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Duration of sovereign debt renegotiation☆

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ABSTRACT

In the period since 1990, sovereign debt renegotiations take an average of five years for bank loans but only one year for bonds. We provide an explanation for this finding by highlighting one key difference between bank loans and bonds: bank loans are rarely traded, while bonds are heavily traded on the secondary market. In our theory, the secondary market plays a crucial information revelation role in shortening renegotiations. Consider a dynamic bargaining game with incomplete information between a government and creditors. The creditors' reservation value is private information, and the government knows only its distribution. Delays in reaching agreements arise in equilibrium because the government uses costly delays to screen the creditors' reservation value. When the creditors trade on the secondary market, the market price conveys information about their reservation value, which lessens the information friction and reduces the renegotiation duration. We find that the secondary market tends to increase the renegotiation payoff of the government but decrease that of the creditors while increasing the total payoff. We then embed these renegotiation outcomes in a simple sovereign debt model to analyze the ex ante welfare implications. The secondary market has the potential to increase the government ex ante welfare when the information friction is severe.

1. Introduction

During the 1980s, developing countries experienced prolonged periods of financial limbo as they renegotiated their debt contracts with foreign commercial banks following sovereign default. Evidence from this period suggests that reaching an agreement with creditors took an average of nine years. During the renegotiation period, governments faced decreased access to global financial markets, hampering economic growth and investment. These costly and protracted renegotiations, arising from the coordination failure between private creditors and sovereign debtors, became a major concern of policymakers.

As sovereign borrowing shifted from using bank loans to using bonds in the 1990s, the concern about the lengthy process of debt renegotiations has intensified. The counterparties of sovereign bank-debt renegotiations are large commercial banks, a fairly concentrated and homogeneous group. In contrast, the counterparties of sovereign bond renegotiations range from individual investors to governments to institutional investors. Renegotiations were expected to become more prolonged because the coordination problem seems even more difficult for bond renegotiations.[1] Surprisingly, the data shows that it takes only an average of about one year to conclude bond renegotiations. Thus, understanding this puzzling reduction in renegotiation durations is of both policy and academic relevance.

We provide a theory that potentially explains the difference in the renegotiation duration between bank and bond debt. Our theory highlights one key difference between bank loans and bonds: bank loans are rarely traded, while bonds are heavily traded on the secondary market. In our theory, the secondary market plays a crucial information revelation role in shortening renegotiations. When negotiating with the creditors whose reservation value for repayments is private information, the government uses costly equilibrium delays to screen the creditors' type. A greater information friction tends to lengthen the renegotiation. When the creditors trade before the renegotiation, the secondary market price conveys information about their reservation, lessening the information friction. Thus, the presence of secondary

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[1] Among many others, Krueger (2002), Kroszner (2003), and Bolton and Jeanne (2007) expressed their concern that bond renegotiations may be more difficult and prolonged than bank loan renegotiations. In one of her addresses in 2002, International Monetary Fund First Deputy Managing Director Anne Krueger said: "The move from commercial bank loans to bond issuance in the 1980s and 1990s has made creditor coordination much more cumbersome. This in turn has made it more difficult for all concerned to predict how the restructuring process will unfold."

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market trading might shorten the renegotiation for bond debt relative to bank debt, consistent with the data.

We start by analyzing sovereign debt renegotiations without the secondary market. Our model builds on a dynamic bargaining game with private information. A government negotiates its defaulted debt with creditors. During the renegotiation, the government suffers a loss in constant output, and the creditors can seize a fraction of the output loss. This fraction is private information of the creditors and becomes their common reservation value. The government is informed only about the distribution of this reservation value. In each period, the government makes a restructuring proposal as a share of the output loss, and the creditors decide whether to accept. If they accept, the government repays the proposed offer and avoids the output loss. Otherwise, the renegotiation continues to the next period.

Private information is key to generating delays in a perfect Bayesian equilibrium. Without private information, the government proposes the reservation value and the creditors accept immediately.[2] With private information, the government would need to propose the highest reservation to ensure an immediate agreement, and would obtain the lowest possible payoff. A lower offer might delay the agreement when the creditors indeed have a high reservation value, but it increases the government payoff if the reservation is low and the agreement is reached today. Thus, costly equilibrium delays arise as a screening device of the creditors' type. Moreover, the maximum renegotiation duration decreases with the government's precision of the information regarding the creditors' reservation value.

We next analyze sovereign debt renegotiation with the secondary market. Specifically, we allow the creditors to trade on the secondary market before the renegotiation starts. When trading, the creditors have not learned their reservation value, but each of them receives a signal about it. The distributions of the reservation and signals, together with the secondary market price, are public information. Each creditor decides whether to buy an additional unit of bond, to sell, or to hold his bond. A random fraction of the creditors are noisy traders who sell regardless of their signals. After trading, the bondholders observe their reservation value and renegotiate with the government.

The maximum renegotiation duration declines with the introduction of the secondary market. The key mechanism is that the price conveys information about the creditors' reservation value, which lessens the information friction and shortens the renegotiation. When the underlying reservation is high, the creditors tend to receive high signals and expect high renegotiation payoffs, which increase the demand for bonds and thus the market price. Upon observing the market price, the government updates its belief about the distribution of the reservation. Thus, the information friction is reduced and the delays are shortened. In the extreme case where the price is fully revealing, perfect information outcomes arise, and there is no delay in renegotiation.

Finally, we conduct welfare analysis both ex post and ex ante. The ex post renegotiation outcomes, endogenously arising from the dynamic bargaining game, depend on the information friction and the presence of the secondary market. For the ex ante welfare analysis, we embed these renegotiation outcomes in a sovereign debt model with default risk. The government borrows from the competitive, risk-neutral creditors to invest in a project with stochastic returns. The government can renege on its debt after the realization of the return, and then negotiate with the creditors. The ex post renegotiation duration determines whether the government can achieve the ex ante efficient welfare.[3] Whenever there are delays, an efficiency loss occurs and the ex ante welfare is lower than the efficient level.

Ex post the secondary market tends to increase the total and government payoff, but lower the creditors' payoff, through reducing delay and the information rent. Ex ante the secondary market has no impact on the creditors' welfare because they always break even, but it does have the potential to increase the government welfare. Lowering the creditors' payoff ex post reduces the government welfare ex ante through a worsening of the terms of contracts. Increasing the total and government payoff ex post tends to increase the government welfare ex ante, since investment increases when the efficiency loss decreases and the default is less costly. The second effect tends to dominate when the information friction is severe.

The model predicts that the duration of sovereign debt renegotiations decreases with the presence of secondary market trading, which is consistent with empirical evidence. When examining the episodes of sovereign debt renegotiations after 1990 in the data, we find that the average renegotiation duration is much shorter for liquid bonds than for illiquid bank loans: one year versus five years. The model also has implications on the haircut rate, which equals one minus the ratio of the present value of restructured repayments to the present value of original debt obligations. The secondary market trading reduces the information rent of the creditors and so lowers the repayment, which tends to increase the haircut rate. On the other hand, the secondary market trading also speeds up the renegotiation process and so increases the present value of the repayments, which tends to decrease the haircut rate. The equilibrium haircut rate depends on the relative strength of the two forces. Under reasonable parameterization, the model implies that the haircut rate is lower with the secondary market trading. This implication is consistent with the data; the average haircut rate is 31% for bank loans and 17% for bonds.

The contribution of this paper is primarily theoretical in terms of making headway toward understanding the duration of sovereign debt renegotiations. Our model highlights a neglected difference between bond and bank debt in terms of the presence of the secondary market to explain the puzzling observation that bond debt renegotiations are much shorter than bank debt renegotiations.[4] To our knowledge, our paper is the first in the literature to propose a concrete explanation for this observation. Our model and the empirics, however, should not be taken as conclusive evidence that our explanation is the underlying cause of the difference in renegotiation length between bond debt and bank debt.

Our work builds on a large body of theoretical work on dynamic bargaining games with incomplete information. These theories have been applied to a broad range of economic issues, such as union strikes and durable goods monopolies.[5] Ausubel et al. (2002) provide an excellent survey of the literature and classify it into two branches — mechanism design and sequential bargaining — based on the approach. We adopt the sequential bargaining approach because empirically, sovereign governments make sequential offers to the creditors during renegotiations. Specifically, our model builds on Fudenberg et al. (1985), where the uninformed party uses costly delays to screen the type of the informed party.

[2] We assume that the creditors always accept whenever they are indifferent.

[3] By "efficient welfare", we mean the optimal welfare in the case of no default risk.

[4] Any other features associated with bonds but not with bank loans can be potential contributors to shorter delays in bond renegotiations as well. That said, two alternative key differences between the two types of debt seem to imply that the renegotiation length would be longer for bond debt. First, the difference in the counter-parties between the two types of debt, as we mentioned earlier, raised the concern that bond renegotiations would be more prolonged due to more difficult coordination problems. Second, creditors of bank debt are generally easy to identify, while bondholders are difficult to identify because the bonds are so actively traded. Given this difference, bond renegotiations are expected to be lengthier because it is much more difficult for borrowing countries to contact the existing bondholders. Peterson (1999) wrote "There are several things that make international bonds much harder to restructure than loans. First, they typically involve many more investors than do loans, even syndicated loans. Second, they may be in bearer form so investors may be untraceable."

[5] Among others, Cramton and Tracy (1992) and Hart (1989) study union strikes; Stokey (1981) and Sobel and Takahashi (1983) examine the durable goods monopoly.

This paper also relates to the theoretical literature on sovereign debt renegotiations. One strand of this literature focuses on the coordination failure between creditors,[6] and the other strand focuses on the coordination failure between a government and creditors. Our work belongs to the latter, most of which is cast in a complete information environment. Bulow and Rogoff (1989) and Fernandez and Rosenthal (1990) analyze the ex ante impact of renegotiation outcomes from a Rubinstein bargaining game with complete information. Yue (2010) introduces a Nash bargaining game into a sovereign debt model developed by Eaton and Gersovitz (1981). In these works, there are no equilibrium delays in reaching agreements. Two recent works, Benjamin and Wright (2008) and Bi (2008), incorporate a dynamic bargaining game with uncertainty into the Eaton–Gersovitz model, as in Merlo and Wilson (1995). Delays arise because both the debtor country and the creditors prefer to wait for a good future shock to split a large "pie." Our work instead focuses on the role of information frictions.

Our work also relates to Broner et al. (2010), who show that the possibility of retrading assets in the secondary market increases borrowing and welfare ex ante. The reason is that by transferring debts from foreign to domestic creditors in periods of financial turmoil, the secondary market reduces the default incentives ex post. Our work highlights that even when defaults do occur in equilibrium, the secondary market still might increase ex ante borrowing and welfare.

The paper is organized as follows. To highlight the mechanisms affecting renegotiation outcomes, we focus on the ex post renegotiation duration in Sections 2 and 3 and study the ex ante and ex post welfare implications in Section 4. In particular, Section 2 studies the renegotiation duration without the secondary market, and Section 3 analyzes the renegotiation duration with the secondary market. We conclude in Section 5.

2. Renegotiation without secondary market

In this section, we analyze the sovereign debt renegotiation without the secondary market in a dynamic non-cooperative bargaining game with one-sided incomplete information as in Fudenberg et al. (1985). The implied renegotiation outcomes will serve as a benchmark for comparison when we analyze the sovereign debt renegotiation with the secondary market in the next section. To highlight the coordination problems between the creditors and the government, we abstract from the coordination problems among the creditors.

2.1. Model

There are two parties in the model: a sovereign government and a continuum of creditors of measure one. The creditors have equal shares of sovereign debt. At date 1, the government defaults on its debt and starts to negotiate with the creditors. Assume that the government has a deterministic output process: $y_t = y$ for any t. In each period, the government proposes a restructuring plan that specifies a per-period payment b to the creditors. The creditors decide whether to accept the proposal. We assume that they accept whenever they are indifferent. If a critical mass of the creditors accept, the renegotiation ends: the government has a per-period payoff $y-b$, and each creditor has a per-period payoff b. Otherwise, the renegotiation continues to the next period. With no agreement, we assume that the government suffers an output loss of fraction γ,[7] and the creditors can capture only a fraction s of this output loss, which is divided among the creditors according to their shares of sovereign debt.[8]

Both parties have a discount factor $\beta<1$ and maximize the present value of future payoffs. The government obtains a per-period payoff $(1-\gamma)y$ regardless of whether the proposal is accepted, and negotiates with the creditors to split per-period payoff γy. Clearly, the creditors never accept an offer lower than $s\gamma y$. We interpret s as the reservation value of the creditors in the renegotiation. We assume that the creditors have private information about s, and the government observes only its distribution: s is uniformly distributed on $[s_l, s_h] \subseteq [0,1)$. The information asymmetry can be understood as follows. The creditors obtain sufficient information about the government before making loans and while monitoring the loans. The government, however, has little information about the reservation value of the creditors.

All the creditors have a common reservation value, and they will either all accept or all reject in renegotiations. Thus, the critical mass, needed to conclude the renegotiation, has little impact on the renegotiation results. Alternatively, one can interpret the model as the renegotiation between one debtor country and one creditor, since all the creditors are identical.

In each period t, the information set of the government is a history of rejected offers $h_t = \{b_1, b_2, \ldots, b_{t-1}\}$, and the information set of the creditors is the same history concatenated with the current offer (h_t, b_t). A system of beliefs for the government is a mapping from its information set into a probability distribution g_t over s (let G_t denote the cumulative distribution). The government's strategy maps its information set h_t into an offer b_t. The creditors' strategy maps their information set into either rejection or acceptance. We define a *perfect Bayesian equilibrium* as follows.

Definition 1. A *perfect Bayesian equilibrium* is a system of beliefs for the government, and a pair of strategies for the government and the creditors, such that the government's beliefs are consistent with Bayes' rule (whenever it is applicable) and the strategies of the government and the creditors are optimal after any history given the current beliefs.

