Fiscal Federalism and Monetary Unions *

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ABSTRACT

We apply ideas from fiscal federalism to rethink standard macro results on how fiscal authority should be delegated in a monetary union. In a real model in which decentralized fiscal authorities have an information advantage about the tastes of their citizens a natural generalization of the Oates Decentralization Theorem holds, in that a fiscal union dominates a decentralized regime and the degree of dominance increases as the signal becomes less informative. Instead with direct fiscal externalities, a decentralized regime is optimal for small unions, but a centralized regime is optimal for large ones. We then consider a monetary economy in which governments finance their expenditure with nominal debt and inflation has a negative effect on productivity. If the monetary authority has commitment the Decentralization Theorem holds but it does not there is an indirect fiscal externality. Instead, a decentralized regime is optimal for a small union but a fiscal union is optimal for a sufficiently large one.

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*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
“We should know over which matters several local tribunals are to have jurisdiction, and in which authority should be centralized.” Aristotle, Politics 4.15

The question of how authority should be delegated between a central authority more local authorities have been a subject of great interest for millenia. In the near term, scholars such as Tabellini (2002), who have studied the European Union, have have argued that questions such as which tasks should be left to a union-wide fiscal authority and which should be left to the Member States is one of the most important ones for the future of Europe. Likewise, scholars such as Saiegh and Tommasi (1999), Nicolini et al. (2002), and Kempf and Cooper (2004) have argued that the complex labyrinth governing the rules of fiscal federalism in Argentina are responsible in large part for its poor performance in both fiscal and monetary policy.

The purpose of this paper is to apply some of the ideas on fiscal federalism in order to rethink standard results in the macroeconomic literature on how fiscal authority should be delegated in a monetary union of member states. In the relatively small literature in this area, the standard models typically imply that it is always weakly optimal to delegate policy to a central fiscal authority. The reason is twofold: the central fiscal authority internalizes any possible fiscal externality that local authorities do not and local authorities have no advantage of any kind in carrying out local decisions over a central authority. We find it interesting that this approach to modeling contrasts sharply with the literature on fiscal federalism which typically builds in assumptions so that, absent large fiscal externalities, it is always optimal to delegate fiscal authority to the local authorities. Specifically, the presumption in much of the literature on fiscal federalism is the centralized authorities are less able to tailor their policies to the preferences of the member states and a local authority could and, hence, policies are more uniform across states than might otherwise be desirable.

This paper builds a simple macro model which formalizes this presumption in a parsimonious way and uses it to rethink the optimal delegation of fiscal policy. One of the main results is that when the size of the monetary union is small the free-riding type of fiscal externalities may also be small so the natural advantage of a decentralized regime in being able to tailor policies to each member state outweighs the costs of the fiscal externalities. But as the size of the monetary union grows, so do the free-riding problems from the fiscal externality.
Hence, the optimal delegation rule has a cutoff form: there is a threshold size of the monetary union, under which it is optimal to have a decentralized regime of fiscal authority but above it a centralized fiscal union is optimal. We characterize in detail the forces that shift this threshold size.

One reading of much of the work in this area is that, loosely, unless there is a compelling case for not doing so authority should be left to the Member States. Indeed, Article 5(3) of the Treaty on European Union (TEU) signed in 1992, enshrined a principle of *subsidiarity* for this delegation, which defines the circumstances in which it is preferable for an action to be take by the Union, rather than the Member States. These include, (i) the competence in this area is shared between the Union and Member States (i.e. non-exclusive competence); (ii) the objectives of the proposed action cannot be sufficiently achieved by the Member States (i.e. necessity); (iii) the action can therefore, by reason of its scale or effects, be implemented more successfully by the Union (i.e. added value).

Even though the European Union is not technically a federation, scholars such as Tabellini (2002) have argued the many of the idea from the classic theory of fiscal federalism can be applied to rethink the answer as to when should the delegation of fiscal authority be for member states of a monetary union. In particular, when should various types of fiscal authority be left to the discretion of member states and when should they be centralized in a union-wide authority. We argue that by doing so we develop some new answers to an old question.

One of the seminal studies of fiscal federalism was developed in a well-known book by Oates (1972). A key presumption of Oates’s theory is that a centralized authority will be less sensitive to the varying preferences among the residents of different communities and hence have a tendency towards uniformity in fiscal matters across such communities (Oates 1972, p.11). To address this question of delegation simply but formally Oates focuses on what he calls an *ideal special case*. In it all individuals in a specific geographic subset of the population are immobile and have identical preferences over public goods. Moreover, the local authority has built in advantage over the central authority in that it has complete knowledge of the tastes of its constituents whereas Oates presumes that any fiscal decision taken by the centralized government must be uniform across member states. Under this assumption and
a few others, Oates argues that a general Decentralization Theorem holds:

“For a public good—the consumption of which is defined over geographical subsets of the total population, and for which the costs of providing each level of output of the good in each jurisdiction are the same for the central or the respective local government—it will always be more efficient (or at least as efficient) for local governments to produce the Pareto-efficient levels of output for their respective jurisdictions than for the central government to provide any specified and uniform level of output across all jurisdictions.” (Oates 1972, p. 35)

A major theme throughout Oates’ work is that local governments have a natural advantage over centralized authorities in being better able to tailor policies to local constituents and that, unless there are compelling offsetting forces, such as very large fiscal externalities, which induce local governments to (typically) underprovide their local public good, most authority should be left to these local governments.

