the trade-off between producing realistic dispersion in wages and wage growth or realistic cyclical fluctuations in unemployment. We can generate very high cyclicality of unemployment, comparable to U.S. data, if shocks to match quality are extremely small and payouts to unemployment are high. But we find this simultaneously implies very little cross-sectional dispersion in wage growth within matches. We consider this implausible, given estimates from micro data of dispersion in wage growth within jobs (especially Topel and Ward). With lower payouts to unemployment, comparable to Shimer's calibration, and considerable match productivity shocks we can generate a realistic dispersion in rates of wage growth. But then the model falls drastically short in capturing cyclical fluctuations in unemployment of the magnitude displayed by the data.

How might the model be extended to overcome this conflict between realistic micro dispersion and realistic aggregate cyclicality, short of dropping wage flexibility? One path to generate more cyclicality would be to modify the model to generate a stronger inverse relationship between a worker's match quality and the worker's marginal utility of consumption–this creates a tighter distribution in the rents from employment and so greater cyclical responses in separations and unemployment. (Our model, because it assumes no insurance and limited borrowing, does generate higher consumption, and lower marginal utility of consumption, for workers who exhibit higher match wages.) But we do not see this as promising. For one, this path would

[15] We considered an intermediate calibration employing a value for B that generates a replacement rate for the unemployed of 70%. This replacement rate is comparable to that employed by Hall (2005b) and by Costain and Reiter (2008). For this intemediate case, the cross-sectional standard deviation of wage growth within matches is still only 7.8%, so well below that reported by Topel and Ward. More importantly, the standard deviation for unemployment only rises modestly compared to the benchmark case, and falls short of that in the data by a factor of five.

We also simulated versions of our benchmark and high-volatility economies with linear utility. For the high-volatility economy risk aversion does not matter much. The replacement rate is so high that wealth effects are largely moot. For the benchmark economy, with plausible dispersion in wage growth, we find that cyclical volatility is even lower under linear utility–by about 25% measured by the standard deviation of unemployment. So our specific conclusion, that we cannot generate both reasonable dispersion in wage growth and much cyclicality, is even more stark under linear utility. The linear utility model has some particularly counterfactual predictions. Because its exhibits no wealth effects, a negative aggregate shock creates a greater spike up in separations. As a result, separations exhibit an autocorrelation of almost zero and are nearly acyclical, with a correlation with the unemployment rate of only 0.08. (For our benchmark model this correlation is 0.54.)

increase the rate of separations to idiosyncratic shocks, so it does not overcome the tradeoff we have highlighted. Secondly, it adds cyclicality in separations, not to vacancies. So it will lead the failure of the models considered to generate a realistic Bevereridge curve. Alternatively one might entertain larger aggregate shocks to labor demand than implied by the volatility of labor productivity. For example, countercyclical price markups generate procyclicality in labor demand not reflected in labor productivity (e.g., Rotemberg and Woodford, 2001). Or firms potentially face shocks, such as disturbances to financing, that make costs of hiring and training more procyclical cost than captured by vacancy posting costs in the model.

A. Computational Algorithm

A.1. Steady-State Equilibrium

In steady state, the aggregate productivity z is constant at its mean and the measures of workers μ and ψ are invariant over time. Computing the steady-state equilibrium amounts to finding *i)* the value functions $W(a,x)$, $U(a)$ and $J(a,x)$, *ii)* the decision rules $a'_e(a,x)$, $a'_u(a)$ and $x^*(a)$, *iii)* the wage schedule $w(a,x)$, *iv)* the labor market tightness θ, *v)* the time-invariant measures $\mu(a,x)$ and $\psi(a)$ that satisfy the equilibrium conditions given in subsection 2.5. The detailed computational algorithm for steady state equilibrium is as follows.

1. Discretize the state space $\mathcal{A} \times \mathcal{X}$ over which the value functions and wages are computed. The stochastic process for the idiosyncratic productivity is approximated by the first-order Markov process of which transition probability matrix is computed using Tauchen's (1986) algorithm.

2. Assume an initial value of θ^0.

3. Given θ^0, we solve the Nash bargaining and individual optimization problems to approximate wages, value functions, and decision rules in the steady state, which will be used to compute the time-invariant measures.

    1. Assume an initial wage schedule $w^0(a,x;\theta^0)$ for each (a,x) node.
    2. Given $w^0(a,x;\theta^0)$, solve for the worker's value functions, $W(a,x;w^0)$ and $U(a;w^0)$, using equations (2.1) and (2.2) in the text. The value functions are approximated using the iterative method. The utility maximization problems in the worker's value functions are solved through the Brent method. The decision rules $a'_e(a,x;w^0)$, $a'_u(a;w^0)$ and $x^*(a;w^0)$ are obtained at each iteration of the value functions.
    3. Compute wages that satisfy the definition of $J(a,x,w^0)$ in (2.3) and the Nash bargaining solution in (2.5) in the text. Specifically, we solve for $w^1(a,x;\theta^0)$ for each (a,x) node that satisfies

    $$w^1(a,x;\theta^0) = zx - J(a,x;w^0) + \beta(1-\lambda)E\left[\max\{J(a'_e,x';w^0),0\}|x\right],$$

    where $J(a,x;w^0)$ is computed using the first order condition for the Nash bargaining problem in (2.5):

$$J(a,x;w^0) = \left(\frac{1-\alpha}{\alpha}\right)\left[W(a,x;w^0) - U(a;w^0)\right] c_e(a,x;w^0).$$

4. If $w^1(a,x;\theta^0)$ and $w^0(a,x;\theta^0)$ are close enough to each other, then move on to the step 4 to compute invariant measures and the corresponding labor market thightness, θ^1. Otherwise, go back to the step 3.1 with a new guess for the wage schedule:

$$w^0(a,x;\theta^0) = \zeta_w w^1(a,x;\theta^0) + (1-\zeta_w)w^0(a,x;\theta^0).$$