Dynamic bargaining games typically have a plethora of equilibria.[9] In our model, however, there exists a unique perfect Bayesian equilibrium. One key assumption behind uniqueness is that the (uninformed) government makes offers to the (informed) creditors. If we allow the informed party to make offers, the signaling mechanism in general leads to multiple equilibria. The other key assumption is $s_h<1$. This implies that the renegotiation is a "gap" game, in which the government can always gain from reaching an agreement. The sure gain of the government is $(1-s_h)\gamma y$. Thus, the renegotiation ends in finite periods T because the potential surplus that the government might hope to extract eventually becomes insignificant compared to the sure gain. Fudenberg et al. (1985) show under these two assumptions that the perfect Bayesian equilibrium is unique.

We now characterize the strategies of the creditors and the government along the equilibrium path. The equilibrium has the Markov property in the sense that the government's strategy depends on its belief, updated by the last rejected offer alone, and the creditors' strategy depends only on the current offer in equilibrium. In the last period T, the government proposes $b_T = s_h\gamma y$. In period $T-1$, suppose the government proposes b_{T-1}. The creditors with a reservation value s decide

[6] See Kletzer (2003), Eichengreen et al. (2003), Haldane et al., (2005), Weinschelbaum and Wynne (2005), and Pitchford and Wright (2007, 2008).

[7] This loss in output could come from various channels: denied access to financial markets, loss of trade credits, or disruption of the domestic financial systems.

[8] Following Bulow and Rogoff (1989), we assume that the creditors seize some payoff during the renegotiation to capture the idea that firms in the debtor country have to pay the creditors higher fees to obtain trade credits or conduct transactions while the government is in arrears on its debt.

[9] See Ausubel et al. (2002) for a detailed discussion.

whether to accept b_{T-1} or to wait for the next period. If the reservation value s is small enough such that

$$\frac{b_{T-1}}{1-\beta} \geq s\gamma y + \frac{\beta s_h \gamma y}{1-\beta},$$

then the creditors find it optimal to accept in period $T-1$.

In general, suppose that the government proposes b_t in period t and is expected to propose b_{t+1} Ênext period if offer b_t is rejected. The creditors will accept the offer if and only if

$$\frac{b_t}{1-\beta} \geq s\gamma y + \frac{\beta b_{t+1}}{1-\beta}.$$

That is, the creditors will accept b_t if and only if their reservation is below a cutoff level $\underline{S}_{t+1}(b_t)$, given by

$$\underline{S}_{t+1}(b_t) = \frac{b_t - \beta b_{t+1}}{(1-\beta)\gamma y}. \tag{1}$$

Thus, with private information, the creditors can pretend to have a higher reservation value than the true one. However, the creditors do not necessarily pretend to have the highest possible reservation value s_h and wait until the last period.

Given the creditors' strategy, the government understands that the creditors' reservation is at least $\underline{S}_{t+1}(b_t)$ if the offer b_t is rejected. Thus, the government truncates the belief from below: the updated belief in period $t+1$ is a uniform distribution on interval $[\underline{S}_{t+1}(b_t), s_h]$. This implies that the government's posterior belief can be characterized with one number $\underline{S}_{t+1}(b_t)$, the lower bound of the reservation interval.

Thus, if the government believes that the creditors' reservation is higher than $\underline{s}$ in period t, then the government's optimal strategy solves the following problem:

$$V_t(\underline{s}) = \max_{b_t} \left\{ \Lambda_t(\underline{s}, b_t)\frac{y-b_t}{1-\beta} + (1-\Lambda_t(\underline{s}, b_t))\left[(1-\gamma)y + \beta V_{t+1}(\underline{S}_{t+1}(b_t))\right] \right\}, \tag{2}$$

where $\Lambda_t(\underline{s}, b_t)$ denotes the acceptance probability of offer b_t, given by $\frac{\underline{S}_{t+1}(b_t)-\underline{s}}{s_h-\underline{s}}$ under the uniform distribution. A higher offer increases the probability of the acceptance but lowers the acceptance payoff of the government. Taking this trade-off into account, the government might find it optimal to delay the agreement. We denote the government's optimal strategy by $B_t(\underline{s})$.

We now summarize features of the equilibrium strategies and outcomes. First, the government's proposal $B_t(\underline{s})$ increases with belief $\underline{s}$, and its posterior belief $\underline{S}_{t+1}(b)$ increases with rejected offer b. Second, in equilibrium the government proposes an increasing sequence of offers $\{b_1, b_2, \ldots, b_T\}$, and the creditors accept in period t when reservation s falls between s_t and s_{t+1}, where $s_1 = s_l$, $s_{T+1} = s_h$, and $s_t = \underline{S}_t(b_{t-1})$ for any $t = 2, \ldots, T$. Clearly, higher reservation values lead to longer renegotiations and higher repayments. Third, the creditors collect information rent; the accepted offer is always at least as high as their reservation.

To illustrate the equilibrium strategies and outcomes transparently, we present a numerical example, where $\beta = 0.95$, $\gamma y = 1$, and $[s_l, s_h] = [0.005, 0.995]$.[10] In Fig. 1, we plot the government's strategy with a solid line and its updated belief with a dashed line, as a function of belief $\underline{s}$.[11] We trace out the equilibrium proposals with dark squares and the equilibrium belief cutoffs with light squares. The maximum renegotiation duration is 29 periods. The government proposes over time 0.77, 0.80, 0.83, etc. The creditors accept in period 1 if their reservation is below 0.18, in period 2 if between 0.18 and 0.29, in period 3 if between 0.29 and 0.38, etc.

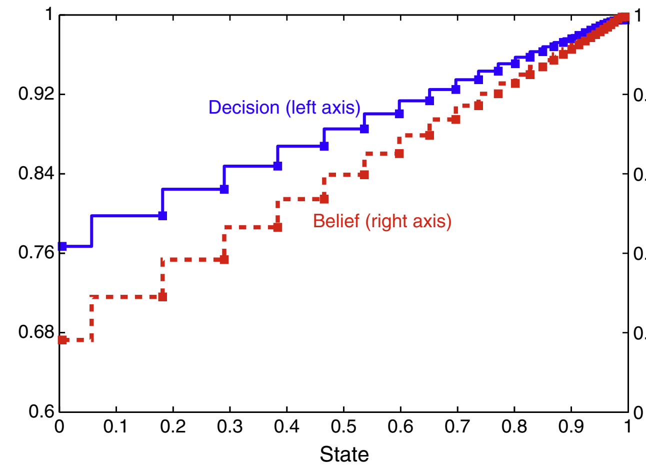

Fig. 1. Government's optimal strategy.

2.2. Renegotiation duration without secondary market

Now we study our key interest: duration of sovereign debt renegotiation. In particular, we are interested in how the information friction impacts the renegotiation length. We measure the degree of information friction, denoted by Ψ, as follows:

$$\Psi = \frac{1-s_l}{1-s_h}, \tag{3}$$

with a higher Ψ indicating a higher degree of information friction.

Let $T(s,[s_l,s_h])$ denote the renegotiation duration when the reservation is s. Denote the maximum renegotiation length by $\hat{T}([s_l,s_h])$. Clearly, the maximum renegotiation length is obtained when the reservation is s_h. That is, $\hat{T}([s_l,s_h]) = T(s_h,[s_l,s_h])$. We find that the maximum renegotiation length increases with the degree of the information friction Ψ. The economic intuition for these results is as follows. The amount $1-s_l$ is the largest possible payoff of the government, and $1-s_h$ is the sure payoff if the government ends the renegotiation immediately. A larger Ψ means that the maximum potential payoff increases relative to the sure payoff. Thus, the government has a greater incentive to use costly equilibrium delays to screen the creditors' type.

Proposition 1. *The maximum renegotiation length $\hat{T}([s_l,s_h])$ increases with the information friction Ψ; it rises as s_h increases, or as s_l decreases, or as interval $[s_l,s_h]$ shifts to the right.*

Proof. See Technical appendix. Q.E.D.

We next look at the expected renegotiation length. We define the expected renegotiation length $\overline{T}$ as

$$\overline{T}([s_l,s_h]) = \int_{s_l}^{s_h} T(s,[s_l,s_h])dG(s). \tag{4}$$

Due to the complexity of the solution, we characterize the expected renegotiation length numerically. We set $\beta = 0.95$ and $\gamma y = 1$, and explore how the expected renegotiation length varies with the information friction Ψ. The results are illustrated in Fig. 2. As Ψ increases, the expected renegotiation length displays an upward trend. That is, as the information friction rises, the renegotiation tends to become longer in expectation. The wiggles are driven by discrete time periods. When an increase in Ψ does not change the maximum renegotiation length,

[10] As we will discuss later in Section 4.3, these parameter values are calibrated to match the average renegotiation duration and the average haircut rate observed for the bank debt renegotiations after 1990.

[11] The detailed solution algorithm is available from the authors upon request.

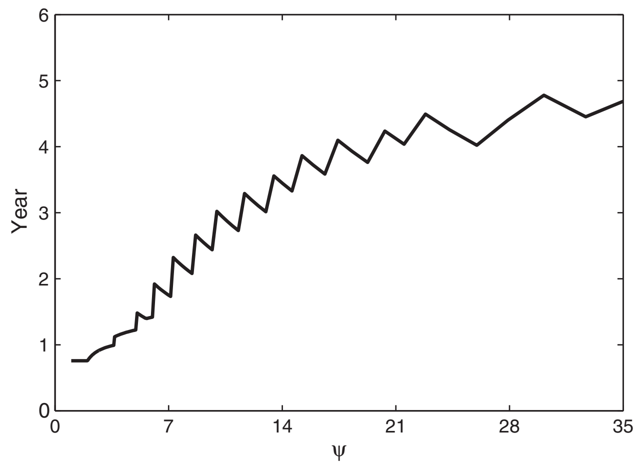

Fig. 2. Expected renegotiation length: no secondary market.

the probability of reaching agreement increases for period 1 and decreases for all future periods. Thus, the expected renegotiation length decreases. When a further increase in Ψ drives up the maximum renegotiation length, the expected renegotiation length increases.

The interpretation of the model renegotiation length in terms of the empirical duration depends on how frequent the government proposes. The frequency of the government proposals is in turn determined by the relative magnitude of the discount factor β and the annual interest rate r: $log(1/\beta)/log(1+r)$. Let's assume the interest rate to be 7%. The example above with β set at 0.95 implies that the government makes about one proposal per year. Thus, each renegotiation period corresponds to a year.

In addition to the information friction, the discount factor β affects the renegotiation duration. A larger β increases not only the expected number of the renegotiation periods but also the frequency of the government proposals. We experiment with different values of β and find that a larger discount factor reduces the renegotiation duration (see Table 1). When β is set at 0.983, the frequency of proposals is four per year, and the corresponding year per period is one quarter. The renegotiation takes 9.88 periods or 2.47 years to complete. When the discount factor further increases to 0.999, the length of renegotiation decreases further to 0.56 years.

In sum, we use a classic dynamic bargaining game with incomplete information to model sovereign debt renegotiation without a secondary market. The information friction plays a crucial role in generating equilibrium delays in reaching agreements. If there is no private information, the agreement is reached immediately. With private information, delays arise in equilibrium because the government does not want to make a proposal too high in the early stages of the renegotiation to miss the likelihood of facing creditors with a low reservation. Furthermore, the renegotiation lengthens as the information friction rises.

3. Renegotiation with secondary market

We now allow the creditors to trade on the secondary market before the renegotiation starts. At the trading stage, the creditors have not learned their reservation value, but each of them receives a signal about it. Based on this signal and the market price, the creditors trade their claims (bonds) on the secondary market. To highlight the role of the secondary market, we model the renegotiation process the same as in the previous section. We demonstrate that the secondary market price mitigates the information friction and reduces the renegotiation duration. We also present some empirical evidence for this prediction.

Table 1
Sensitivity analysis over β.

Discount factor	Delay in periods	Year-per-period	Delay in years
0.87	3.66	2.0	7.32
0.95	6.00	0.76	4.53
0.983	9.88	0.25	2.47
0.999	37.95	0.01	0.56

3.1. Model with secondary market

The government defaults at the beginning of period 1. Trading occurs immediately after default; the creditors buy or sell bonds on the secondary market. The government starts to renegotiate with the remaining bondholders at the end of this period. The detailed timing is presented in the ex-post stage portion of the Timing Appendix (Fig. 6). The creditors' reservation s is uniformly distributed on $[s_l, s_h]$, which is public information. Each creditor receives a signal z about s, where $z = s + \sigma_z \varepsilon$ with ε uniformly distributed on $[-1, 1]$. Each creditor can either hold, sell, or buy one unit of bonds.[12] The payoff of selling is the market price p. The payoff of holding or buying depends on the expected payoff from the renegotiation, conditional on the private signal z and the public information p. A random fraction α of the creditors are noisy creditors; they sell their bonds regardless of their signals. The ratio of noisy and non-noisy creditors $\alpha/(1-\alpha)$ is given by

$$\frac{\alpha}{1-\alpha} \equiv \sigma_\eta \eta, \tag{5}$$

where η is a random variable and uniformly distributed on $[0, 1]$, and $0 < \sigma_\eta < 1$.

The renegotiation starts after trading in period 1. The reservation value s is revealed to all the creditors with bonds, but not to the government. The government makes a proposal each period until a critical fraction κ of the creditors accept. The payoffs during the renegotiation are the same as before. Our model results again are independent of the critical mass κ because all the creditors are ex post identical.

We restrict the trading strategy of the creditors to be *monotonic*: the creditor buys when his signal z is more than $\hat{z}(p)$, and sells otherwise. We first define the *monotonic perfect Bayesian equilibrium*. We then establish that the monotonic perfect Bayesian equilibrium exists and is unique.

Definition 2. A *monotonic perfect Bayesian equilibrium* consists of a market price, the beliefs of the government and the creditors, a monotonic trading strategy of the creditors in the trading stage, and a pair of strategies for the government and the creditors and a system of beliefs for the government in the renegotiation stage, such that (i) in the renegotiation stage, the government's beliefs are consistent with Bayes' rule and the strategies of the government and the creditors are optimal at any history; (ii) in the trading stage, the monotonic trading strategy is optimal given the creditors' belief, and the beliefs of the government and the creditors are consistent with Bayes' rule; and (iii) the secondary market clears.