Even though Oates’ work is very influential, it is not without detractors. In a survey of the benefits of decentralization Lockwood (2005, p.2) characterizes Oates thesis as a preference-matching argument through which “goods provided by governments in localities will be better matched to the preferences of residents in those localities”. But Lockwood (2005 p.3) criticizes Oates for capturing the preference-matching benefit by making the “ad-hoc assumption of policy uniformity: central government is assume to set a uniform level of local good provision in all regions.” Moreover, he argues that even if local governments have some better information about local preferences than the central government there is a large incentive literature that shows how a benevolent central government could, under some reasonable assumptions, design an incentive scheme to elicit this information from the regions and then implement the efficient outcome.

Besley and Coate (2003, p. 2612) also criticize the idea that centralization implies uniformity and argue that this assumption is neither empirically nor theoretically satisfactory. They argue that from an empirical point of view, there are many examples of goods provided unequally by the federal government and from a theoretical point of view, it is unclear why a government charged with providing public goods in a centralized system cannot differentiate
the levels according to the heterogeneous tastes in each district.”

We find it interesting that both Lockwood (2002) in his survey and Besley and Coate (2002) in their work argue that a more satisfactory way to capture the idea the local governments may be able to better tailor their decisions to the benefits of their constituency is to retreat from the idea the governments try to maximize the welfare of their residents and instead try to generate such outcomes by various political economy considerations such as rent-seeking, lobbying, worries about reelection, together with various detailed rules about how legislative bargaining works.

For macroeconomists who would like to develop broad principles governing many different countries one view on this political economy literature is that it is attractive because of its the clear theoretical foundations underpinning it. However, as the survey of Lockwood (2002) makes clear, the predictions of the various political economy models in the literature are often highly dependent on the precise details of the legislative bargaining, the election process for officials and so on. As such, one might worry that when applying these theories to a large number of member states in a union one might need to delve into the differing political structures of each member state in order to apply it.

Here we adopt a much simpler approach that we view as complementary to that literature. First, we assume throughout that any legislative body is trying to maximize the ex-ante utility of the citizens it is responsible for. In particular, the fiscal authorities of the member states maximize the utility of the citizens in their states and union-wide fiscal authorities and monetary authorities maximize the utility of the citizens of the union. Second, we formalize some of the ideas of Oates by assuming that local authorities have a natural advantage in being able to tailor their policies to the preferences of their constituents because they have more precise information about their citizens. In particular, in a centralized regime, the union-wide fiscal authority only receives a noisy signal about the citizens preferences in each of the member states. In this sense, we formalize in a simple way that even if the member states have no incentives to conceal this information and attempt to communicate their true preferences to the centralized authority there is some built-in noise in this transmission process.

Some empirical support for the idea that even well-meaning agents often have a hard
time communicating their preferences to others has been found by Waldfogel (1993). In an interesting study Waldfogel argues that an important economic aspect of gift-giving, say around the holidays, is that the gifts may be mismatched with the recipients’ preferences. Indeed he argues that in the standard microeconomic framework of consumer choices, the best a gift-giver can do with, say, $10 is to duplicate the choice that the recipient would have made. But if the gift-giver is less than perfectly informed it is likely that the gift will leave the recipient worse off than if the recipient had made the consumption choice with an equal amount of cash. Hence, gift-giving is a potential source of deadweight loss. Based on this idea, Waldfogel surveyed undergraduates and found the holiday gift-giving from significant others destroys about 10 percent of the value of the value of gifts whereas gifts from members of the extended family destroy about a third of their value.

In short, the main defense of the assumption that member states can only communicate a noisy signal about their preferences is motivated by the idea that if even spouses— with zero incentive problems— cannot seem to communicate their preferences to each other very well, the same may well apply to officials of a member state communicating about their citizens preferences to a central authority. The secondary defense is that when coupled with the assumption that all legislative authorities try to maximize the welfare of the citizens they represent, we keep the forces governing the advantage of the local authorities both transparent and we do not need to delve into the weeds of the clean but very complicated political economy approach.

In terms of organization, we begin with a real model with no externalities that highlights the ideas behind Oates’ Decentralization Theorem. In it we dispense with the ad-hoc assumption of policy uniformity by the central authority of Oates and instead have the degree of optimal uniformity of this authority naturally arise from a noisy signal that authority receives. Here a natural generalization of the Decentralization Theorem holds, in that a fiscal union dominates a decentralized regime and the degree of dominance increases as the signal becomes less informative.

We then extend the real model to have a direct fiscal externality in which the spending of any member state has a component which contributes to the public goods in every other member state. Our main result here is that since the fiscal externality gets worse as the size
of the union grows, there is a cutoff size for the union such that a decentralized regime is optimal for any smaller-sized union but a centralized regime is optimal for any larger-sized union.

We then turn to a simple two-period monetary model in which countries finance their government expenditure with nominal debt and inflation has a negative effect on productivity. We find that when the monetary authority has commitment the optimal inflation is zero and the economy reduces to a real economy in which the Decentralization Theorem holds.

The most interesting case is when the monetary authority lacks commitment. Here if the monetary authority faces a larger amount of nominal debt it finds it optimal to inflate more so as lessen the need to raise distortionary taxes in the second period. Because of this feature, when deciding its own level of spending and debt, each member state only considers the negative consequences of the inflation it induces on itself and not on other member states. Hence, in the decentralized regime there is an indirect fiscal externality in that even though the spending of any member state does not enter the utility or production of any other state, the increased inflation its action generates does. The main result echoes the earlier result in the real economy with a direct externality. Since the fiscal externality gets worse as the size of the union grows, there is a cutoff size for the union such that a decentralized regime is optimal for any smaller-sized union but a centralized regime is optimal for any larger-sized union.

