A Note on ‘Money Is Memory’: A Counterexample*

Yu Awaya† and Hiroki Fukai‡

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Abstract

A counter-example to the notion that *money is memory* is provided—one that relies on incomplete information. For it, there exists an implementable allocation with money which is not implementable with memory. The result arises because money conveys only a limited amount of information about past actions which can be beneficial in settings with incomplete information.

Keywords: money, incomplete information, imperfect monitoring

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†Department of Economics, Penn State University, yxa120@psu.edu.

‡Department of Economics, Penn State University, hzf110@psu.edu.

1 Introduction

Kocherlakota [3] studies the role of money in a class of economies in which people are matched into groups and then trade. In such economies, *memory* means that there is knowledge of past actions of direct and indirect partners, while *money and no memory* means that players observe none of the past actions of other players and there is a fixed supply of fiat money, an intrinsically useless object. For that class, he shows that any implementable allocation with money and no memory is implementable with memory. The class of environments he studies does not contain information asymmetries. Moreover, he conjectures that the result would not extend to an environment with *persistent asymmetries of information*. We confirm his conjecture. We provide an example with persistent information asymmetries about a player's type and show that there is an allocation that is implementable with money and no memory, but not with memory.

In this paper, as in Kocherlakota [3], money conveys coarser information about past actions than memory does. As is well-known in the reputation literature (see for example, Fudenberg and Levine [1] and the survey in Mailath and Samuelson [4]), in the presence of persistent incomplete information an equilibrium outcome under no memory may not be an equilibrium under memory. We show that money, by providing an intermediate level of information between memory and no memory, gives rise to an equilibrium outcome that cannot arise under either memory or no-memory.[1]

Our counter-example is related to analysis in Fudenberg and Levine [1] with a long-lived player who has private information about her permanent type and a sequence of short-lived players. The long-lived player is either a strategic type or a behavioral type. They show that the set of equilibria shrinks relative to the case where there is no behavioral type, because the strategic type has an option to mimic the behavioral type. We adopt those features of Fudenberg and Levine [1], and add an absence-of-double-coincidence feature so that money plays a role when our economy has no memory.

Each date is divided into two parts, day and night. In the day part, the short-lived player can produce, but has no consumption opportunity, while at night he has a potential consumption opportunity, but cannot produce. The long-lived player is essentially in the opposite situation. In the economy with memory, as in Fudenberg and Levine [1], the strategic long-lived player wants to mimic the behavioral type. In an economy with money and no memory,

[1] Kahn et al. [2] also study a positive role of money related to the coarsening of information. However, in their setting there is no persistent incomplete information and some players can choose what to reveal.

when a new-born short-lived player observes that the long-lived player has money, he cannot distinguish between two histories: (i) trade has occurred in the past using money; (ii) trade has not occurred. This feature of money prevents the strategic long-lived player from mimicking the behavioral type.

Two remarks are in order about our result. First, Kocherlakota [3] provides an example which shows that his main result does not hold if memory is replaced by *perfect monitoring*—knowledge of the past actions of *all* players (not only direct and indirect trading partners). Our counter-example is not of that type because memory and perfect monitoring coincide in it. Second, there is a different conjecture which states that *imperfect monitoring is necessary for money to play a role* (see, for example, Wallace [5]). This conjecture says that in an economy with perfect monitoring, the presence of money does not enlarge the set of implementable allocations. Our economy is *not* a counter-example to that claim.

2 Model

Time is discrete, lasts forever, and is indexed by $t = 0, 1, 2, \cdots$. There is one long-lived ("she") player and a sequence of short-lived ("he") players, each of whom lives for one date, a day followed by a night. The long-lived player, who has discount factor $\beta \in (0, 1)$, is one of two types, *strategic* or *behavioral*. The long-lived player's type is permanent and privately known to her. Her type is drawn at the beginning of date 0. She is the strategic type with probability $1 - \pi$ and is the behavioral type with probability π.

In each of day and night, there is one indivisible, perishable, produced good. In each day (resp. night), only the long- (resp. short-) lived player can consume the good. In day, there are three technologies: joint production by both players, solo production by the short-lived player, and non-production.[2] At night, there are two technologies: solo production by the long-lived player and non-production. With the cost and utility of non-production normalized to be zero at day and at night, the payoffs are given in Table 1, where all the parameters in the table are positive. Production is costly and there are gains from trade for both players at each date under either day-time technology. However, conditional on production at night, the long-lived player prefers solo production during the day, while the short-lived player prefers joint production.