Proposition 2. *There exists a unique monotonic perfect Bayesian equilibrium.*

Proof. See Technical appendix. Q.E.D.

We now demonstrate the key reasonings for this proposition and delegate the formal proof to the Technical appendix. The secondary market trading influences the renegotiation outcomes because the

[12] The assumption on the upper bound of trading makes our analysis simple and transparent, but it is not essential for our main findings. For details, see Section 3.4.

government updates its belief using the secondary market price. For any given (s,η), the creditors' monotonic trading strategy $\hat{z}(p)$ implies that the excess demand of non-noisy creditors $X(p;s,\eta)$ is given by

$$X(p;s,\eta) = (1-\alpha)\left[P(z>\hat{z}(p)|s) - P(z\leq\hat{z}(p)|s)\right], \tag{6}$$

where $P(z>\hat{z}(p)|s)$ denotes the probability of signals above $\hat{z}(p)$, i.e., the amount of bonds demanded, and $P(z\leq\hat{z}(p)|s)$ denotes the amount of bonds supplied. The excess supply of noisy creditors is α. Since z is uniformly distributed on $[s-\sigma_z, s+\sigma_z]$, the market clearing condition implies

$$\frac{s-\hat{z}(p)}{\sigma_z} = \frac{\alpha}{1-\alpha} = \eta\sigma_\eta. \tag{7}$$

Therefore, the government infers that s is uniformly distributed on $\left[\hat{z}(p), \hat{z}(p)+\sigma_z\sigma_\eta\right]$ when observing the market price p. Together with its prior, the government updates its belief about s to be uniform on the interval $[s_l^g, s_h^g]$, where $s_l^g = max\{\hat{z}(p), s_l\}$ and $s_h^g = min\{\hat{z}(p)+\sigma_z\sigma_\eta, s_h\}$. Note that for each realization of (s,η), there is a unique cutoff signal $\hat{z}$ that clears the bond market. The government then forms its renegotiation strategy according to its updated belief $[s_l^g, s_h^g]$, as discussed in the previous section.

At the beginning of the renegotiation, the reservation value is known to all the bondholders. For each possible reservation value $s\in[s_l^g, s_h^g]$, the bondholders will obtain the following renegotiation payoff $W(s,\hat{z}(p))$, given the government strategy,

$$W(s,\hat{z}(p)) = \frac{\beta^{T(s,[s_l^g,s_h^g])-1} b_{T(s,[s_l^g,s_h^g])}}{1-\beta} + \sum_{t=1}^{T(s,[s_l^g,s_h^g])-1} \beta^{t-1} s\gamma y, \tag{8}$$

where $T(s,[s_l^g,s_h^g])$ is the period in which the creditors with reservation s accept the government proposal. The renegotiation payoff $W(s,\hat{z}(p))$ increases with reservation s because high-reservation creditors can always imitate low-reservation creditors' strategy.

In the trading stage, each creditor calculates the expected renegotiation payoff based on both the market price p and his own signal z. This implies that the creditors have better information about the underlying reservation value than the government. Specifically, the updated belief of a creditor with signal z is uniform on $[s_l^c(z), s_h^c(z)]$, where $s_l^c(z) = \max\{z-\sigma_z, s_l^g\}$ and $s_h^c(z) = \max\{z+\sigma_z, s_h^g\}$. Thus, the expected payoff, denoted by $W^e(\hat{z}(p), z)$, is given by

$$W^e(\hat{z}(p), z) = \int_{s_l^c(z)}^{s_h^c(z)} \frac{W(s,\hat{z}(p))}{s_h^c(z)-s_l^c(z)} ds. \tag{9}$$

A higher signal z implies that reservation s is likely to be higher. Thus, the expected renegotiation payoff increases with signal z.

Based on their expected renegotiation payoffs, the creditors decide on the trading strategy. For a creditor with signal z, the payoff to sell is p, the payoff to hold is $W^e(\hat{z}(p), z)$, and the payoff to buy is $-p+2W^e(\hat{z}(p), z)$. Clearly, holding is always weakly dominated by either selling or buying. Moreover, creditor z will sell if and only if $p\geq W^e(\hat{z}(p), z)$. At price p, the cutoff creditor $\hat{z}(p)$ is indifferent between selling or buying, i.e.,

$$p = W^e(\hat{z}(p), \hat{z}(p)). \tag{10}$$

Since the expected payoff from renegotiation increases monotonically with signal z, the creditors with higher signals (weakly) prefer buying to selling, and vice versa.

Thus, Eqs. (7) and (10) characterize the monotonic perfect Bayesian equilibrium of the model. For any given p, there might be multiple cutoff signals which solve Eq. (10) due to the discrete time periods. For each realization of (s,η), however, there is a unique cutoff signal $\hat{z}$ which satisfies Eq. (7) and clears the secondary market. This cutoff signal $\hat{z}$ characterizes the equilibrium monotonic trading strategy and also determines the equilibrium price p^*. Fixing η, an increase in s will lead to a rise in p^* because a high s generates high signals and increases the expected renegotiation payoff. Fixing s, an increase in η will lead to a decrease in p^* because a large supply of bonds from noisy traders drives down the secondary market price.

3.2. Renegotiation duration with secondary market

We now characterize the maximum renegotiation duration with the secondary market. For each pair of realization (s,η), the government updates its belief of the creditors' reservation to the interval $[s_l^g, s_h^g]$, where $s_l^g = max\{\hat{z}(p), s_l$ and $s_h^g = min\{\hat{z}(p)+\sigma_z\sigma_\eta, s_h\}$, and $\hat{z}(p) = s-\eta\sigma_z\sigma_\eta$ is given by the market clearing condition. Let's denote this interval with $\Omega(s,\eta)$. The renegotiation duration is given by $T(s,\Omega(s,\eta))$ accordingly. We define the maximum renegotiation duration with the secondary market as

$$\hat{T}^M([s_l, s_h]) = \max_{(s,\eta)}\{T(s,\Omega(s,\eta))\}.$$

We now evaluate the impact of the secondary market on the maximum renegotiation duration. We find that the maximum renegotiation duration is shorter with the secondary market than without the secondary market, that is, $\hat{T}^M([s_l, s_h])\leq\hat{T}([s_l, s_h])$. The key to this result is that as long as the secondary market price is somewhat informative, the government will form an updated belief which is more precise than its ex ante belief. Thus, the maximum renegotiation length is shortened. Moreover, as creditors' signals become more precise or as the amount of noise decreases, the maximum renegotiation length decreases. Consider an extreme case of the *perfect secondary market* where there is no noise in the market. In this case, the secondary market price is fully revealing, and the renegotiation always ends in one period. We summarize these findings in the following proposition.

Proposition 3. (*i*) *The maximum renegotiation duration with the secondary market is shorter than or equal to that without the secondary market.* (*ii*) *The maximum renegotiation duration with the secondary market decreases as σ_η and σ_z decreases. In particular, there is no renegotiation delay when there is no noise*, i.e., $\sigma_\eta = 0$ *or* $\sigma_z = 0$.

Proof. See Technical appendix. Q.E.D.

We next illustrate the impacts of the secondary market on the expected renegotiation length, given by

$$\overline{T}^M([s_l, s_h]) = \int_0^1\int_{s_l}^{s_h} T(s,\Omega(s,\eta))dG(s)d\eta.$$

The expected renegotiation duration depends only on the ex ante information friction Ψ when there is no secondary market trading. With the secondary market, this duration also depends on the distribution of the reservation value. Consider two non-overlapping intervals with the same Ψ. After trading, the government updates its belief to be $\Omega(s,\eta)$, which in general has a fixed length $\sigma_z\sigma_\eta$. As the underlying state (s,η) shifts $\Omega(s,\eta)$ to the right, the information friction rises, as does the renegotiation duration. Thus, the expected renegotiation tends to be longer for the interval on the right. To isolate the role of the information friction, we keep the mean reservation fixed while varying the variance of the reservation value.

In the following numerical example, we set the mean reservation at 0.5, $\beta = 0.95$, $\gamma y = 1$, and $\sigma_z\sigma_\eta = 0.43$ as our benchmark parameter values, and vary $s_l = 1-s_h$ to change the information friction Ψ. Fig. 3 plots the expected renegotiation length $\overline{T}^M$ (labeled "noisy secondary market") over Ψ. To highlight the role of the secondary market, we

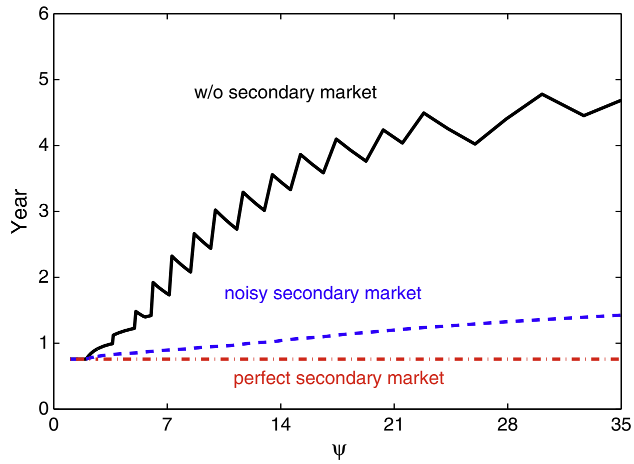

Fig. 3. Expected renegotiation length: secondary market.

also plot the expected renegotiation length for the case without the secondary market and with the perfect secondary market. Clearly, the presence of the secondary market greatly reduces the expected renegotiation duration. Moreover, the smaller the secondary market noise, the shorter the expected renegotiation duration. In one extreme case where the secondary market is perfect with no noise, the renegotiation concludes immediately. In the other extreme case where the market is so noisy that the price offers no information, the renegotiation duration is the same as in the case without the secondary market.

3.3. Empirical duration of sovereign debt renegotiations

In this subsection, we present empirical evidence on the duration of sovereign debt renegotiations. In the data, the renegotiation duration is shorter for bond debt than for bank debt. As bond debt is traded on the secondary market and bank debt is rarely traded, our model implications on the renegotiation duration are consistent with the data.

Private lending to developing countries has evolved over time. Before World War II, developing countries borrowed predominantly in terms of sovereign bonds. Between World War II and 1990, they borrowed mainly in the form of syndicated bank loans from commercial banks in advanced economies. In the 1990s, bonds again became the dominant form of borrowing by developing countries. Correspondingly, sovereign defaults were exclusively on bond debt before World War II and exclusively on bank debt in the 1970s and 1980s. We observe sovereign defaults on both bank debt and bond debt only after 1990. When comparing the duration of the bank loan renegotiations with that of the bond renegotiations, we thus focus on the default episodes after 1990 tocontrol for the time effect.

The main data come from Benjamin and Wright (2008), which provides the starting and ending dates of each default episode after 1990. The starting date is defined as the date when a sovereign country fails to make payments within a grace period specified in the contract. The ending date is defined as the date when a settlement occurs. We supplement the data with the form of sovereign borrowing, bank debt or bond debt, using Standard & Poor's (Beers and Chambers (2004)). There are 23 default episodes on bank debt and 15 default episodes on bond debt. We report the summary statistics on the renegotiation length in Table 2.[13] The renegotiation is on average longer for bank debt than for bond debt: 4.65 versus 1.21 years.

There are three countries who defaulted on both bank debt and bond debt in our sample: Russia, Uruguay, and Venezuela. We find

Table 2
Duration of sovereign debt renegotiation after 1990, years.
Data Source: Benjamin and Wright (2008) and Standard & Poor's (Beers and Chambers (2004)).

	Mean	Median	Std	Maximum	Minimum	Episodes
Bank debt	4.65	4.70	3.04	12.00	0.70	23
Bond debt	1.21	1.10	1.33	4.00	0.00	15

for each of these countries that the bond renegotiation was shorter than the bank loan renegotiation. Russia defaulted on its bank debt in 1991 and on its bond debt in 1998. It took 6 years for Russia to conclude the bank debt renegotiation, but only 2.3 years to finish the bond renegotiation. Similarly for Uruguay, it took 1.1 years to restructure the defaulted bank debt in 1990, but less than two months to restructure the defaulted bond debt in 2003. Venezuela took 1 year to renegotiate its defaulted bank loans in 1990 and an average of 0.7 years to renegotiate its defaulted bond debt in 1995, 1998 and 2005.

Our finding that the renegotiation duration is shorter for bond debt than for bank debt is supported by an alternative data source. Trebesch (2009) traces the starting date of renegotiations using financial news and finds that renegotiation durations in general are shorter than those constructed by Standard & Poor's.[14] Nonetheless, he also confirms that the renegotiation duration is much shorter for bank loans than for bond loans: 30.9 months versus 13.1 months.

We also look at the duration of sovereign debt renegotiations before 1990. The median renegotiation duration for bond-debt defaults in the period from 1800 to 1945 is about six years as documented in Reinhart and Rogoff (2009). The median renegotiation duration for bank-debt defaults in the period from 1970 to 1990 is about eleven years according to the data from the Standard & Poor's. However, one needs to be careful in drawing conclusions from comparing the renegotiation durations across periods due to the lack of controls for the time effect. International financial markets evolved dramatically across periods, which might drive the difference in the renegotiation duration. For more details see the Data appendix.

3.4. Discussions

To illustrate the model mechanisms transparently, the model makes many simplifying assumptions. We want to highlight two seemingly strong assumptions in particular. First, the model imposes the trading limit to be one unit. Second, the model assumes that the degree of the information friction is the same for the bond and bank debt renegotiations. We now discuss whether our model implications are robust when we relax these assumptions.