1. Fiscal Federalism and the Oates Decentralization Theorem

Consider a simple static model that highlights the ideas behind the Oates Decentralization Theorem. The basic idea of Oates is that there is a natural advantage to having local fiscal authorities decide on the local fiscal actions rather than having a centralized fiscal authority deciding on these actions. Specifically, Oates argues that a basic shortcoming of a centralized system is its “probable insensitivity to varying preferences among the residents of different communities” (Oates 1999, p. 11). In contrast, “A decentralized form of government therefore offers the promise of increasing economic efficiency by providing a range of output of certain public goods that corresponds more closely to the differing tastes of groups of consumers” (Oates, 1999, p. 12).
In his seminal text, Oates simply assumes that a centralized fiscal authority must provide the same level of public goods to all member states even though these states have differing preferences for them. We both generalize and provide a microfoundation for this presumption by assuming that the centralized fiscal authority observes a noisy signal from each of the member states so that it is incapable of precisely tailor the policies it chooses for each member states to its preferences. The extreme case in which the signal from each member state is completely uninformative then naturally nests the case discussed by Oates.

The Decentralization Theorem focuses on what Oates calls the case of *perfect correspondence* in the provision of public goods. This case involves several assumptions. The first is that individuals with the same tastes for this public good are grouped into a geographic region. The second is that the only citizens who benefit from the local public good are those in this region. The third is that the cost of providing this good are the same at the local level and the centralized level. The next is that the local government possesses “complete knowledge of the tastes of its constituents”. The last is that the local governments have been set up so that each jurisdiction corresponds to a geographic region for which the tastes of citizens coincide. Under these assumptions, the theorem states that “it will always be more efficient (or at least as efficient) for local governments to provide the Pareto-efficient levels of output for their respective jurisdictions than for the central government to provide any specified and uniform level of output across all jurisdictions”.

We set up our baseline real model to be consistent with all of these assumptions except that we allow for a more general information structure for the centralized authority. We show that a Generalized Decentralization Theorem holds for that case. In this sense, we address the criticism that Besley and Coate (2003, p. 2612) who criticize the idea that “centralization implies uniformity” and argue that this assumption is “neither empirically nor theoretically satisfactory” by showing that a version of the theorem holds in this case.

We then turn to the case with externalities, namely a case in which Oates’ second assumption fails. We focus on a case in which the externality implies for any given member state its government spending has direct benefits but also that there are indirect benefits to that state that accrue through a fraction of the government expenditure of all the other member states. We focus on this case because as the fraction of other states spending varies
from zero to one, it covers the case of a pure local public good and a purely national public good. We find a new result, namely under certain conditions there is a cutoff rule in the size of the federation of countries such that for small enough federation, a decentralized regime is optimal but for a sufficiently large federation a fiscal union is optimal.

A. A Real Economy

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B. A Generalized Decentralization Theorem

Each of $i = 1, ..., I$ countries has a representative consumer and a government. There is also a central fiscal authority. The utility of a representative consumer in country $i$ is

$$u(c_i) + \theta_i h(g_i)$$

where $c_i$ is consumption and $g_i$ is government spending in country $i$ and $u$ and $h$ are increasing concave functions. The variable $\theta_i$ is the taste of country $i$ consumers for government spending. Here at the beginning of the period $\theta_i$ is randomly drawn independently across countries. For simplicity of notation only, we assume $\theta_i \in \{\theta_H, \theta_L\}$ with $\theta_H > \theta_L$ where $q = \Pr(\theta_H)$ and we denote the mean of the $\theta$ by

$$\mu_\theta = q\theta_H + (1 - q)\theta_L.$$
Each consumer in country $i$ has an endowment $y$ and the resource constraint for that country is

\begin{equation}
    c_i + g_i = y.
\end{equation}

The budget constraints for the consumer and the government of country $i$ are

\begin{equation}
    c_i + T_i = y \text{ and } g_i = T_i
\end{equation}

where $T_i$ is lump sum taxes. The consumer of country $i$ maximizes utility subject to the budget constraint.

A key feature of the setup motivated by the analysis of Oates is that the government of country $i$ has better information about the tastes of its citizens than does the central fiscal authority. In particular, the government of country $i$ perfectly observes $\theta_i$, whereas the central fiscal authority observes only a noisy signal about each country $i$’s tastes $s_i \in \{s_H, s_L\}$. Specifically, let the signal be symmetric in that

\begin{equation}
    \phi = \Pr(s_H|\theta_H) = \Pr(s_L|\theta_L) \text{ and } 1 - \phi = \Pr(s_L|\theta_H) = \Pr(s_H|\theta_L)
\end{equation}

and $\phi \in [1/2, 1]$ denotes the informativeness of the signal. Here $\phi = 1/2$ means that the signal is uninformative, in that $E(\theta|s_H) = E(\theta|s_L) = \mu_\theta$ whereas $\phi = 1$ means that the signal is perfectly informative in the $E(\theta|s_H) = \theta_H$ and $E(\theta|s_L) = \theta_L$. More generally, for arbitrary level of informativeness, Bayes’ rule implies that, given the prior $q$ that the taste is $\theta_H$, the posterior probabilities that the taste is $\theta_H$ after a high and low signal respectively are

\begin{equation}
    Q(q, s_H) = \frac{q \phi}{p_H} \text{ and } Q(q, s_L) = \frac{q(1 - \phi)}{1 - p_H},
\end{equation}

where

\begin{equation}
    p_H = q \phi + (1 - q)(1 - \phi)
\end{equation}

is the unconditional probability of receiving a high signal. Clearly, in (5), $q \phi / p_H$ is simply
the probability of receiving a high signal conditional on being of type $\theta_H$ relative to the unconditional probability of receiving a high signal.