[2]Here, in the joint production the long-lived player helps the short-lived player in production by incurring a cost. Solo production in day by the long-lived player is not possible.

Table 1: Payoffs

	joint in day and solo at night		solo in day and solo at night	
	long-lived	short-lived	long-lived	short-lived
day	$\nu - \gamma$	$-c_j$	ν	$-c_s$ $(< -c_j)$
night	$-\kappa$	u	$-\kappa$	u
net	$\nu - \gamma - \kappa > 0$	$u - c_j > 0$	$\nu - \kappa$	$u - c_s > 0$

We also assume that

$$\pi \leq \frac{c_s - c_j}{u - c_j}$$

While the strategic type maximizes her lifetime payoff multiplied by $(1 - \beta)$ (which is just normalization, following a convention of the literature on repeated games and reputation), we assume that the behavioral type (i) never agrees to joint production, and (ii) produces the good in the night of a date whenever she consumed the good in the day of the date.

3 Mechanisms

We take into account all possible ways of trading. Thus, both mechanisms with one-stage commitment and those without commitment are allowed. As is standard, a mechanism is defined to be a sequence of games that can depend on the history.

Mechanisms with No Commitment

The day-time stage of a mechanism with no commitment has three sub-stages.

1. A lottery over use of the technologies is proposed.

2. (i) If joint production is realized, then simultaneously both players choose from $\{yes, no\}$. If both players choose yes, then joint production is executed. If at least one player chooses no, then joint production is rejected. (ii) If solo or non-production is realized, then only the short-lived player chooses from $\{yes, no\}$.[3]

3. If any rejection occurs, then the short-lived player chooses from $\{solo, non\}$. If *solo* is chosen, then it is executed.

[3]In general, when solo or non-production is realized, the long-lived player could also reply yes or no. In that case, however, there is trivially a dominant strategy for the long-lived player. Hence, our formulation is without loss of generality.

The night-time stage proceeds in the same way except that there is no joint production and *short-lived* is replaced by *long-lived* in the above description.

Mechanisms with One-date Commitment

The day-time stage of a mechanism with one-date commitment proceeds as follows.

1. A lottery over use of the technologies is proposed.

2. (i) If joint production is realized, then simultaneously the long-lived player chooses from $\{yes, no\}$ and the short-lived player chooses from $\{yes, no\} \times \{solo, non\}$. Again, if both players choose yes then joint production is executed, while if at least one player chooses no then joint production is rejected. (ii) If solo or non-production is realized, then only the short-lived player chooses from $\{yes, no\} \times \{solo, non\}$.

3. If any rejection occurs, then the short-lived player does what he has chosen from the set $\{solo, non\}$.

Again, the night-time stage proceeds in the same way except that there is no joint production and *short-lived* is replaced by *long-lived* in the above description.

In mechanisms with one-stage commitment, the second element of the Cartesian product $\{yes, no\} \times \{solo, non\}$ represents the decision conditional on any rejection. That is, before knowing whether rejection occurs, the short-lived player commits to whether he executes solo or non-production if rejection occurs. For either case, each short-lived player is free to execute solo production during the day-time.

Note that the class of one-date commitment mechanisms includes take-it-or-leave-it offers by both the long and short-lived players. We will use this fact in the proof with money. Note also that, because each player can always reply no, any mechanism we consider is sequentially individually rational for each player.

Memory

Following Kocherlakota [3], we mean by memory complete observation of the history. A history consists of (i) previous planner proposals, (ii) previous lottery realizations, (iii) previous actions including whether people have executed solo or non-production when rejections have occured. A planner can choose a proposal that depends on the history up to that time, though no commitment between day and night stages is possible.

A short-lived player's strategy consists of two things: (i) a reply to each day-time proposal and (ii) a day-time choice after any rejection. Each short-lived player has a belief about the long-lived player's type. The long-lived player's strategy consists of three things: (i) a reply to each day-time proposal, (ii) a reply to each night-time proposal, and (iii) a night-time choice after any rejection.

Money

Fiat money is an indivisible, durable, intrinsically useless object. We give the long-lived player one unit of money at the beginning of the initial date. Money is disposable. In particular, if a short-lived player holds money at the end of a night, he simply disposes of the money when he dies. We assume that people publicly observe who has money.

Because there is no memory, the short-lived player at a given date only knows money holdings. The long-lived player remembers what people have done in the past. A public history with money at some moment of a date consists only of (i) who is the current money holder and (ii) what has happened in the earlier stages of the date. Also, players now have strategies about money disposal, too. At any point in time, each player chooses from $\{dispose, not\}$.