4. Using the converged decision rules $a'_e(a,x;w^0)$, $a'_u(a;w^0)$ and $x^*(a;w^0)$ given the converged wage schedule $w^0(a,x;\theta^0)$ from the step 3.2 and 3.1, compute the time-invariant measures $\mu(a,x;\theta^0)$ and $\psi(a;\theta^0)$ by iterating the laws of motion for measures given in (2.6) and (2.7). Then, compute the labor market tightness θ^1 that satisfies the free-entry condition using equation (2.4) and the converged measures:

$$\kappa = \beta q(\theta^1)\int J(a'_u,\bar{x};\theta^0)d\widetilde{\psi}(a'_u;\theta^0).$$

5. If θ^1 and θ^0 are close enough to each other, then we found the steady state. Otherwise, go back to the step 3 with a new guess for the labor market tightness:

$$\theta^0 = \zeta_\theta\theta^1 + (1-\zeta_\theta)\theta^0.$$

A.2. Equilibrium with Aggregate Fluctuations

Approximating the equilibrium in the presence of aggregate fluctuations requires us to include the aggregate productivity, z, and the measures of workers, μ and ψ, as state variables for agents' optimization problems. In order to make match separation decisions at the end of a period, agents need to know their matching probabilities in the next period, $p(\theta_{t+1})$ and $q(\theta_{t+1})$, which in turn depends on the next period's measures of workers, $\mu_{t+1}(a,x)$ and $\psi_{t+1}(a)$. The laws of motion for the measures are given in equations (2.6) and (2.7). It is impossible to keep track of the evolution of these measures. We employ Krusell-Smith's (1998) "Bounded Rationality" method which approximates the distribution of workers by a number of its moments. We assume that agents in the economy make use of two first moments of the measures: the average asset holdings of the economy, $K = \int a\, d\mu(a,x) + \int a\, d\psi(a)$, and the number of employed workers,

$E = \int d\mu(a,x)$. Let $\hat{\mathbf{s}}$ denote a vector of aggregate state variables in the approximation of equilibrium with fluctuations. Then $\hat{\mathbf{s}} = (K, E, z)$. In addition we assume that the agents use log-linear rules in predicting the current θ, the future K and the future E.

1. Guess a set of prediction rules for the equilibrium labor market tightness (θ) in the current period, the average asset of the economy (K') and the number of employed workers (E') in the next period. This step amounts to setting the coefficients of the log-linear prediction rules:

$$\log \theta = b^0_{\theta,0} + b^0_{\theta,1} \log K + b^0_{\theta,2} \log E + b^0_{\theta,3} \log z$$

$$\log K' = b^0_{K,0} + b^0_{K,1} \log K + b^0_{K,2} \log E + b^0_{K,3} \log z$$

$$\log E' = b^0_{E,0} + b^0_{E,1} \log K + b^0_{E,2} \log E + b^0_{E,3} \log z.$$

   As is the case in the steady state computation, we approximate the stochastic process for the aggregate productivity by the first-order Markov process of which transition probability matrix is computed using Tauchen's (1986) algorithm.

2. Given these prediction rules, we solve the individual optimization and wage bargaining problems. This step is analogous to step 3 in the steady state computation, so we omit the detailed description of computational procedure. However, the dimension of state variables is now much larger: $(a, x, \hat{\mathbf{s}})$. Computation of the conditional expectations involves the evaluation of the value functions not on the grid points along K and E dimensions since K' and E' are predicted by the log-linear rule above. We polynomially interpolate the value functions along the K dimension when necessary.

3. We generate a set of artificial time series data $\{\theta_t, K_t, E_t\}$ of the length of 9,000 periods. Each period, these aggregate variables are calculated by summing up 50,000 workers' decisions on asset accumulation and match separation, which are simulated using the converged value functions, $W(a, x, \hat{\mathbf{s}})$, $U(a, \hat{\mathbf{s}})$, and $J(a, x, \hat{\mathbf{s}})$, the decision rules, $a'_e(a, x, \hat{\mathbf{s}})$, $a'_u(a, \hat{\mathbf{s}})$ and $x^*(a, \hat{\mathbf{s}})$ from the step 2, and the assumed prediction rules for θ, K' and E' from the step 1.

4. We obtain the new values for the coefficients (b^1's) in the prediction functions through the OLS using the simulated data from the step 3. If b^0 and b^1 are close enough to each other, then we find the (limited information) rational expectations equilibrium with aggregate fluctuations. Otherwise, go back to the step 1 with a new guesses for the coefficients in

the prediction functions:

$$b^0_{i,j} = \zeta_b b^1_{i,j} + (1-\zeta_b) b^0_{i,j},$$

where $i = \theta, K, E$ and $j = 0, \cdots, 3$.

The converged prediction rules and their accuracy, measured by R^2, for the benchmark calibration with $h = 1$ are as follows.

- Prediction for labor market thightness in the current period:

$$\log\theta = 1.934 - 0.05810 \log K + 0.4220 \log E + 0.14804 \log z, \qquad R^2 = 0.9971$$

- Prediction for average asset holdings in the next period:

$$\log K' = 0.0096 + 0.9965 \log K - 0.0071 \log E + 0.0457 \log z, \qquad R^2 = 0.9999$$

- Prediction for number of employed workers in the next period:

$$\log E' = -0.0182 - 0.0015 \log K + 0.6361 \log E + 0.0276 \log z, \qquad R^2 = 0.9538$$

Overall, the estimated prediction rules are fairly precise as R^2's are close to 1, while the prediction rule for average asset holdings provides the highest accuracy.

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