Notice that this economy has been purposely set up to be consistent with Oates’ four assumptions. The way we set up the utility function in (1) implies that all the consumers in a given country have the same tastes and the jurisdiction of the country $i$ government are on these same consumers. Moreover, it implies that there are no external benefits to country $i$ from another other state’s spending $g_j$ for $j \neq i$. The way we wrote the resource constraint and the budget constraints imposed that the cost of providing $g_i$ is the same for country $i$ and the fiscal union. Finally, in terms of information we assumed that the government of country $i$ has complete knowledge about the tastes $\theta_i$ of its citizens. The generalization from Oates is that we allow the central fiscal authority to receive a noisy signal about $\theta_i$ and then optimally choose $g_i$, which will in general imply that $g_i$ responds to the signal rather than simply imposing that $g_i$ is the same for all countries, regardless of their underlying tastes.

The idea behind this formulation is that each local government tries its best to communicate to the central authority its true $\theta_i$ but the communication system is noisy so that what arrives is a noisy signal $s = (s_1, ..., s_I)$. It is worth noting that in the fiscal federalism literature the motivation is not that agents have private information that they choose not to truthfully reveal, but rather that the central authorities have a natural tendency towards uniformity across member states. Our formulation generalizes the standard case in Oates (1972, 1999) which can be thought of as the uninformative case.

Let us turn to evaluating the outcomes under two ways of delegating fiscal authority.

**Decentralized Regime.** In the decentralized regime, the government of each country $i$, given perfect information on the tastes of its citizens, chooses the level of government spending for its country. Hence it solves

$$(7) \quad W^D(\theta_i) = \max_{c_i, g_i} u(c_i) + \theta_i h(g_i)$$

subject to $c_i + g_i = y$. The ex-ante utility of country $i$ is then

$$(8) \quad V^D = qW^D(\theta_H) + (1 - q)W^D(\theta_L)$$
Since all countries are ex-ante symmetric, under an equal weighting scheme $V^D$ is also the ex-ante value for the union, that is,

$$V^D = EW^D(\bar{\theta}) = \frac{1}{I} \sum_{i} EW^D(\theta_i)$$

**Fiscal Union.** In the fiscal union, the central fiscal authority sees the vector of signals $\bar{s} = (s_1, \ldots, s_I)$ and then chooses the vector of spending $\bar{g} = (g_1, \ldots, g_I)$. To derive ex-ante welfare in the fiscal union, the first step is to solve the problem of the central fiscal authority allocation of spending by calculating the maximized value of welfare for each vector of signals $\bar{s}$. The second step is to calculate ex-ante welfare as the expected value of these maximized values.

For the first step consider an arbitrary vector signal $\bar{s}$. Let $W^C(\bar{s})$ denote the problem of maximizing an equally weighted normalized sum of expected utilities of the $I$ countries, given the vector of signals $\bar{s}$, namely to solve

$$W^C(\bar{s}) = \max_{\{c_i, g_i\}} \frac{1}{I} \sum_{i=1}^{I} E \left[ u(c_i) + \theta_i h(g_i) | \bar{s} \right]$$

subject to $c_i + g_i = y$ for all $i$. Observe that since the utility of each country $i$ does not depend on what other countries do, the problem of the fiscal union (9) reduces to one of maximizing the utility of each country separately given the signal it receives for that country. This implies that that

$$W^C(\bar{s}) = \frac{1}{I} \sum_{i=1}^{I} W^C(s_i)$$

where for each $i$,

$$W^C(s_i) = \max_{c_i, g_i} E \left[ u(c_i) + \theta_i h(g_i) | s_i \right]$$

subject to $c_i + g_i = y$ for all $i$. Also, given the multiplicative form of $\theta_i$, this problem further
reduces to

\[\text{\(W^C(s_i) = \max_{c_i,g_i} \left[ u(c_i) + E(\theta_i|s_i)h(g_i) \right]\)}\]

For the second step we calculate the ex-ante utility in the fiscal union. To do so we need to take expectations of \(W^C(s_i)\) over the signals \(s_H\) and \(s_L\). Given the probability of receiving a high and low signal is \(p_H\) and \(1 - p_H\) where \(p_H\) is defined by (6), ex-ante utility under the fiscal union

\[(10) \quad V^C = p_HW^C(s_H) + (1 - p_H)W^C(s_L).\]

When \(\phi = 1\) the signal is perfectly informative and the welfare in the two regimes coincides. The Oates’ Decentralization Theorem corresponds to the completely uninformative case with \(\phi = 1/2\). As will be obvious, in this case the fiscal authority chooses a uniform level of spending for each country. As \(\phi\) increases from \(1/2\) the fiscal authority increasingly tailors the level of spending to the underlying tastes of the country and does so perfectly when \(\phi\) reaches one.

**Proposition 1.** (A Generalized Decentralization Theorem) As long as signals are not perfectly informative, in that \(\phi < 1\), the decentralized regime yields higher ex-ante welfare than the fiscal union. The difference in welfare between these regimes increases as the informativeness of the signal falls.

**Proof.** To prove this result we will use Blackwell’s Theorem on the information structures (Blackwell (1951)) twice: first to show that the welfare in the centralized regime is strictly lower than that in the decentralized regime and second to show that the welfare in the centralized regime worsens as the signal in that regime becomes less informative. To that end, we setup some notation in order to map our economy into that considered in the theorem. Consider two information structures \(\sigma_1\) and \(\sigma_2\) represented by maps \(\sigma_k : \Theta \rightarrow \Delta(S)\) for \(k = 1, 2\) where \(\theta_i \in \Theta = \{\theta_H, \theta_L\}\) are the set of states, \(s \in S = \{s_H, s_L\}\) are the set of signals, and \(\sigma_k(s|\theta)\) is the conditional probability of observed signal \(s\) given state \(\theta\) under information structure \(k\). Recall that \(\sigma_2\) is a garbling of \(\sigma_1\) if an agent who knows \(\sigma_1\) could replicate \(\sigma_2\) by randomly drawing a signal \(s' \in S\) after each observation \(s \in S\), that is, there
exists a garbling function $\gamma : S \to \Delta(S)$ such that

$$
(11) \quad \sigma_2(s'|\theta) = \sum_{s \in S} \gamma(s'|s)\sigma_1(s|\theta)
$$

where $\gamma(s'|s) < 1$ for either $\gamma(s_H|s_H)$ or $\gamma(s_L|s_L)$, that is to (strictly) garble the signal, the garbling function must sometimes report that the signal is high when it is actually low or report that the signal is low when it is actually high. Blackwell’s Theorem states that if $\sigma_2$ is a garbling of $\sigma_1$ then any Bayesian decision maker prefers $\sigma_1$ to $\sigma_2$.