Let's first relax the trading limit assumption. Suppose each creditor can buy at most $M \geq 1$ units of bonds in the secondary market. In this case, the creditors either sell their one-unit bond or buy M units of bonds. The excess demand of the non-noisy traders is given by

$$X(s,p) = (1-\alpha)\left(P\left(z > \hat{z}(p)|s\right)M - P\left(z \leq \hat{z}(p)|s\right)\right).$$

In equilibrium, the excess demand of non-noisy traders equals the supply of the noisy traders, α. Thus, we have the following relation between the underlying state (s,η) and the cutoff reservation $\hat{z}(p)$:

$$s = \hat{z}(p) - \frac{M-1}{M+1}\sigma_z + \frac{2}{M+1}\sigma_z\sigma_\eta\eta.$$

[13] See Data appendix for details.

[14] Trebesch (2009) argues that Standard & Poor's tends to overestimate the renegotiation length, since it codes default episodes mainly using information on arrears and missed payments, thus disregarding negotiation processes.

Under the assumption that the noise η is uniformly distributed on $[0,1]$, the government updates its belief of s to $[s_l^g, s_h^g]$, where $s_l^g = \max\left\{s_l, \hat{z}(p) - \frac{M-1}{M+1}\sigma_z\right\}$ and $s_h^g = \min\left\{s_h, \hat{z}(p) - \frac{M-1}{M+1}\sigma_z + \frac{2}{M+1}\sigma_z\sigma_\eta\right\}$. Thus, the information of the government becomes more precise with the secondary market trading, which reduces the maximum duration of renegotiation. Moreover, the maximum renegotiation duration decreases as the trading limit M rises. Intuitively, when M goes to infinity, the creditor with the highest signal buys all the bonds. Thus, the noisy trader effect disappears, and the secondary market price reveals that $s = \hat{z}(p) - \sigma_z$. Under this belief, the government proposes $s\gamma y$ in the renegotiation period, and the agreement is reached immediately.

We next allow the degree of the information friction to differ across the renegotiation scenarios with and without the secondary market. As long as the secondary market is sufficiently efficient to overcome the difference in the information friction, the secondary market will still reduce the renegotiation duration. Consider an extreme case where the secondary market price is perfect revealing. The renegotiation process with the secondary market will always conclude right way, and the degree of the information friction becomes irrelevant.

Despite little evidence of the market efficiency for defaulted sovereign bonds,[15] a large empirical literature documented the evidence for defaulted corporate bonds, traded in similar markets as sovereign bonds. In theory, a relatively efficient secondary market implies that the secondary market price should reflect well the recovery value of bonds. Eberhart and Sweeney (1992) documented that the bond price at the time of bankruptcy is an unbiased estimate of the bond payoff upon settlement. Khieu and Mullineaux (2009) showed that the 30-day post-default trading price is highly correlated with the settlement recovery value. These findings suggest that the secondary market is relatively efficient and the price is informative. In practice the bond price at default is commonly used to estimate the recovery rate of defaulted bonds (see Acharya et al. (2007) and Schuermann (2004) for example). The influential rating agencies, Moody's and Standard & Poor's, use the post-default trading price to predict the recovery rate of defaulted bonds.

4. Ex post and ex ante welfare

In this section, we first analyze the ex post welfare implications in the case with and without the secondary market. The secondary market tends to increase the total ex post welfare by reducing costly renegotiation delays. It also tends to reduce the expected welfare of the creditors while increasing that of the government. We then introduce these renegotiation outcomes into a simple sovereign debt model with default risk. The ex ante efficient investment and welfare are achieved only when there is no renegotiation delay ex post. Moreover, the presence of the secondary market has the potential to increase the ex ante welfare of the government when the information friction is severe.

4.1. Ex post welfare implications

We now examine the ex post renegotiation welfare. Since both the creditors and the government are risk-neutral, welfare is equivalent to the payoff in the model. The maximum ex post renegotiation payoff is the present value of the potential output loss, i.e., $\frac{\gamma y}{1-\beta}$. We analyze both the total expected renegotiation payoff and the division of the total payoff between the creditors and the government for the case with and without the secondary market trading.

[15] This lack of evidence is mainly driven by the limited sample of defaulted sovereign bonds.

We first consider the case without the secondary market trading. For any ex ante prior $[s_l, s_h]$, the total renegotiation payoff, as a share of the maximum renegotiation payoff, is given by

$$w = \int_{s_l}^{s_h} \left[s + \beta^{T(s,[s_l,s_h])-1}(1-s)\right] dG(s),$$

where $T(s,[s_l,s_h])$ denotes the agreement period for reservation s under the optimal strategy. The total renegotiation payoff equals the maximum payoff when there is no renegotiation delay for any s. Otherwise, the total payoff is lower than the maximum payoff. We refer to the difference between these two payoffs as the *ex post efficiency loss*.

The expected renegotiation payoff of the government, as a share of the maximum renegotiation payoff, is given by

$$w_g = \int_{s_l}^{s_h} \beta^{T(s,[s_l,s_h])-1}\left(1 - \frac{b_{T(s,[s_l,s_h])}}{\gamma y}\right) dG(s),$$

where $b_{T(s,[s_l,sh])}$ denotes the per-period repayment for each reservation value s. The expected payoff of the creditors, as a share of the maximum renegotiation payoff, is given by

$$w_c = \int_{s_l}^{s_h} \left[s + \beta^{T(s,[s_l,s_h])-1}\left(\frac{b_{T(s,[s_l,s_h])}}{\gamma y} - s\right)\right] dG(s).$$

For each s, the first term in the integral is the payoff to the creditors regardless of whether the agreement is reached, and the second term is the additional payoff when the renegotiation concludes. We refer to the second term as the *information rent* of the creditors.

We now consider the renegotiation welfare for the case with the perfect secondary market, where the price is fully revealing and the renegotiation concludes right away. The total renegotiation payoff equals the maximum renegotiation payoff, and so there is no ex post efficiency loss. Moreover, the creditors receive a per-period payoff equal to their reservation ex post under our assumption that the creditors accept an offer whenever they are indifferent. This implies that the information rent is zero. Therefore, the creditors receive a lower expected payoff, while the government receives a higher expected payoff, under the perfect secondary market than without the secondary market. We summarize these results in the following proposition.

Proposition 4. *The perfect secondary market has no ex post efficiency loss. Moreover, the government expects a higher payoff, while the creditors expect a lower payoff, under the perfect secondary market than without the secondary market.*

Proof. See Technical appendix. Q.E.D.

Finally, we examine the renegotiation welfare for the case with an imperfect secondary market. The government's expected renegotiation payoff, as a share of the maximum renegotiation payoff, is given by

$$w_g^M = \int_0^1 \int_{s_l}^{s_h} \beta^{T(s,\Omega(s,\eta))-1}\left(1 - \frac{b_{T(s,\Omega(s,\eta))}}{\gamma y}\right) dG(s) d\eta,$$

where $\Omega(s,\eta)$ denotes the updated government belief of the creditors' reservation. The creditors' expected payoff, as a share of the maximum renegotiation payoff, is

$$w_c^M = \int_0^1 \int_{s_l}^{s_h} \left[s + \beta^{T(s,\Omega(s,\eta))-1}\left(\frac{b_{T(s,\Omega(s,\eta))}}{\gamma y} - s\right)\right] dG(s) d\eta.$$

The total renegotiation payoff w^M is the sum of w_c^M and w_g^M.

We study the impact of an imperfect secondary market on the renegotiation payoff numerically, since there are no closed-form solutions for the optimal strategies. Fig. 4 plots w_c^M and w_g^M under the

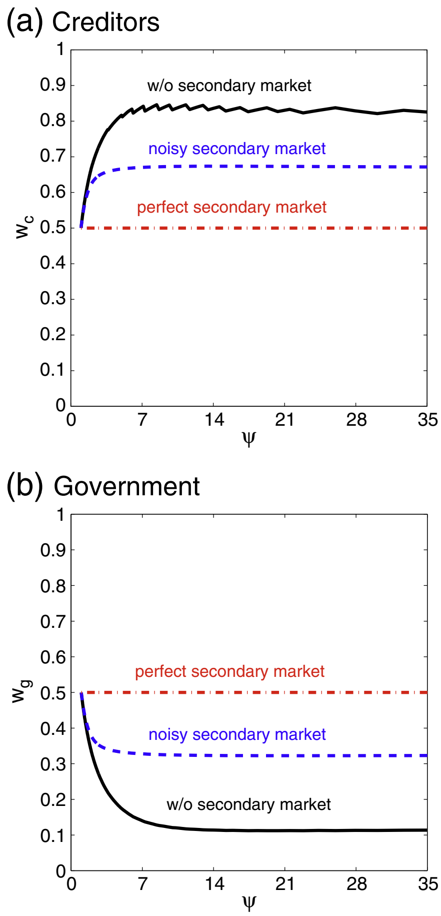

Fig. 4. Ex post expected renegotiation payoff.

benchmark parameters. We also plot the welfare implications without the secondary market and with the perfect secondary market. Since the secondary market reduces their information rent, the creditors' expected payoff is the highest in the case without the secondary market, lowest in the case with the perfect secondary market, and intermediate in the case with an imperfect secondary market. The opposite is true for the government's expected renegotiation payoff. Since it tends to reduce the renegotiation duration, the secondary market increases the total renegotiation payoff.

4.2. Ex ante welfare implications

The secondary market affects the ex post renegotiation outcomes, which in turn affect the lending incentives of the creditors and the borrowing and default incentives of the government ex ante. In models without equilibrium default (e.g. Bai and Zhang (2010)), a lower ex-post payoff unambiguously leads to a higher ex-ante payoff for the government. This implication, however, is not necessarily true in models with equilibrium default. In these models, the ex-ante welfare comprises both the repayment welfare and the defaulting welfare. A lower ex-post payoff improves the terms of borrowing and thus the repayment welfare, but hurts the defaulting welfare. We analyze these ex ante impacts in a simple sovereign debt model with equilibrium default.[16]

[16] Different from Cole and Kehoe (2000) and Conesa and Kehoe (2011), our ex-ante game abstracts from strategic interactions between creditors and governments.

Consider a sovereign country that has access to a risky project in period 0. The random project productivity $a \in [0, \bar{a}]$ with a density function $g(a)$ realizes in period 1 and remains constant afterward. For an investment level k in period 0, the project generates output $y = ak^\alpha$ every period. In period 0, the government has no resources, and finances investment k from foreign creditors using long-term contracts, (k, b_r), which specifies an annuity repayment b_r for each loan k. The contract has limited enforceability in that the government can renege on its repayment after the realization of a. To avoid the output loss after default, the government renegotiates with the creditors, whose reservation value is drawn from a uniform distribution on $[s_l, s_h]$. For details, see the Timing Appendix. The ex post renegotiation game either has the secondary market trading or not, and (w_c, w_g) summarizes the ex postrenegotiation outcomes.

Let's first look at the government's default decision at the beginning of period 1. Given contract (k, b_r) and project productivity a, the government obtains a per-period payoff of $ak^\alpha - b_r$ if it repays and of $(1-\gamma+w_g\gamma)ak^\alpha$ if it defaults. The government decides whether to default to maximize its payoff $V(a, k, b_r; w_c, w_g)$, given by

$$V\left(a, k, b_r; w_c, w_g\right) = \max\left\{ak^\alpha - b_r, \left(1-\gamma+w_g\gamma\right)ak^\alpha\right\}. \tag{11}$$

Default tends to occur when productivity a and investment k are low and repayment b_r is large. Specifically, for any contract (k, b_r), there exists a cutoff level of productivity $\hat{a}(k, b_r)$, which solves

$$\gamma\left(1-w_g\right)\hat{a}(k, b_r)k^\alpha = b_r, \tag{12}$$

such that the government chooses to default if and only if $a \leq \hat{a}(k, b_r)$.

We next examine the government's borrowing and investment decisions in period 0. The government chooses a contract to maximize the expected welfare $\hat{V}(w_c, w_g)$ given by

$$\hat{V}\left(w_c, w_g\right) = \max_{(k, b_r)} \frac{\beta}{1-\beta} \int_0^{\bar{a}} V\left(a, k, b_r; w_c, w_g\right) g(a) da. \tag{13}$$

The optimal investment, borrowing, and default are denoted by $\hat{k}(w_c, w_g)$, $\hat{b}_r(w_c, w_g)$, and $\hat{a}(w_c, w_g)$, respectively.

We finally study the lending decisions of the risk-neutral creditors. They provide a set of long-term contracts which take into account the government's default incentive and the expected renegotiation payoff w_c. The creditors have access to funds at the risk-free rate r, where $\beta(1+r) = 1$. Under perfect competition, they have to break even for each contract (k, b_r):

$$rk = \int_{\hat{a}(k, b_r)}^{\bar{a}} b_r g(a) da + \int_0^{\hat{a}(k, b_r)} w_c \gamma a k^\alpha g(a) da. \tag{14}$$

The left-hand side is the opportunity cost of fund k in annuity value, and the right-hand side is the expected payoff in annuity value. The creditors receive non-contingent repayment b_r when the government chooses not to default, i.e., $a > \hat{a}(k, b_r)$, and contingent repayment $w_c \gamma a k^\alpha$ when the government defaults, i.e., $a \leq \hat{a}(k, b_r)$.

We will focus on the ex ante welfare of the government, since the creditors always break even ex ante. To highlight the impact of default risk on investment and welfare of the government, we define the efficient investment level k^* as $k^* = t\left(\frac{\alpha a^e}{r}\right)^{\frac{1}{1-\alpha}}$, where a^e denotes the expected productivity level. Under the efficient investment, the expected marginal return of funds equals the marginal cost of funds in each period. Accordingly, we define the efficient welfare V^* as $V^* = a^e {k^*}^\alpha - rk^*$. In what follows, we illustrate how the renegotiation outcomes affect the optimal investment and welfare relative to the efficient investment and welfare.

Delays, associated with the efficiency loss, reduce the expected return of the project and lower ex ante investment and welfare below the efficient levels. Without delays, the efficiency loss disappears and the government chooses the efficient investment whenever the creditors are willing to offer such a contract. The sufficient condition for the creditors to offer such an investment loan is $w_c\gamma \geq \alpha$. Proposition 5 highlights the findings.