Equilibrium and Implementability

A profile of strategies and beliefs is a *(perfect Bayesian) equilibrium for a mechanism* if at any time and any stage and for any history, each player maximizes his or her objective given the other's strategy and belief, and the belief is updated using Bayes' rule whenever possible. We say that an outcome is *implementable* if for some mechanism, there exists an equilibrium which achieves the outcome on the equilibrium path.

4 Result

Our result involves a particular outcome called a *cooperative outcome*.[4] A cooperative outcome is defined to be joint production in every day and solo production (by the long-lived player) in every night whenever the long-lived player is strategic. Notice that if the long-lived player is the behavioral type, she, by assumption, never accepts joint production. We have

Theorem 1. *Suppose the strategic long-lived player is sufficiently patient. Then, a cooperative outcome is implementable with money, but not with memory.*

Proof. There are two parts of the theorem. We first show that a cooperative outcome is implementable with money. The proof of that part uses a

[4]Because the behavioral-type long-lived player does not have a utility function, direct welfare comparison between a cooperative outcome and a non-cooperative outcome, would not be plausible. For our purpose, however, it suffices to show that there is *an* implementable outcome with money that is not implementable with memory.

commitment proposal, one in which, in effect, the short-lived player makes a take-it-or-leave-it offer involving joint production. The proof of the second part shows that if there is memory, then the strategic long-lived player wants to mimic the behavioral type if joint production is realized. This is profitable for her (if she is patient enough) because all future short-lived players will, then, believe that the long-lived player is the behavioral type and will want to execute solo production. Here are the details for each part.

Implementability with Money

This part of the proof uses a commitment proposal, one in which, in effect, the short-lived player makes a take-it-or-leave-it offer involving joint production. We first describe the mechanism and then a candidate for equilibrium strategies. Then, we show that the strategies are an equilibrium.

(i) The mechansim. In day, the short-lived player proposes joint production and a monetary transfer if the long-lived player has money. The short-lived player proposes non-production and no monetary transfer if the long-lived player does not have money. At night, the long-lived player proposes solo production and monetary transfer if the short-lived player has money. The long-lived player proposes non-production and no monetary transfer if the short-lived player does not have money.

(ii) Candidate strategies. In day, (d.i) the strategic long-lived player replies yes (the short-lived player also replies yes) if a short-lived player proposes joint production and monetary transfer. By assumption, the behavioral type replies no to this proposal, and neither production nor monetary transfer takes place, (d.ii) the strategic long-lived player replies yes if a short-lived player proposes solo production and monetary transfer, and (d.iii) the long-lived player replies no if the short-lived player proposes non-production. At night, (n.i) a short-lived player replies yes if the long-lived player proposes solo production and monetary transfer, and (n.ii) a short-lived player disposes of money if the long-lived player makes any proposal involving non-production. Otherwise he gives money back to the long-lived player.

Now we show the optimality of the candidate strategy profile.

Short-lived player: A short-lived player cannot distinguish between the following two histories in both of which the long-lived player has money at the beginning of a date: (i) the long-lived player accepted joint production in day and produced good at night, at some point in time, and (ii) the long-lived player rejected joint production, at some point in time (and of course, any combination of these). Therefore, if the long-lived player has money, a short-lived player has the prior belief $(1-\pi, \pi)$ about her type.

Suppose that a short-lived player proposes joint production in day (which is supposed to happen on the equilibrium path). If the long-lived player is the strategic type, then the proposal is accepted in day and solo production is executed at night. In that case, the short-lived player gets $u - c_j$. If the long-lived player is the behavioral type, then the proposal is rejected in day and non-production is executed at night. In that case, the short-lived player gets 0. Therefore, given his belief, the short-lived player's expected payoff is $(1 - \pi)(u - c_j)$.

Suppose that the short-lived player proposes solo production in day. Then, either type of long-lived player accepts this proposal and the short-lived player's expected payoff is given by $u - c_s$. If the short-lived player proposes non-production in day, then his expected payoff is 0.

Because $\pi \leq (c_s - c_j)/(u - c_j)$ by assumption, $(1 - \pi)(u - c_j) \geq u - c_s$. Therefore, neither proposing non-production (which gives 0) nor proposing solo production (which gives $u - c_s > 0$) is a profitable deviation for the short-lived player.