To apply this result in our context, let $\alpha_k$ denote the signal structure associated with a signal with informativeness parameter $\phi_k \in [1/2, 1]$ where

$$
\phi_k = \sigma_k(s_H|\theta_H) = \sigma_k(s_L|\theta_L) \quad \text{and} \quad 1 - \phi_k = \sigma_k(s_L|\theta_H) = \sigma_k(s_H|\theta_L)
$$

and denote ex-ante welfare of an agent in the centralized regime under information structure $\sigma_k$ as

$$
(12) \quad V^C_k = p_{Hk}W^C_k(s_H) + (1 - p_{Hk})W^C_k(s_L).
$$

where $W^C_k(s) = \sum_{i=1}^{I} W^C_k(s_i)$ and for each $i$,

$$
W^C_k(s) = \max_{c_i,g_i} Q_k(q,s_H) [u(c_i(s_H) + \theta_H h(g_i(s_H))] + Q_k(q,s_L) [u(c_i(s_L) + \theta_L h(g_i(s_L))]
$$

subject to $c_i + g_i = y$ where $Q_k(q,s_H) = q\phi_k/p_{Hk}, Q(q,s_L) = q(1 - \phi_k)/(1 - p_{Hk})$, and $p_{Hk} = q\phi_k + (1 - q)(1 - \phi_k)$.

To show that welfare in the fiscal union decreases when the informativeness of the signal falls from $\phi_1$ to $\phi_2$ where $1/2 \leq \phi_2 < \phi_1$, we need only show that the associated information structures satisfy the garbling condition (11). To do so define the symmetric garbling function $\gamma$ that takes original signals $s_H$ and $s_L$ in $S$ into the garbled signals $\tilde{s}_H$ and $\tilde{s}_L$ in $S$ via

$$
\gamma = P(\tilde{s}_H|s_H) = P(\tilde{s}_L|s_L) \quad \text{and} \quad 1 - \gamma = P(\tilde{s}_L|s_H) = P(\tilde{s}_H|s_L)
$$
Then we can write the less informative signal as a garbled version of the original signal by

$$\phi_2 = P(s_H|\theta_H)P(\tilde{s}_H|s_H) + P(s_L|\theta_H)P(\tilde{s}_H|s_L) = \phi_1 \gamma + (1 - \phi_1)(1 - \gamma).$$

Solving for $\gamma$ gives that it is a strict garbling in that

$$\gamma = \frac{\phi_1 + \phi_2 - 1}{2\phi_1 - 1} < 1 \text{ if } \frac{1}{2} < \phi_2 < \phi_1 \quad (13)$$

So by Blackwell’s Theorem, the ex-ante welfare in the centralized regime satisfies $V_2^C < V_1^C$.

To show that welfare in the decentralized regime is strictly greater than that in the fiscal union, note that the value of the centralized regime equals that of the fiscal union when the fiscal union has a perfectly informative signal. Since we know that the value in the fiscal union strictly decreases when the informativeness of the signal varies, this latter result then follows. Q.E.D.

**C. Adding a fiscal externality**

Let us now suppose that there is a direct fiscal externality, in the sense that the value from government spending to citizens in country $i$ depends not only on what its government spends but also on what other governments $j \neq i$. We capture this by letting the $h$ function of country $i$ consumers also depend on the vector of spending of other countries $\bar{g}_{-i} = (g_1, ..., g_{i-1}, g_{i+1}, ..., g_I)$ and $\bar{g} = \{g_i\}$ for all $i$. Specifically, assume that the utility function of a consumer in country $i$ is now

$$u(c_i) + \theta_i h(g_i, \bar{g}_{-i}).$$

We want to set up the economy so that it produces symmetric equilibrium in the relevant sense. By this we mean that in the decentralized regime, all countries with the same realized type $\theta_i$ choose the same level of spending $g_i$. Likewise, we mean that in the centralized regime all countries with realized signal $s_i$ are assigned the same level of spending $g_i$. We begin with a simple case in which signals are perfectly correlated across countries and then turn to the case in which signals are drawn independently in each country. Since we
want this result to hold in both the perfectly correlated signal and the independent signal cases we assume that function $h$ has the symmetry property that for any two vectors of $\bar{g}_{-i}$ and $\bar{g}'_{-i}$ of length $I - 1$ with $n_H$ entries of $g_H$ and $n_L$ entries of $g_L$ with $n_H + n_L = I - 1$, in any arbitrary order then

$$h(g_i, \bar{g}_{-i}) = h(g_i, \bar{g}'_{-i})$$

When find it convenient to focus on the case in which

$$h(g_i, \bar{g}_{-i}) = h(g_i + \gamma \sum_{j \neq i} g_j).$$

Here the total effective government consumption in country $i$ is the sum of its own spending $g_i$ and a fraction $\gamma$ of the total spending of all other countries in the union. To give some intuition for this externality imagine that a fixed fraction $1 - \gamma$ of any country’s spending benefits only that country and the remaining fraction $\gamma$ benefits all countries in the union equally. So the spending of any country $i$ can be decomposed into these two parts, $g_i = (1 - \gamma) g_i + \gamma g_i$ and we can rewrite the total effective spending in $h$ as $(1 - \gamma) g_i + \gamma \sum_{j=1}^{I} g_j$. For later use it is convenient to let

$$G_i(\bar{g}) = g_i + \gamma \sum_{j \neq i} g_j,$$

denote the total effective government spending in country $i$ when countries spend $\bar{g}$.