Proposition 5. *The government achieves the ex ante efficient investment k^* and welfare V^* if and only if there is no renegotiation delay, i.e., $w_c + w_g = 1$, and $w_c\gamma \geq \alpha$.*

Proof. See Technical appendix. Q.E.D.

We now analyze how an increase in the expected payoff of each party affects ex ante investment and welfare when $w_c + w_g < 1$. We first consider an increase in the creditors' payoff w_c, while fixing the government's payoff w_g. In this case, the creditors demand a lower non-contingent payment b_r for each investment level k when their renegotiation payoff increases. Thus, the government faces a more favorable set of contracts, invests more, and achieves higher welfare.

We consider an increase in the government's share w_g while fixing the creditors' share w_c. A higher w_g implies a smaller efficiency loss and a higher expected return of the project, and the government has more incentive to borrow and invest. At the same time, the government's default incentive also rises with a higher w_g. Hence, the creditors require a higher non-contingent payment b_r for any investment k to compensate for the higher default likelihood. This loan schedule effect tends to lower investment. Thus, the investment response depends on which effect dominates — the demand or loan effect.

When w_c is small, the loan effect tends to dominate when w_g increases. This is because the creditors expect a large decline in repayment and so require a large repayment b_r. Thus, the optimal investment decreases with w_g. When w_c is large, the demand effect tends to dominate when w_g increases. In this case, the cost of default is relatively low for creditors, and the response of the loan schedule to an increase in w_g is small. This implies that the optimal investment rises with w_g. For intermediate levels of w_c, which effect dominates is ambiguous. We demonstrate these results under a uniform distribution of a in the following proposition.

Proposition 6. *The ex ante investment and welfare increase with w_c under a constant w_g. When productivity a is uniformly distributed on $[0, \bar{a}]$, there exists $(\underline{w}_c, \overline{w}_c)$ with $0 \leq \underline{w}_c < \overline{w}_c < 1$ such that ex ante investment decreases with w_g when $w_c \leq \underline{w}_c$ and increases with w_g when $w_c \geq \overline{w}_c$.*

Proof. See Technical appendix. Q.E.D.

From the previous subsection, we know that the presence of the secondary market increases the expected renegotiation payoff of the government but lowers that of the creditors. We now illustrate numerically the ex ante investment and welfare for different degrees of the information friction with and without the secondary market. In this numerical example, we set α at 0.3, r at 0.07, and γ at 0.5 and assume that a is uniformly distributed on $[0, 2]$, in addition to the benchmark parameters. In the left panel of Fig. 5, we plot the the ex ante investment $\hat{k}$, as a fraction of the efficient investment k^*, as a function of Ψ for three different scenarios. In the right panel, we plot the corresponding welfare, as a fraction of the efficient welfare V^*.

The perfect secondary market does not necessarily lead to efficient investment and welfare, though it eliminates the renegotiation delays and generates the maximum ex post renegotiation welfare. In this case, the creditors receive an expected renegotiation payoff of 0.5, independent of the information friction. This payoff w_c is not large enough to support the efficient investment and welfare. The optimal ex ante investment is about 20% lower than the efficient level, and the optimal welfare is about 1% lower than the efficient welfare. Of

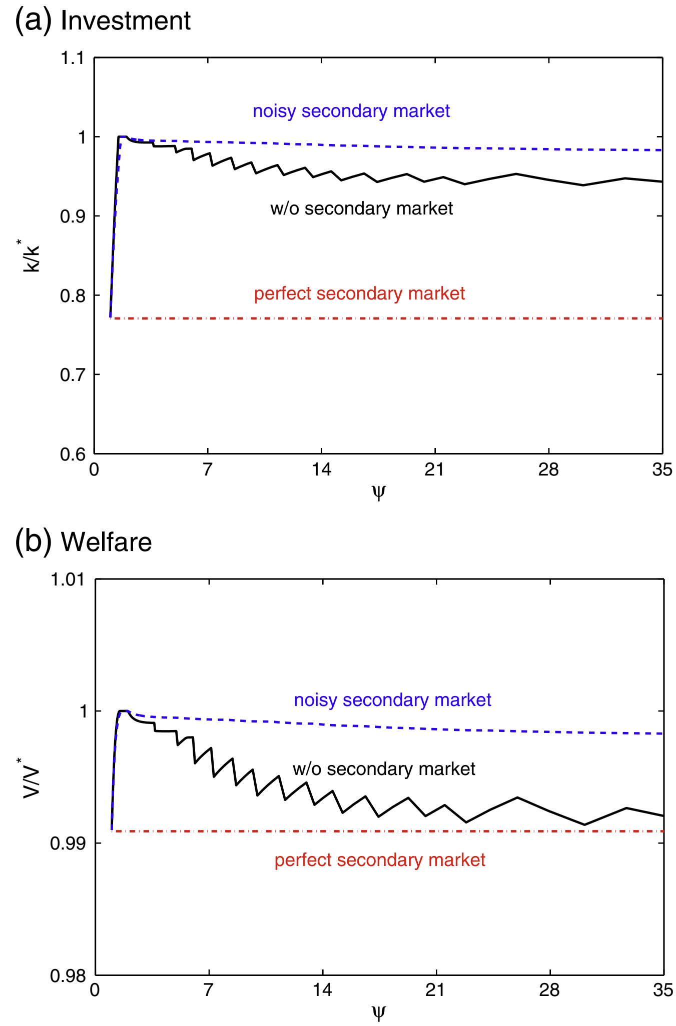

Fig. 5. Ex ante implications.

course, under an alternative parameterization where the creditors derive a higher expected renegotiation payoff, the optimal investment will rise to the efficient level.

In the case without the secondary market, the optimal investment and welfare first increase and then decrease with the information friction. When Ψ is close to zero, the outcomes are similar to those in the case with the perfect secondary market. When Ψ rises from zero, the creditors obtain more information rents and offer a larger set of contracts. At the same time, the renegotiation duration is still short and the ex post efficiency loss is low. Thus, the optimal investment and welfare rise. When Ψ continues to rise, the renegotiation duration becomes long and the ex post efficiency loss gets large. This implies that the expected return of the project is low. Thus, the government lowers the optimal investment even though a higher payoff to the creditors generates a more favorable loan schedule.

The noisy secondary market delivers a pattern of optimal investment and welfare that is similar to that in the case without the secondary market. Moreover, the optimal investment and welfare are higher than those under the perfect secondary market. The reason is that the noisy secondary market still permits the information rent, which increases the creditors' renegotiation payoff and allows them to offer a larger set of contracts. Compared with the case without the secondary market, the noisy secondary market increases both investment and welfare only when the information friction is high. It increases the ex post efficiency by reducing renegotiation delays, which leads to an increase in investment. At the same time, it also decreases the creditors' renegotiation payoff, which tends to reduce the optimal investment. The first effect becomes dominant when the information friction is large.

4.3. Haircut rate: model and data

In addition to the renegotiation duration, a key outcome in the sovereign debt renegotiations is the haircut rate, which measures the creditors' loss. By definition, the haircut rate equals one minus the recovery rate, which is the ratio of the present value of rescheduled debt repayments to the present value of debt obligations. Our model generates predictions on the haircut rate in the renegotiations with and without the secondary market. Thus, we can test our model along this dimension by comparing the empirical evidence with our model's predictions.

For empirical estimates of the haircut rate, we rely on the most comprehensive study by Benjamin and Wright (2008), who estimate the haircut rate for all default episodes after 1970. To control for time effect, we focus on the defaulting episodes after 1990. We compute the average haircut rates for bond debt and bank debt renegotiations. The haircut rate is on average smaller in bond renegotiations than in bank loan renegotiations: 17% versus 31%.[17] One important caveat worth mentioning is that it is generally difficult to obtain a precise empirical estimate of the haircut rate due to the complex structure of the old and new loan payments. As a result, the empirical estimates of the haircut rate for a default episode might vary across studies with different methodologies. For example, the estimated haircut rate for 1998 Russian debt restructuring is 32% in Benjamin and Wright (2008), 53% in Sturzenegger and Zettelmeyer (2005), and 38% in GCAB (2004).

The haircut rate in the model is denoted by H and given by

$$H = 1 - \int_{a_L}^{\hat{a}} \frac{w_b \gamma a k^\alpha}{r b_r} \frac{g(a)}{G(\hat{a})} da, \tag{15}$$

where $w_b \gamma a k^\alpha$ is the expected per-period debt repayment after the renegotiation, rb_r is the per-period debt repayment conditional on not defaulting, and $\hat{a}$ is the cutoff default productivity. In particular, we have in the bank loan renegotiation

$$w_b \gamma a k^\alpha = \int_{s_l}^{s_h} \beta^{T(s,[s_l,s_h])-1} b_{T(s,[s_l,s_h])} dG(s),$$

and in the bond renegotiation

$$w_b \gamma a k^\alpha = \int_0^1 \int_{s_l}^{s_h} \beta^{T(s,\Omega(s,\eta))-1} b_{T(s,\Omega(s,\eta))} dG(s) d\eta.$$

Clearly, the haircut rate depends on the final accepted government proposal and the renegotiation length, both of which in turn depend on the information friction. A reduction in the information friction shortens the renegotiation process, and at the same time lowers the accepted proposal due to a lower information rent. A lower accepted proposal tends to decrease the haircut rate, while a shorter renegotiation tends to increase the haircut rate since the accepted offer is discounted by less. Thus, the information friction has two opposite effects on the haircut rate, and the equilibrium haircut rate depends on which effect dominates.

To examine whether our model's prediction of the haircut rate is consistent with the data, we parameterize our model with reasonable parameter values. The annual interest rate is set at 7% to match the average yield of US 5-year bonds in 1980–2001.[18] The mean reservation value is set at 0.5. We calibrate the discount factor β at 0.95 and the standard deviation of the reservation value at 0.495 such that the model without the secondary market matches the average renegotiation duration and the average haircut rate of the bank loan renegotiations after 1990. We then set the noise in the secondary market $\sigma_z \sigma_\eta$ to be 0.43 to match the average duration of 1.21 years in the bond renegotiations. The model with the secondary market generates a haircut rate of 0.23, lower than the haircut rate predicted by the model without the secondary market of 0.31. Thus, our model's prediction of the haircut rate is consistent with the data.

So far we have taken the financing choice of borrowing countries between bank loans and bonds to be exogenous. Obviously, the financing choice is endogenous. The empirical literature attributes the shift from bank loan to bond financing to a rise in effective interest rates of bank loans due to a declining need to recycle petrodollars and a changing US tax environment for foreign lending. When the government can choose from bank loan and bond financing given the observed pattern of the interest rates, our model can rationalize the changing financing choice over time. For a detailed discussion see the Data Appendix.

5. Conclusion

With respect to sovereign debt restructuring, on average it takes a long time for creditors and a sovereign government to reach an agreement. Lengthy renegotiations are costly: during renegotiation, governments cannot resume international borrowing and creditors cannot realize their investment returns. Thus, deepening our understanding about the causes of renegotiation delays is important for both academic and policy purposes.

This paper emphasizes the effect of information frictions on renegotiation delays and highlights the role of the secondary market in reducing these delays. When renegotiating with creditors to restructure debt, the government might prefer to have costly delays if the reservation value of the creditors is private information. Though a low restructuring proposal might cause costly delays in reaching agreements, it might also increase the government payoff if the creditors turn out to have a low reservation value. The more severe the information friction, the longer the maximum renegotiation duration. The presence of a secondary market might then reduce the renegotiation duration by lessening the information friction through price revelation. This implication is consistent with the empirical finding that sovereign debt renegotiations are on average much shorter for liquid bonds than for illiquid bank loans.

We also find that the secondary market has important welfare implications both ex post and ex ante. From the ex post point of view, the secondary market increases the total payoff by reducing delays and the efficiency loss. It also increases the government's payoff while decreasing the creditors' payoff through reducing the information rent of the creditors. From the ex ante point of view, the secondary market might increase the ex ante welfare of the government while allowing the creditors to break even ex ante. Thus, bond financing on the secondary market seems to be a potentially better means of sovereign borrowing. Finally, the model highlights that to achieve higher welfare and more efficient allocations, the creditors have to receive a certain level of renegotiation payoff.

[17] For the full sample of the bank loan renegotiations after 1970, the average haircut rate is 43%.

[18] Similar values for the interest rate are used in the quantitative sovereign debt literature, for example, Arellano (2007).

A. Timing appendix

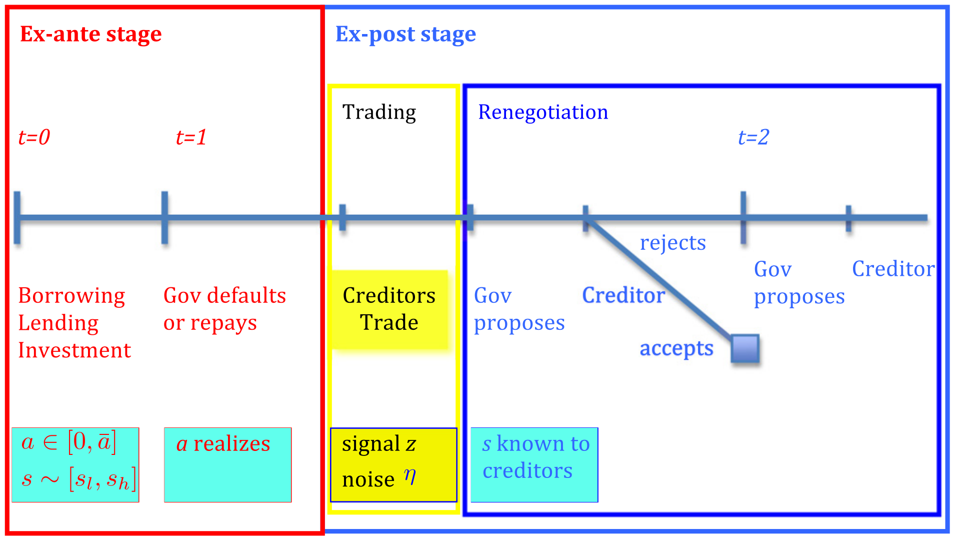

Fig. 6. Timing.