Notice also that, at night, short-lived players are indifferent between disposing of money and giving it to the long-lived player, so any action on monetary transfer (on and off path) is optimal.

Long-lived strategic player: Suppose that the long-lived player accepts joint production (which is supposed to happen on the equilibrium path). In that case, the stage-game payoff to the long-lived player is given by $\nu - \gamma - \kappa$. And, she starts the next date with money. Suppose that the long-lived player rejects joint production. In that case, non-production is executed in day and the short-lived player cannot buy the good at night. In that case, the stage-game payoff to the long-lived player is given by 0, and she starts the next date with money (and thus, the future payoff is the same as the above case). So, this is not a profitable deviation.

Next consider her night-time incentives. Suppose that the long-lived player proposes solo production at night (which is supposed to happen on the equilibrium path). In that case, she incurs cost κ for production today, and she gets $\nu - \gamma - \kappa$ from the next date on. So, the continuation payoff to her is given by $-(1-\beta)\kappa + \beta(\nu - \gamma - \kappa)$. Suppose that the long-lived player proposes non-production at night. In that case, she evades cost κ, but instead, the short-lived player disposes of money and the stage-game payoff to the long-lived player is 0 from the next date on. So, the continuation payoff to her is given by 0. For β sufficiently close to one, $-(1-\beta)\kappa + \beta(\nu - \gamma - \kappa) \geq 0$, which implies that non-production at night is not a profitable deviation for the long-lived player.

Non-implementability with Memory

Suppose, by way of contradiction, that a cooperative outcome is implementable. That is, suppose that for some mechanism there is an equilibrium which supports the cooperative outcome. It will be shown that the strategic long-lived player has a profitable deviation irrespective of the mechanism.

If the strategic type deviates by rejecting joint production at $t = 0$, then all future short-lived players believe that they are facing the behavioral type—because only the behavioral type is supposed to reject joint production along the equilibrium path. Given that belief, the best response of all future short-lived players is to execute solo production whether or not one-date commitment is possible. Thus, the strategic type gets $\nu - \kappa$ from date 1 on regardless of the mechanism. The date-0 payoff to the strategic type is bounded from below by $-\kappa$ irrespective of the mechanism. This is because the worst case scenario for the long-lived player is to get no good in day and produce at night which would give her $-\kappa$. Therefore, the continuation payoff to the strategic type from the deviation is bounded below by $-(1 - \beta)\kappa + \beta(\nu - \kappa)$, while that from not deviating is given by $\nu - \gamma - \kappa$. Because $\gamma > 0$, the deviation is profitable for all β sufficiently close to one. $\square$

5 Concluding remarks

In our example, incomplete information plays a crucial role. If, instead, the long-lived player's type were observable, then Kocherlakota's money-is-memory result would hold. In particular, a cooperative outcome would be implementable with memory, because, with complete information, the strategic long-lived player could not mimic the behavioral type any longer.

The class of mechanisms we study includes take-it-or-leave-it offers by the short-lived players—which of course are the best mechanism for the short-lived players. This assumption plays a crucial role in showing that the co-operative outcome is implementable with money. For example, if one simply imposes that the *long-lived* player makes a take-it-or-leave-it offer at day stage, then, even with money, the strategic long-lived player can mimic the behavioral type by proposing solo production.

Our result also holds when goods are divisible. Suppose the long-lived player chooses an effort level from $[0, \infty)$ and the short-lived players chooses a production level from $[0, \infty)$ and that the higher the costly effort level of the long-lived player, the lower is the short-lived players' cost of production. The behavioral type long-lived player is assumed to always choose zero effort level, an analogue of the assumption that the behavioral type always rejects joint production.

It also holds when there are many long-lived players and many short-lived players at a date and when they are randomly paired off with each other. It is crucial for our result to have a long-lived player and a sequence of short-lived players à la Fudenberg and Levine [1]. Thus, we take the minimal element of modeling, one long-lived player and one short-lived player at a date, to obtain our main result. Note however that, in such an extention with many agents, it is also crucial to assume that the *same* two people interact for two sub-dates and production technologies in day and night are like the ones that we studied above.

As a final remark, the money-is-memory claim is different from the claim which says that imperfect monitoring is necessary for money to be essential (see, for example, Wallace [5]). Our economy is *not* a counterexample to that essentiality claim. A proof of it would take an economy with memory *and* money and show that the presence of money does not enlarge the set of implementable allocations. In our economy, if there is both memory and money, then no cooperative outcome is implementable. The proof is the same as we gave above for the memory part of Theorem 1.

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