**Perfectly Correlated Preferences Across Countries**

As we shall see below, when there are fiscal externalities the case we have been considering so far in which each country independently draws a signal leads to some complicated combinatorics because of the large number of realizations of possible patterns of signals that can occur and evaluating these involves adding up over the cases. To build intuition for this more general case, we start with a simple case in which all countries draw the same type, either $\theta_H$ with probability $q$ or $\theta_L$ with probability $1 - q$. In the decentralized regime the government of each country observes the type of its citizens and in the fiscal union the
centralized authority sees one signal $s$ about the common type and that signal satisfies the assumptions (4)-(6) above.

Here of course, the information assumption that all countries have the same type but the central government cannot figure that out is less attractive than in the independent case. Nonetheless, we can think that in this case the information content of the signal the central authority receives is fairly high, in that $\phi$ is fairly high. In fact, the main point we want to establish below holds as long as there is a strict information advantage to the decentralized authorities in so that $\phi$ can be close to perfectly informative as long as it is strictly less than 1. The reason is that we are most interested in how the relevant benefits of centralization versus decentralization changes as the number of countries in the union grow. As such we focus on how policies and welfare change with the number of countries $I$ holding fixed the information advantage of the decentralized governments over the fiscal authority at whatever level it has.

**Equilibrium in a Fiscal Union.** Consider now how to set up the equilibrium in the fiscal union. By the symmetry property of the externality function and concavity we know that it will never be optimal for the fiscal union to treat countries with the same signal asymmetrically. Hence, we restrict attention to allocations of the form such that for each given number of countries $I$, $g_H(I) = g(s_H, I)$ and $g_L(I) = g(s_L, I)$.

Here, in contrast to the case without externalities, we cannot solve for the allocations for each country separately. But by symmetry we can first solve for the ex-post utility by positing an arbitrary symmetric allocation of the class $\{g_H(I), g_L(I)\}$ which is the allocation for given number of countries $I$ for the case in which all countries are assigned $g_H$ or all countries are assigned $g_L$. We can then take the relevant expected value of these allocations to solve for ex-ante welfare in the union.

We also focus on a specific utility function of the log form

\begin{equation}
\log c_i + \theta_i \log G_i
\end{equation}

subject to $c_i + g_i = y$ for all $i$. After imposing the symmetry property discussed above the
ex-post welfare in the fiscal union is

\[(17) \quad W^C(\theta_i, I) = \log(y - g_i) + \theta_i \log(G_i) \]

where \(G_i(\bar{y}) = g_i(1 + \gamma(I - 1))\). Given the signal \(s \in \{s_H, s_L\}\), the fiscal union maximizes the interim welfare

\[\max_g E(W^C(\theta, I)|s) \]

It is immediate that the solution is

\[(18) \quad c^C(s_i) = \frac{1}{1 + \bar{\theta}_i} y \quad \text{and} \quad g^C(s_i) = \frac{\bar{\theta}_i}{1 + \bar{\theta}_i} y, \]

where \(\bar{\theta}_i \equiv E(\theta|s_i)\) is the posterior mean of \(\theta\) under the signal \(s_i\). Note for later that the optimal fraction of output that is devoted to government consumption does not vary with the number of countries but, importantly, the effective government spending

\[(19) \quad G^C(s_i, I) = (1 + \gamma(I - 1))g^C(s_i) \]

tends to infinity as the number of countries in the union \(I\) gets large. Ex-ante welfare in the fiscal union is then

\[(20) \quad V^C(I) = p_H W^C(\bar{\theta}_H, I) + (1 - p_H) W^C(\bar{\theta}_L, I) \]

where \(p_H = q\phi + (1 - q)(1 - \phi)\) is the probability of observing the signal \(s_H\). Substituting (18) into (20) and using the form of the utility in (17) gives that

\[(21) \quad V^C(I) = \sum_{i=H,L} p_i \left[ \log c^C(\theta_i, I) + \theta_i \log G^C(\theta_i, I) \right]. \]

Clearly, as the number of countries in the union grows, the ex-ante utility from consumption is constant but the ex-ante utility from effective government spending tends to infinity (where we used that \(\log x\) tends to infinity as \(x\) does). Clearly, this occurs because even though
each country is spending a constant amount of its endowment on its country’s government consumption, the spillovers from other countries’ spending through the externality term $\gamma(I - 1)$ in (19) eventually becomes infinitely valuable.