B. Technical appendix

To solve the equilibrium, we start with the last period T. The government proposes $B_T(\underline{s}) = s_h\gamma y$ to end the renegotiation right away. The payoff of the government is $V_T(\underline{s}) = (y - s_h\gamma y)/(1-\beta)$, and the creditors' payoff is $W_T(s) = s_h\gamma y/(1-\beta)$.

We next proceed backward. In any period $t<T$, the creditors accept a proposal b if and only if their reservation value is below a cutoff level $\underline{S}_{t+1}(b)$ that solves

$$\frac{b}{1-\beta} = \underline{S}_{t+1}(b)\gamma y + \frac{\beta B_{t+1}\left(\underline{S}_{t+1}(b)\right)}{1-\beta},$$

where $B_{t+1}(\underline{S}_{t+1}(b))$ denotes the government's optimal offer under belief $\underline{S}_{t+1}(b)$ in period $t+1$. Given the function $\underline{S}_{t+1}(b)$, the government with current belief $\underline{s}$ chooses b to maximize its payoff

$$\hat{V}_t(\underline{s}) = \max_b \Lambda_t(\underline{s}, b)\frac{y-b}{1-\beta} + (1-\Lambda_t(\underline{s}, b))\left[(1-\gamma)y + \beta V_{t+1}\left(\underline{S}_{t+1}(b)\right)\right], \quad (16)$$

where $\Lambda_t(\underline{s}, b)$ denotes the acceptance probability of offer b, given by $\frac{\underline{S}_{t+1}(b) - \underline{s}}{s_h - \underline{s}}$ under the uniform distribution. The optimal offer is denoted by $\hat{B}_t(lines)$.

At any period t, the government also faces the choice of whether to propose $\hat{B}_t(\underline{s})$ or to propose $B_{t+1}(\underline{s})$ to end the game earlier. There exists a cutoff belief $\hat{\underline{s}}_{t+1}$ that solves $V_{t+1}(\hat{\underline{s}}_{t+1}) = \hat{V}_t(\hat{\underline{s}}_{t+1})$. For any $\underline{s} \geq \underline{s}_{t+1}$, the government prefers to end the renegotiation earlier, and proposes $B_t(\underline{s}) = B_{t+1}(\underline{s})$. For any $\underline{s} < \hat{\underline{s}}_{t+1}$, the government prefers to propose $\hat{B}_t(\underline{s})$. On the other hand, the government has to offer at least $\hat{b}_t$, given by $\underline{S}_{t+1}\left(\hat{b}_t\right) = \hat{\underline{s}}_{t+1}$, to ensure ending the renegotiation within $T-t$ periods. Thus, the government's optimal proposal $B_t(\underline{s})$ is given by

$$B_t(\underline{s}) = \begin{cases} B_{t+1}(\underline{s}) & \text{if } \underline{s} \geq \hat{\underline{s}}_{t+1} \\ \hat{B}_t(\underline{s}) & \text{if } \underline{s}_t < s < \hat{\underline{s}}_{t+1} \\ \hat{b}_t & \text{if } s \leq \underline{s}_t \end{cases} \quad 17$$

were $\underline{s}_t$ is given by $\hat{B}_t(\underline{s}_t) = \hat{b}_t$. We also update the government's welfare accordingly and denote it by $V_t(\underline{s})$. We proceed with the above process until we have $\underline{s}_1 \leq s_l$.

According to this computation algorithm, the solution to the dynamic bargaining game is characterized by a sequence of the government's piecewise proposing functions $\{B_t(s)\}_{t=1}^T$, the belief functions $\{\underline{S}_{t+1}(b)\}_{t=1}^T$, and the cutoff beliefs $\{\hat{\underline{S}}_{t+1}\}_{t=1}^T$.

Proof of Proposition 1

To prove Proposition 1, we establish two lemmas. Lemma A.1 shows that the belief, the strategy, and the welfare of the government have the homogeneity property. Lemma A.2 demonstrates that the cutoff belief is a linear function of s_h. For convenienceof the proofs, we write the welfare and the optimal strategy of the government as functions of $(1-s;\ 1-s_h)$ instead of $(s;\ s_h)$, and the belief as a function of $(1-b;\ 1-s_h)$.

Lemma A.1. The government's welfare V_t, optimal strategy B_t, and belief function $\underline{S}_{t+1}$ have the homogeneity property. Specifically, for any $\lambda \in \left(0, \frac{1}{1-s_h}\right)$,

$$V_t(\lambda(1-s);\lambda(1-s_h)) = \lambda V_t(1-s;1-s_h) \quad (18)$$

$$1 - \frac{B_t(\lambda(1-s);\lambda(1-s_h))}{\gamma y} = \lambda\left[1 - \frac{B_t(1-s;1-s_h)}{\gamma y}\right] \quad (19)$$

$$1 - \underline{S}_{t+1}(\lambda(1-b);\lambda(1-s_h)) = \lambda\left[1 - \underline{S}_{t+1}(1-b;1-s_h)\right]. \quad (20)$$

Proof. We prove the homogeneity by induction. We first show that homogeneity holds for the last two periods, $t=T$ and $T-1$. We then prove that homogeneity holds for period n, assuming that it holds for period $n+1$, for any $n \leq T-1$.

For simplicity of the proofs, we normalize the government welfare V_t to$\tilde{V}_t$, where$\tilde{V}_t(1-s;1-s_h) = \frac{(1-\beta)V_t(1-s;1-s_h) - (1-\gamma)y}{\gamma y}$, and the government optimal strategy B_t to$\tilde{B}_t$, where$\tilde{B}_t(1-s;1-s_h) = \frac{B_t(1-s;1-s_h)}{\gamma y}$. Thus, proving Eqs. (18) and (19) is equivalent to proving the following two equations:

$$\tilde{V}_t(\lambda(1-s);\lambda(1-s_h)) = \lambda\tilde{V}_t(1-s;1-s_h), \tag{21}$$

$$1-\tilde{B}_t(\lambda(1-s);\lambda(1-s_h)) = \lambda\left[1-\tilde{B}_t(1-s;1-s_h)\right]. \tag{22}$$

In period T, the government's strategy is $1-\tilde{B}_T(1-s;1-s_h) = 1-s_h$, and the welfare is $\tilde{V}_T(1-s;1-s_h) = 1-s_h$. If the bank rejects proposal b at period $T-1$, the government updates its belief according to $1-\underline{S}_T(1-b;1-s_h) = \frac{(1-b)-\beta(1-s_h)}{1-\beta}$. Hence, homogeneity holds for period T.

Given the optimal strategy and the belief in period T, we solve the problem in period $T-1$. The solutions for $\tilde{V}_{T-1}$, $\tilde{B}_{T-1}$, and $\underline{S}_{T-1}$ are as follows:

$$\tilde{V}_{T-1}(1-s;1-s_h) = \begin{cases} 1-s_h & \text{if } 1-s\le 2(1-s_h) \\ \dfrac{(1-\beta)(1-s)^2+4\beta(1-s_h)(1-s)-4\beta(1-s_h)^2}{4(1-s)-4(1-s_h)} & \text{if } 2(1-s_h)<1-s\le 4(1-s_h) \\ \dfrac{2+\beta(1-s_h)(2-\beta)(1-s_h)(1-s)-(4-3\beta)(1-s_h)^2}{(1-s)-(1-s_h)} & \text{if } 1-s>4(1-s_h) \end{cases}$$

$$1-\tilde{B}_{T-1}(1-s;1-s_h) = \begin{cases} 1-s_h & \text{if } 1-s\le 2(1-s_h) \\ \dfrac{(1-\beta)(1-s)}{2}+\beta)(1-s_h) & \text{if } 2(1-s_h)<1-s\le 4(1-s_h) \\ (2-\beta)(1-s_h) & \text{if } 1-s>4(1-s_h) \end{cases}$$

$$1-\underline{S}_{T-1}(1-b;1-s_h) = \begin{cases} \dfrac{(1-b)-\beta(1-s_h)}{1-\beta} & \text{if } 1-b\le(2-\beta)(1-s_h) \\ \dfrac{2\left(1-b-\beta^2(1-s_h)\right)}{(1-\beta)(2+\beta)} & \text{if}(2-\beta)(1-s_h)<1-b\le 1-\overline{b}_T \\ \dfrac{(1-b)-\beta(2-\beta)(1-s_h)}{1-\beta} & \text{if } 1-b>1-\overline{b}_T, \end{cases}$$

where $1-\overline{b}_T = \left(4-2\beta-\beta^2\right)(1-s_h)$. Thus, homogeneity holds for period $T-1$.

We now assume that Eqs. (20), (21), and (22) hold for period $n+1$ and prove that they also hold for period n. Define the probability function Λ_n as follows:

$$\Lambda_n(1-s,1-b;1-s_h) = \frac{(1-s)-(1-\underline{S}_{n+1}((1-b);(1-s_h)))}{(1-s)-(1-s_h)}.$$

Clearly, Λ_n is homogeneous of degree zero in its arguments given the homogeneity property of $\underline{S}_{n+1}$. Using the homogeneity of V_{n+1} and Λ_n, we rewrite $\tilde{V}_n$ as, for any $\lambda\in\left(0,\frac{1}{1-s_h}\right)$,

$$\tilde{V}_n(1-s;1-s_h) = \frac{1}{\lambda}\max_{1-\tilde{b}}\Big\{\Lambda_n\left(1-\tilde{s},1-\tilde{b};1-\tilde{s}_h\right)\left(1-\tilde{b}\right) + \left(1-\Lambda_n\left(1-\tilde{s},1-\tilde{b};1-\tilde{s}_h\right)\right)\beta\tilde{V}_{n+1}\left(1-\underline{S}_{n+1}\left(1-\tilde{b};1-\tilde{s}_h\right);1-\tilde{s}_h\right)\Big\},$$

where $1-\tilde{s}_h\equiv\lambda(1-s_h)$, $1-\tilde{s}\equiv\lambda(1-s)$, and $1-\tilde{b}\equiv\lambda(1-b)$. Therefore, we have

$$\tilde{V}_n(1-s;1-s_h) = \frac{1}{\lambda}\tilde{V}_n(\lambda(1-s);\lambda(1-s_h)),$$

which gives Eq. (21). The homogeneity of the optimal strategy $\tilde{B}_n$ and the belief function $\underline{S}_n$ easily follows. Q.E.D.

Lemma A.2. In any period n, the cutoff belief $\hat{s}_n$ is a linear function of s_h with a slope depending on the discount factor β, i.e.,

$$1-\hat{s}_n = g_n(\beta)(1-s_h), \text{ with } g_n(\beta)>1. \tag{23}$$

Proof. Under the belief $\hat{s}_n$, the government is indifferent between ending the game in period n and period $n+1$. That is, $\tilde{V}_n\left(1-\hat{s}_n;1-s_h\right) = \tilde{V}_{n+1}\left(1-\hat{s}_n;1-s_h\right)$. According to the homogeneity of $\tilde{V}_n$ and $\tilde{V}_{n+1}$, we have

$$\tilde{V}_n\left(1;\frac{1-s_h}{1-\hat{s}_n}\right) = \tilde{V}_{n+1}\left(1;\frac{1-s_h}{1-\hat{s}_n}\right).$$

This implies that the ratio of $(1-s_h)$ and $(1-\hat{s}_n)$ only depends on the underlying parameter β. We summarize this result with $1-\hat{s}_n = g_n(\beta)(1-s_h)$, and clearly $g_n(\beta)>1$. Q.E.D.

Proof of Proposition 1. We need to prove that the maximum negotiation length increases with the degree of information friction Ψ. Let's consider two intervals $[s_l^1,s_h^1]$ with Ψ^1 and $[s_l^2,s_h^2]$ with Ψ^2. Assume $\Psi^1\le\Psi^2$. To compare the maximum renegotiation length, we normalize the interval $[s_l^1,s_h^1]$ to $\left[\tilde{s}_l^1,s_h^2\right]$ with $1-\tilde{s}_l^1 = \frac{(1-s_h^2)(1-s_l^1)}{1-s_h^1}$. According to the homogeneity properties in Lemma A.1 and Lemma A.2, interval $[s_l^1,s_h^1]$ and interval $\left[\tilde{s}_l^1,s_h^2\right]$ have the same maximum renegotiation length. It is easy to see that $\tilde{s}_l^1\ge s_l^2$ since $\Psi^1\le\Psi^2$. This implies that the maximum renegotiation length is shorter under interval $\left[\tilde{s}_l^1,s_h^2\right]$ than under interval $[s_l^2,s_h^2]$. Thus, the maximum negotiation length is shorter under $[s_l^1,s_h^1]$ than under $[s_l^2,s_h^2]$. Q.E.D.

Proof of Proposition 2. We need to prove the uniqueness of the monotonic perfect Bayesian equilibrium. We start by establishing the monotonicity of the creditors' payoff in their reservation value in the renegotiation stage in Lemma A.3.

Lemma A.3. *The creditors' payoff increases with their reservation value s for any given uniform distribution of* $s\sim[s_l,s_h]$.

Proof. For a given distribution $[s_l,s_h]$, the government proposes $\{b_t\}_{t=1}^{T}$ and updates its belief to $\{c_t\}_{t=1}^{T+1}$ if offer b_t is rejected at period t. The payoff of the creditors with reservations is therefore given by

$$W(s) = \sum_{t=1}^{T(s)-1}\beta^{t-1}s\gamma y + \frac{\beta^{T(s)-1}}{1-\beta}b_{T(s)}, \tag{24}$$

where $T(s)$ denotes the period that the creditors accept the government's proposal, i.e., $T(s)=\min\{t: s\le c_{t+1}\}$. We now prove that $W(s)$ increases with s.