**Equilibrium Under Decentralization.** The problem of the decentralized fiscal authority given the observed state $\theta$ when all other countries are spending the same amount $g_{-i}$ is

$$W^D(\theta, g_{-i}, I) = \max_{g} \log(y - g) + \theta \log(g + \gamma(I - 1)g_{-i})$$

Taking the first order condition holding fixed $g_{-i}$ and then imposing symmetry gives that optimal levels of $c$ and $g$ are

$$c^D(\theta, I) = \left(\frac{1 + \gamma(I - 1)}{1 + \theta + \gamma(I - 1)}\right) y \quad \text{and} \quad g^D(\theta, I) = \left(\frac{\theta}{1 + \theta + \gamma(I - 1)}\right) y$$

Note that in stark contrast to the centralized case, as the number of countries gets large the fraction of output devoted to government spending by each country tends to zero so that consumption eventually becomes all of output, so $c = y$. But at the same time effective government spending

$$G^D(\theta, I) = (1 + \gamma(I - 1)) g^D(\theta, I) = \left(\frac{1 + \gamma(I - 1)}{1 + \theta + \gamma(I - 1)}\right) \theta y$$

converges to a constant $\theta y$. Substituting these optimal policies into the welfare function gives that as the number of countries $I$ in the union grows large interim welfare given $\theta$ in the decentralized regime converges to a constant

$$W^D(\theta, I) = \log c^D(\theta, I) + \theta \log G^D(\theta, I) \to \log y + \theta \log \theta y.$$  

to

Clearly then the ex-ante welfare under decentralization also converges to a constant when $I$ tends to infinity,

$$V^D(I) = E(W^D(\theta, I)) \to \log y + E(\theta \log \theta y).$$
Comparison of Regimes. We summarize our discussion above with a proposition.

Proposition 2. (A Cutoff Rule for When a Fiscal Union is Preferred, Correlated Signals) Given any level of the spillover parameter $\gamma > 0$, for any level of informativeness of the signal $\phi$, there is a cutoff number of countries $I^* = I^*(\phi, \gamma)$ such that for $I \geq I^*$ the fiscal union is preferred. Likewise, for any finite number of countries $I^*$ and informativeness level $\phi < 1$ there is a sufficiently small spillover parameter $\gamma(I^*)$ such that the decentralized regime is preferred for $I < I^*$.

When comparing regimes there are clearly costs and benefits of both. Given its superior information, the decentralized regime can better tailor the policies to the preferences of member states, but it does not internalize the externalities. In contrast, the fiscal union internalizes these externalities, but it does a worse job of tailoring policies to the preferences of its member states. Which regime is preferred then clearly depends on the details of these two sets of forces.

Consider first the role of the quality of the information of the centralized authority, as indexed by the informativeness $\phi$ of the signal. Obviously, as this gets better, which happens as $\phi$ increases towards 1, so does its ability to tailor its policies. In the limit when $\phi = 1$, the fiscal union has the same information as countries in the decentralized regime so it is strictly preferred to any decentralized regime (that is when $I \geq 2$). That this is true does not depend on the exactly details of the nature of the fiscal externality. Clearly, it will hold as long as there is any nontrivial externality when there are at least two countries. In particular, here we have considered positive externalities, which implies that each member state is devoting too small a fraction of its output to government spending, relative to the socially efficient amount. But it obvious the analogous result holds if the externality is negative, think of common air pollution from government projects.

In the proposition we write $I^*$ as a function of the information content of signals and the externality parameter $\gamma$ to emphasize that the cutoff depends on them. Of course, if $\gamma = 0$, there is no tradeoff and the decentralized regime is optimal regardless of the number of countries and if $\phi = 1$ there is also no tradeoff and the fiscal union is optimal regardless of the number of countries, where both notions of optimality hold in a weak sense.

In Figures 1A and 1B we illustrate how the size of the spillover, $\gamma$, and the level of
informativeness, \( \phi \), determine the cutoff level of countries after which a fiscal union is preferred to the decentralized regime. In Figure 1A as the information becomes less precise, in that \( \phi \) decreases, the cutoff level \( I^* \) for when the fiscal union is preferred increases. Indeed, as we decrease the informativeness of the signal from a perfectly informative one to a moderately informative one \( (\phi = 3/4) \) to an uninformative one \( (\phi = 1/2) \), a fiscal union goes from always being optimal, to being optimal only after 8 countries and after 10 countries respectively.

In Figure 1B we see that as the externality \( \gamma \) decreases, the cutoff level \( I^* \) for when the decentralized equilibrium is preferred increases: for example, at a high spillover of 30\% \( (\gamma_h = .3) \) the decentralized union is preferred only for 5 or fewer countries, but for a spillover less than the medium level of 14\% \( (\gamma_m = .14) \) the decentralized regime is preferred for up to 11 countries.

Clearly the results we have established depend on the nature of the spillovers. That ex-ante welfare of each country in the fiscal union converges to infinity as the size of the union grows clearly depends on the exact nature of both the spillover encoded in the form of effective government spending and the utility function over such spending. For the utility function, even holding fixed the form of effective government spending, if we allowed for satiation at some finite value then this welfare would tend to converge as the size grows large. For the spillover, even holding fixed the utility function, if we assumed a fraction of the average spending in other countries spilled over, as in

\[
(23) \quad h(g_i, \bar{g}_{-i}) = h(g_i + \frac{\gamma}{I-1} \sum_{j \neq i} g_j).
\]

rather than a fraction of the total spending in other countries did as in (15) then ex-ante welfare in the fiscal union would converge to a finite value as the union grows. Hence, in either of these cases, it can easily be that as long as the decentralized regime has a strict information advantage over the union \( (\phi < 1) \) and the externality is small but positive, the decentralized regime is preferred regardless of the number of countries in the union.
Independent Preferences Across Countries

We want to set up the economy so that it produces symmetric equilibrium in the relevant sense. By this we mean that in the decentralized regime, all countries with realized type $H$ choose the same level of spending $g_H$ and all countries with realized type $L$ choose the same level of spending $g_L$. Likewise, we mean that in the centralized regime all countries with realized signal $s_H$ are assigned the same level of spending $g_H$ and all countries with realized signal $s_L$ are assigned the same level of government spending $g_L$.