For any s_1 and s_2 such that $s_l\le s_1<s_2\le s_h$ and $T(s_1)=T(s_2)$, clearly $W(s_1)\le W(s_2)$. For any s_1 and s_2 such that $s_1<s_2$ and $T(s_1)=T(s_2)-1$, the difference between $W(s_2)$ and $W(s_1)$ is

$$W(s_2)-W(s_1) = \sum_{t=1}^{T(s_1)-1}\beta^{t-1}(s_2-s_1)\gamma y + \beta^{T(s_1)-1}s_2\gamma y + \beta^{T(s_1)-1}\frac{\beta b_{T(s_2)}-b_{T(s_1)}}{1-\beta}. \tag{25}$$

In equilibrium, the creditors with reservation $c_{T(s_2)}$ are indifferent between accepting the offer in period $T(s_1)$ and accepting it in period $T(s_2)$. This implies that

$$b_{T(s_1)} = (1-\beta)\gamma y c_{T(s_2)} + \beta b_{T(s_2)}.$$

Substituting the above relation into Eq. (25), we have

$$W(s_2)-W(s_1)=\sum_{t=1}^{T(s_1)-1}\beta^{t-1}(s_2-s_1)\gamma y+\beta^{T(s_1)-1}\left(s_2-c_{T(s_2)}\right)\gamma y.$$

By the definition of $T(s_2)$, we have $s_2 \geq c_{T(s_2)}$. As a result, we prove $W(s_1) \leq W(s_2)$, since the first term is non-negative. Given the generality of $T(s_1)$, we essentially proved that $W(\cdot)$ increases in s. Q.E.D.

Proof of Proposition 2. We prove this proposition in two steps. First, taking the monotonic trading strategy $\hat{z}(p)$ as given, we show in the negotiation stage that the government has a unique optimal strategy and an associated belief function, and the creditors have a unique optimal strategy. Second, we show in the trading stage that a unique market price p clears the market for any underlying parameters (s,η) and the monotonic trading strategy $\hat{z}(p)$ is optimal for the creditors.

Suppose there exists a monotonic trading strategy, which all the creditors follow in the trading stage. According to this trading strategy and the observed market price p, the government updates its belief on reservation s with the market clearing condition, which implies that s is uniformly distributed on $[s_l^g, s_h^g]$ with $s_l^g = \max\{s_l, \hat{z}(p)\}$ and $s_h^g = \min\{s_h, \hat{z}(p)+\sigma_z\sigma_\eta\}$. With this updated prior, the government's and the creditors' negotiation strategies are uniquely pinned down as proved by Fudenberg et al. (1985).

We now show the existence and the uniqueness of the monotonic trading strategy and the market price. When observing market price p, all the creditors expect the government to propose according to a new prior updated with $\hat{z}(p)$. Each creditor, however, forms his own belief of reservation s with the market price and his signal. Specifically, creditor z updates his belief of reservation s to be uniform on $[s_l^c(z), s_h^c(z)]$ where, $s_l^c(z)=\max\{s_l^g, z-\sigma_z\}$ and $s_h^c(z)=\min\{s_h^g, z+\sigma_z\}$. In particular, the creditor with signal $\hat{z}(p)$ has the same belief as the government. Each creditor therefore has a different expected renegotiation payoff given by

$$W^e(z;\hat{z}(p))=\int_{s_l^c(z)}^{s_h^c(z)}\frac{W(s;\hat{z}(p))}{s_h^c(z)-s_l^c(z)}ds.$$

As shown in Lemma A.3, $W(s;\hat{z}(p))$ increases with s for any government's belief determined by $\hat{z}(p)$. Moreover, a creditor with a higher signal expects that the reservation tends to be higher. Thus, the expected payoff weakly increases with signal z. If creditor $\hat{z}(p)$ is indifferent between buying and selling, i.e., $p=W^e(\hat{z}(p);\hat{z}(p))$, then the creditors with $z<\hat{z}(p)$ weakly prefer to sell, and the creditors with $z>\hat{z}(p)$ weakly prefer to buy.

For any underlying parameters (s,η), there exists a pair of $(p,\tilde{z})$ such that the market clearing condition holds and the cutoff creditor $\tilde{z}$ is indifferent between buying or selling. Specifically, we choose $\tilde{z}$ to clear the market: $\tilde{z}=s-\sigma_z\sigma_\eta\eta$, and p to make the cutoff creditor indifferent: $p=W^e(\tilde{z};\tilde{z})$. Such a pair of $(p,\tilde{z})$ exists and is unique for each realization of (s,η). These pairs form a correspondence from p to $\tilde{z}$. We choose a function $\hat{z}(p)$ as a selection from $\tilde{z}(p)$ such that $(p,\hat{z}(p))$ are equilibrium under any (s,η). Q.E.D.

Proof of Proposition 3

Without the secondary market, the government proposes according to its prior $[s_l, s_h]$. With the secondary market trading, the government's belief is updated to $[s_l^g, s_h^g]$, where $s_l^g=\max\{\hat{z}(p), s_l\}$ and $s_h^g=\min\{\hat{z}(p)+\sigma_z\sigma_\eta, s_h\}$. Clearly, the updated belief $[s_l^g, s_h^g]$ is a subset of the prior belief $[s_l, s_h]$. According to Proposition 1, both a lower s_h and a higher s_l shorten the maximum negotiation length.

When σ_η or σ_z decreases, s_h^g weakly decreases. According to Proposition 1, the maximum renegotiation duration becomes shorter. In particular, when there is no noise, i.e., $\sigma_\eta=0$ or $\sigma_z=0$, we have $s=\hat{z}(p)$ in equilbrium. This implies that the government figures out the reservation s, and so proposes $s\gamma y$ in the renegotiation stage. The creditors accept immediately, and there is no renegotiation delay. Q.E.D.

Proof of Proposition 4

The total renegotiation payoff without the secondary market trading, as a share of γy, is given by the sum of w_g and w_c as follows:

$$w=\int_{s_l}^{s_h}\left[s+\beta^{T(s;[s_l,s_h]]-1(1-s)]dG(s)}\right.$$

The total renegotiation payoff with the secondary market trading, as a share of γy, is given by

$$w^M=\int_0^1\int_{s_l}^{s_h}\left[s+\beta^{T(s;\Omega(s,\eta))(1-s)]dG(s)d\eta}\right.$$

Clearly, the total payoff is linked to the renegotiation duration. The longer the renegotiation, the lower the total expected payoff. When the secondary market is perfect, i.e., no noise, the government knows the creditors' reservation value, and so proposes this reservation value in the renegotiation. The renegotiation concludes immediately. The renegotiation duration shortens, and the government pays no information rent. The government therefore derives a higher payoff. The creditors' expected payoff decreases even though that the total renegotiation payoff increases when they lose all the information rent.

Characterization of ex ante equilibrium

Proof of Proposition 5. The equilibrium optimal investment k, default cutoff $\hat{a}$, and optimal repayment b_r satisfy the following three equations:

$$k=\left(\frac{\alpha E\hat{a}}{r}\frac{1-\eta(\hat{a})}{1-\alpha\eta(\hat{a})}\right)^{\frac{1}{1-\alpha}}, \tag{26}$$

$$\alpha E\,\hat{a}\,\frac{1-\eta(\hat{a})}{1-\alpha\eta(\hat{a})}=\gamma\left[w_c\int_0^{\hat{a}}ag(a)da+\left(1-w_g\right)\int_{\hat{a}}^{\overline{a}}\hat{a}g(a)da\right], \tag{27}$$

$$\gamma\left(1-w_g\right)\hat{a}k^\alpha=b_r,$$

where $E\hat{a}=a^e-\gamma\left(1-w_g-w_c\right)\int_{0\hat{a}}ag(a)da$ and $\eta(\hat{a})=\frac{(1-w_c-w_g)\hat{a}g(\hat{a})}{(1-G(\hat{a}))(1-w_g)}$. When there are delays, i.e., $w_c+w_g<1$, we have $E\hat{a}<a^e$ and $1-\eta(\hat{a})<1-\alpha\eta(\hat{a})$. Therefore, the optimal investment is smaller than the efficient investment. When there are no delays, the optimal investment equals the efficient investment if the condition $\alpha \leq \gamma w_c$ holds. This condition ensures that the creditors are willing to offer the efficient investment in their contracts. Q.E.D.

Proof of Proposition 6. We first prove that when w_c increases, the ex ante investment increases. Let's define the left-hand side of Eq. (27) as

$$f\left(a,w_c,w_g\right)=\alpha E\hat{a}\,\frac{1-\eta(\hat{a})}{1-\alpha\eta(\hat{a})},$$

and the right-hand side of Eq. (27) as

$$c\left(a,w_c,w_g\right)=\gamma\left[w_c\int_0^{\hat{a}}ag(a)da+\left(1-w_g\right)\int_{\hat{a}}^{\overline{a}}\hat{a}g(a)da\right].$$

Since in equilibrium k and $\hat{a}$ satisfy conditions (26) and (27), we take the derivative of k with respect to w_c using these equations:

$$\frac{dk}{dw_c} = \frac{k}{(1-\alpha)f\left(\hat{a}, w_c, w_g\right)} \frac{\frac{\partial f}{\partial \hat{a}}\frac{\partial c}{\partial w_c} - \frac{\partial f}{\partial w_c}\frac{\partial c}{\partial \hat{a}}}{r\frac{\partial f}{\partial \hat{a}} - \frac{\partial c}{al\hat{a}}},$$

where $h(a)$ is the hazard rate $\frac{g(a)}{1-G(a)}$. It is easy to show that $h'(\hat{a})>0$. We thus have

$$\frac{\partial c}{\partial \hat{a}} = \gamma\left(1-w_g\right)(1-G(\hat{a}))(1-\eta(\hat{a}))\geq 0,$$

$$\frac{\partial c}{\partial w_c} = \gamma\int_0^{\hat{a}} ag(a)da\geq 0,$$

$$\frac{\partial f}{\partial w_c} = \frac{1-\eta(\hat{a})}{1-\alpha\eta(\hat{a})}\alpha\gamma\int_0^{\hat{a}} ag(a)da + \alpha E\hat{a}\frac{1-\alpha}{1-\alpha\eta(\hat{a})}\frac{\hat{a}g(\hat{a})}{(1-G(\hat{a}))\left(1-w_g\right)}\geq 0,$$

$$\frac{\partial f}{\partial \hat{a}} = -f\left(\hat{a}, w_c, w_g\right)\left(1-w_c-w_g\right)\left[\frac{\gamma\hat{a}g(\hat{a})}{E\hat{a}} + \frac{(1-\alpha)\left(h(\hat{a})+ah'(\hat{a})\right)}{(1-\eta(\hat{a}))(1-\alpha\eta(\hat{a}))\left(1-w_g\right)}\right]\leq 0.$$

According to the sign of these derivatives, it is clear that $\frac{dk}{dw_c}\geq 0$. The optimal investment increases with w_c.

Now we prove the second part of the proposition, the relation between w_g and the optimal investment. Similar to the case of varying w_c, we have

$$\frac{dk}{dw_g} = \frac{k}{(1-\alpha)f\left(\hat{a}, w_c, w_g\right)} \frac{\frac{\partial f}{\partial \hat{a}}\frac{\partial c}{\partial w_g} - \frac{\partial f}{\partial w_g}\frac{\partial c}{\partial \hat{a}}}{r\frac{\partial f}{\partial \hat{a}} - \frac{\partial c}{l\hat{a}}},$$

$$\frac{d\hat{a}}{dw_g} = -\frac{r\frac{\partial f}{\partial w_g} - \frac{\partial c}{\partial w_g}}{r\frac{\partial f}{\partial \hat{a}} - \frac{\partial c}{\partial \hat{a}}},$$

where

$$\frac{\partial f}{\partial w_g} = f\left(\hat{a}, w_c, w_g\right)\left[\frac{\gamma\int_0^{\hat{a}} ag(a)da}{E\hat{a}} + \frac{1-\alpha}{(1-\eta(\hat{a}))(1-\alpha\eta(\hat{a}))}\frac{w_c\hat{a}h(\hat{a})}{\left(1-w_g\right)^2}\right]\geq 0,$$

$$\frac{\partial c}{\partial w_g} = -\gamma\hat{a}(1-G(\hat{a}))\leq 0.$$

It is straightforward to show that $\hat{a}$ increases with w_g. The higher the government renegotiation payoff, the more default incentive the government has. The necessary and sufficient condition for a positive $\frac{dk}{dw_g}$ is thus

$$\frac{\partial f}{\partial \hat{a}}\frac{\partial c}{\partial w_g} \leq \frac{\partial f}{\partial w_g}\frac{\partial c}{\partial \hat{a}},$$

since the denominator in the expression of $\frac{dk}{dw_g}$ is negative. After simplifying, we need

$$\frac{\eta\hat{a}(1-G(\hat{a}))-(1-\eta)\int_0^{\hat{a}} ag(a)da}{(1-\alpha)\hat{a}E\hat{a}} \leq \frac{w_c h(1-\eta)-\left(1-w_c-w_g\right)\left(h+zh'\right)}{\gamma(1-\eta)(1-\alpha\eta)\left(1-w_g\right)^2}. \tag{28}$$

When $w_c=0$, the right-hand side of the above inequality is negative, and the left-hand side is positive under the uniform distribution. Thus, the above inequality does not hold when $w_c=0$, which implies that $\frac{dk}{dw_g}<0$. Note that the left-hand side under the uniform distribution is always positive. Given the continuity of the uniform distribution, both the left-hand and right-hand sides are continuous. Hence, there exists a $\underline{w}_c$ such that $\frac{dk}{dw_g}<0$ when $w_c<\underline{w}_c$. When $w_c=1$, the left-hand side is negative and the right-hand side is positive. According to the continuity, there exists $\overline{w}_c<1$ such that the optimal investment increases with w_g when $w_c\geq\overline{w}_c$ Q.E.D.

Table 3
Duration of sovereign debt renegotiation, bond loans.

Country	Default start	Default end	Length (years)
Argentina	2001	2005	3.6
Ecuador	1999	2000	1.7
Ecuador	2000	2001	1.1
Guatemala	1989	1989	0.0
Ivory Coast	2000	2004	4.0
Moldova	1998	1998	0.0
Moldova	2002	2002	0.5
Nigeria	2002	2002	0.0
Paraguay	2003	2004	1.4
Russia	1998	2000	2.3
Ukraine	1998	2000	1.4
Uruguay	2003	2003	0.0
Venezuela	1995	1997	2.0
Venezuela	1998	1998	0.0
Venezuela	2005	2005	0.1

B. Data appendix

Duration of sovereign debt renegotiation

Benjamin and Wright (2008) collect the starting date, the ending date, and the negotiation length for 90 episodes of sovereign debt restructuring. However, their data contain no information about the form of sovereign debt. We document the form of sovereign debt for each debt restructuring using Standard & Poor's (2004). Among the 90 episodes reported by Benjamin and Wright, 68 episodes are in the form of bank loans and 15 episodes are in the form of bonds. We exclude 7 defaulting episodes on domestic debt. We summarize the renegotiation duration of bank loans and bonds in the two tables below (Table 3).

Changing international financial market

We list below three major changes of the international financial market from the pre-war to post-war periods. First, the legal environment for sovereign debt has evolved over time. For example, the concept of sovereign immunity changed dramatically after World War II. Before 1950, the principle of absolute sovereign immunity was prevalent in the international community, that is, sovereigns cannot be sued in foreign courts without their consent. The interpretation of sovereign immunity started to change after 1950, in part a consequence of the Cold War. The US government, however, encouraged a restrictive sovereign immunity, which allows private entities to sue a foreign government in US courts for commercial activities. Under US law, for example, sovereign bonds are almost always considered commercial activities independent of the purpose of the issue.[19] Second, the ability of creditors to seize collateral has weakened over time. Before World War I, 17 out of a total of 57 renegotiation settlements included the transfer of property (such as land) or income streams (such as custom or railway revenues), as documented by Suter (1992).[20] After World War II, we don't observe any episodes of the seizure of collaterals by creditors. Third, the key players

[19] For details see Sturzenegger and Zettelmeyer (2005).

[20] Examples include 1889 Peru, 1861 and 1873 Colombia, 1885 Costa Rica, 1895 Ecuador, 1855 Paraguay, 1876 Egypt, 1881 Turkey, 1895 Serbia, 1898 Greece, 1903 Morocco, 1912 Liberia default episodes, and etc.

Table 4
Duration of sovereign debt renegotiation, bank loans.

Country	Default start	Default end	Length (years)
Albania	1991	1995	4.6
Algeria	1991	1996	5.2
Angola	1985	2004	19.0
Argentina	1980	1990	11.2
Bolivia	1980	1993	12.4
Brazil	1983	1994	11.2
Bulgaria	1990	1994	4.3
Burkina Faso	1983	1996	13.0
Cameroon	1985	2003	18.0
Cape Verde	1981	1996	15.7
Central African Republic	1983	2004	21.0
Chile	1983	1990	7.4
Colombia	1985	1991	5.3
Costa Rica	1983	1990	6.7
Croatia	1992	1996	4.0
Dominica	2003	2004	1.0
Dominican Republic	1983	1994	10.9
Ecuador	1982	1995	12.3
Ethiopia	1991	1999	8.1
Gabon	1986	1994	7.4
Gabon	1999	2004	4.7
Gambia	1986	1990	4.2
Guinea	1986	1988	2.3
Guinea	1991	1998	8.0
Guinea-Bissau	1983	1996	13.0
Guyana	1982	2004	21.5
Haiti	1982	1994	12.0
Honduras	1981	2004	23.0
Ivory Coast	1983	1998	15.2
Jamaica	1987	1993	6.1
Jordan	1989	1993	4.1
Kenya	1994	2004	10.0
Macedonia	1992	1997	5.2
Madagascar	1981	2002	20.1
Mauritania	1992	1996	4.7
Mexico	1982	1990	7.9
Morocco	1986	1990	4.6
Mozambique	1983	1992	10.0
Myanmar	1997	2003	6.0
Nicaragua	1979	2003	24.0
Niger	1983	1991	7.9
Nigeria	1982	1992	10.4
Pakistan	1998	1999	1.6
Panama	1983	1996	12.7
Paraguay	1986	1993	7.6
Peru	1980	1980	0.9
Peru	1983	1997	14.4
Philippines	1983	1992	9.6
Poland	1981	1994	12.9
Romania	1981	1983	1.5
Russia	1991	1997	6.0
Sao Tome and Principle	1987	1994	7.7
Senegal	1990	1990	0.7
Senegal	1992	1996	5.0
Serbia and Montenegro	1992	2004	12.0
Seychelles	2000	2002	2.0
Sierra Leone	1986	1995	9.7
South Africa	1993	1993	0.7
Tanzania	1984	2004	20.3
Togo	1991	1997	7.0
Trinidad and Tobago	1988	1989	2.0
Uganda	1980	1993	13.2
Uruguay	1990	1991	1.1
Venezuela	1990	1990	1.0
Vietnam	1985	1998	14.0
Yemen	1985	2001	16.5
Zambia	1983	1994	10.5
Zimbabwe	2000	2004	4.0

in the sovereign renegotiation processes has changed over time. In particular, the presence of the IMF and World Bank makes a critical difference for sovereign debt renegotiations after the 1970s.

These changes affect the renegotiation duration differently. Weakening of sovereign immunity might strengthen creditors' position in renegotiations because unsatisfied creditors can take the sovereigns to the court. Losing the abilities to seize collateral over time, however, might worsen creditors' position because they have fewer outside options. Intervention of the super-national organizations, like the IMF, in the renegotiations might help speed up the process. In sum, the changing international financial market itself might have changed the renegotiation durations over time. Without a control for the time effect, it is thus hard to compare the renegotiation durations of different types of debt across periods (Table 4).

Changing financing choice of borrowing countries

In the late 1970s and early 1980s, the price of oil rose dramatically, and OPEC producers amassed billion dollars in their oil revenues, which were deposited in European and US banks. Banks, flush with money, had to re-lend to pay interest to their depositors, and turned to developing countries. Moreover, US commercial banks could claim a foreign tax credit against US income tax liabilities under the old US tax rules in this period. These tax rules were exploited by both the governments of developing countries and US commercial banks to extract interest subsidies from the US government.[21] The US commercial banks earned substantial profits, which provided huge incentives for lending to developing countries. For the borrowers, an excessive supply of funds and the foreign tax credit made these bank loans so cheap that real interest rates were actually negative for several years. Thus, we observe that commercial bank loans were the dominant form of developing countries' debt financing during the 1970s and early 1980s.

From the early 1980s, the oil price started to decline. Also, the US 1986 tax reform changed the rules of the foreign tax credit, and the commercial banks lost their profit opportunities from lending to developing countries. Bank lending to developing countries started to dry out. Bond financing by developing countries started to take off, promoted in part by the Brady plan to restructure defaulted bank debt. Over time, bond financing became the dominant borrowing form of developing countries.

Our model can rationalize the evolution of this financing pattern of developing countries. Consider the financing choice of a borrowing government. To mimic the data, we assume that the interest rate offered by banks is lower than that offered by bondholders in the first period, while it is the same as that offered by bondholders in the second period. Under a wide range of parameter values, the bank loan financing will be preferred due to its lower interest rate in the first period. In the secondperiod where both choices require the same interest rate, the bond financing will be preferred as long as the creditors' payoff is not too low, as shown in Proposition 6.[22]

[21] In an extreme case, the foreign tax credit from the US government became the US commercial banks' profit because they have already increased the interest rate charged to the developing countries to fully cover the foreign tax. The developing countries faced the same effective interest rate after adjusting for the tax income. In general, the tax subsidies from the US government were negotiated and split between the commercial banks and the borrower countries. For details see Frankel (1985).

[22] In a related paper, Bolton and Jeanne (2007) argue that bond financing may have increased in the 1990s because bonds are more difficult to restructure and are believed to be de facto senior to bank loans. However, the evidence supporting their hypothesis is inconclusive: as we document above, bond debt restructuring was on average much more rapid than bank loan restructuring.

References

Acharya, Viral V., Bharath, Sreedhar T., Srinivasan, Anand, 2007. Does industry-wide distress affect defaulted firms? Evidence from creditor recoveries. Journal of Financial Economics 85, 787–821.

Arellano, Cristina, 2007. Default risk and income fluctuations in emerging economies. The American Economic Review 98 (3), 690–712 (June).

Ausubel, Lawrence M., Cramton, Peter, Deneckere, Raymond J., 2002. Bargaining with incomplete information. In: Aumann, Robert J., Hart, Sergiu (Eds.), Handbook of Game Theory with Economic Applications, Vol. 3. Elsevier, Amsterdam.

Bai, Yan, Zhang, Jing, 2010. Solving the Feldstein–Horioka puzzle with financial frictions. Econometrica 78 (2), 603–632 (March).

Beers, David T., Chambers, John, 2004. "Sovereign Defaults: Heading Lower Into 2004," Standard & Poor's Research.

Benjamin, David, Wright, Mark L.J., 2008. Recovery before redemption? A theory of delays in sovereign debt renegotiations. UCLA Working Paper.

Bi, Ran, 2008. Beneficial delays in debt restructuring negotiations. IMF Working Paper WP/08/38.

Bolton, Patrick, Jeanne, Olivier, 2007. Structuring and restructuring sovereign debt: the role of a bankruptcy regime. Journal of Political Economy 115 (6), 901–924.

Broner, Fernando A., Martin, Alberto, Ventura, Jaume, 2010. Sovereign risk and secondary markets. The American Economic Review 4, 1523–1555 (September).

Bulow, Jeremy, Rogoff, Kenneth, 1989. A constant recontracting model of sovereign debt. Journal of Political Economy 97 (1), 155–178 (February).

Cole, Harold L., Kehoe, Timothy, 2000. Self-fulfiling debt crises. The Review of Economic Studies 67 (1), 91–116.

Conesa, Juan Carlos, Kehoe, Timothy, 2011. Gambling for redemption and self-fulfilling debt crises. Federal Reserve Bank of Minneapolis Staff Report.

Cramton, Peter C., Tracy, Joseph S., 1992. Strikes and holdouts in wage bargaining: theory and data. The American Economic Review 82 (1), 100–121 (March).

Eaton, Jonathan, Gersovitz, Mark, 1981. Debt with potential repudiation: theoretical and empirical analysis. The Review of Economic Studies 48 (2), 289–309 (April).

Eberhart, Allan C., Sweeney, Richard J., 1992. Does the bond market predict bankruptcy settlements? Journal of Finance 47 (3), 943–980 (July).

Eichengreen, Barry, Kletzer, Kenneth, Mody, Ashoka, 2003. Crisis resolution: next steps. IMF Working Paper WP/03/196.

Fernandez, Raquel, Rosenthal, Robert W., 1990. Strategic models of sovereign-debt renegotiations. The Review of Economic Studies 57 (3), 331–349 (July).

Frankel, Allen, 1985. Some implications of the president's tax proposals for U.S. banks with claims on developing countries. International Finance Discussion Papers. September, (263).

Fudenberg, Drew, Levine, David, Tirole, Jean, 1985. Infinite-horizon models of bargaining with one-sided incomplete information. In: Roth, Alvin E. (Ed.), Game-Theoretic Models of Bargaining. Cambridge University Press, Cambridge, pp. 73–98. chapter 5.

GCAB, 2004. Investor Roadshow Presentation. Global Committee of Argentina Bondholders, New York.

Haldane, Andrew G., Penalver, Adrian, Saporta, Victoria, Shin, Hyun Song, 2005. Analytics of sovereign debt restructuring. Journal of International Economics 65 (2), 315–333 (March).

Hart, Oliver, 1989. Bargaining and strikes. Quarterly Journal of Economics 104 (1), 25–43 (February).

Khieu, Hinh, Mullineaux, Donald, 2009. The determinants of bank loan recovery rates. University of Southern Indiana Working Paper.

Kletzer, Kenneth M., 2003. Sovereign bond restructuring: collective action clauses and official crisis intervention. IMF Working Paper WP/03/134.

Kroszner, Randall S., 2003. Soverign debt restructuring. American Economic Review, Papers and Proceedings 75–79.

Krueger, Anne O., 2002. A New Approach To Sovereign Debt Restructuring. International Monetary Fund.

Merlo, Antonio, Wilson, Charles, 1995. A stochastic model of sequential bargaining with complete information. Econometrica 2, 371–399 (March).

Peterson, Michael, 1999. A crash course in default. Euromoney 366, 47–50 (October).

Pitchford, Rohan, Wright, Mark L.J., 2007. Restructuring the sovereign debt restructuring mechanism. UCLA Working Paper.

Pitchford, Rohan, Wright, Mark L.J., 2008. Holdouts in sovereign debt restructuring: a theory of negotiation in a weak contractual environment. UCLA Working Paper. .

Reinhart, Carmen M., Rogoff, Kenneth S., 2009. This Time is Different: Eight Centuries of Financial Folly. Princeton University Press.

Schuermann, Til, 2004. What do We Know About Loss Given Default? Wharton Financial Institutions Center. 04–01.

Sobel, Joel, Takahashi, Ichiro, 1983. A multistage model of bargaining. The Review of Economic Studies 50 (3), 411–426 (July).

Stokey, Nancy L., 1981. Rational expectations and durable goods pricing. Bell Journal of Economics 12 (1), 112–128 (Spring).

Sturzenegger, Federico, Jeromin, Zettelmeyer, 2005. Haircuts: estimating investor losses in sovereign debt restructurings, 1998–2005. IMF Working Paper WP/05/137.

Suter, Christian, 1992. Debt Cycles in the World Economy: Foreign Loans, Financial Crises, and Debt Settlements, 1820–1990. Westview Press, Boulder, Colorado.

Trebesch, Christoph, 2009. Delays in sovereign debt restructuring. Should we really blame the creditors. Working Paper. Free University of Berlin.

Weinschelbaum, Federico, Wynne, Jose, 2005. Renegotiation, collective action clauses and sovereign debt markets. Journal of International Economics 67 (1), 47–72 (September).

Yue, Vivan Z., 2010. Sovereign default and debt renegotiation. Journal of International Economics 80 (2), 176–187 (March).