To make this property clear, consider the simple case in which the externality is of the form

(24) $h(g_i, g_{-i}) = h(g_i + \gamma \sum_{j \neq i} g_j)$.

which is a slight generalization of (16). Here we are assuming that the sum of the value of all other countries $j \neq i$ government spending increases the utility from government spending in country $i$ with the spillover parameter $\gamma \geq 0$. Clearly, (24) which has the symmetry property (14) because $\sum_{j \neq i} g_j = n_H g_H + n_L g_L$ and does not depend on the order in which these entries appear.

Given this property we can write the $h$ function for a country with government spending $g_H$ as $h_H(g; g_H, g_L, n)$ and for a country with government spending $g_L$ as $h_L(g; g_H, g_L, n)$ where $n$ denotes the number of countries with $g_H$. To illustrate these functions, suppose the externality has the form (24) then

$$h_H(g; g_H, g_L, n) = h(g + \gamma [((n - 1)g_H + (I - n)g_L)]$$

and,

$$h_L(g; g_H, g_L, n) = h(g_L + \gamma [ng_H + (I - n - 1)g_L])$$

In short, if $n$ is the total number of countries with $g_H$ then in case in which the country $i$ we are considering has $g_H$, then there are $n - 1$ other countries with $g_H$ and $I - n$ other countries with $g_L$, whereas if the country we are considering has $g_L$ then we have $n$ others that have
Equilibrium in a Fiscal Union. Consider now how to set up the equilibrium in the fiscal union. By the symmetry property of the externality function (and concavity) we know that it will never be optimal for the fiscal union to treat countries with the same signal asymmetrically, we restrict attention to allocations of the form \( g_H = g(s_H) \) and \( g_L = g(s_L) \). Here, in contrast to the case without externalities, we cannot solve for the allocations for each country separately. But by symmetry we can first solve for the ex-post utility by positing an arbitrary symmetric allocation of the class \( \{g_H(n), g_L(n)\} \) which is the allocation for any realization of \( I \) signals with \( n \) that are \( s_H \) and \( I - n \) that are \( s_L \) and solve that problem for fixed \( n \). We can then take the relevant expected value of these allocations to solve for ex-ante welfare under centralization.

So the first step is to solve the ex-post problem of the fiscal union given a realization of signals with \( n \) that are \( s_H \), namely

\[
W^C(n, I) = \max_{g_H \neq g_L} \left\{ n \left[ u(c_i) + \tilde{\theta}_H h_H(g_H; g_H, g_L, n, I) \right] + (I - n) \left[ u(c_i) + \tilde{\theta}_L h_L(g_L; g_H, g_L, n, I) \right] \right\}
\]

where \( \tilde{\theta}_H \equiv E(\theta|s_H) \) and \( \tilde{\theta}_L = E(\theta|s_L) \). Denote the solution to this problem as \( g^C_H(n, I) \) and \( g^C_L(n, I) \).

The second step is to construct ex-ante utility for this regime. To calculate that utility we need to take expectations both over the signals \( s_i \) given the type, say \( \theta_H \), and then over the types. Recall that the unconditional probability of any country receiving a signal \( s_H \) is \( p_H \). Hence, with \( I \) countries, the probability of receiving \( n \) high signals and \( I - n \) low signals is given by the binomial formula

\[
\binom{I}{n} p^n_H (1 - p_H)^{I-n}
\]

where \( \binom{I}{n} = \frac{I!}{n!(I-n)!} \) is the binomial coefficient. Hence, the ex-ante welfare under the central-
ized fiscal union is given by

\begin{equation}
V_C(I) = \frac{1}{I} \sum_{n=0}^{I} \binom{I}{n} p_H^n(1 - p_H)^{I-n} W_C(n, I).
\end{equation}

**Equilibrium in a Decentralized Regime.** Consider next how to set up the equilibrium in the decentralized regime. By the symmetry property of the externality function we restrict attention to allocations of the form \(g_H = g(\theta_H)\) and \(g_L = g(\theta_L)\). By symmetry we can first solve for the ex-post utility by positing an *almost symmetric* allocation \((g; g_H(n), g_L(n))\) in which we impose that all other types, besides that of the country \(i\) we are considering are choosing symmetric allocations. We need to do so because we have to allow a single country of either type to best respond to the actions of others by considering what happens if chooses an asymmetric action for its type, given what the other \(I - 1\) countries have chosen.

So the first step is to solve the ex-post problem in the decentralized regime for a country with type \(\theta_H\), given a realization of types with \(n\) countries that have type \(\theta_H\), namely

\begin{equation}
W^D_H(n, g_H, g_L, I) = \max_g u(c_i) + \theta_H h_H(g; g_H, g_L, n, I)
\end{equation}

The solution to this problem gives the best response of a country of type \(\theta_H\) to an almost symmetric situation in which there are \(n - 1\) other countries choosing \(g_H\) and \(I - n\) other countries choosing \(g_L\), denoted \(G^D_H(n, g_H, g_L)\). The analogous ex-post problem for a country with type \(\theta_L\) is

\begin{equation}
W^D_L(n, g_H, g_L, I) = \max_g u(c_i) + \theta_L h_L(g; g_H, g_L, n, I)
\end{equation}

which defines the best response \(G^D_L(n, g_H, g_L, I)\).

An *equilibrium in the decentralized regime* is a pair of numbers \((g^D_H(n, I), g^D_L(n, I))\) defined as the solution to the two-dimensional fixed point problem

\[ g^D_H(n) = G^D_H(n, I, g^D_H(n, I), g^D_L(n, I)) \text{ and } g^D_L(n) = G^D_L(n, I, g^D_H(n, I), g^D_L(n, I)) \]

Substituting these equilibrium allocations back into the ex-post problems (26) and (27) gives
the ex-post values given $n$ realizations of type $\theta_H$ for a types $\theta_H$ and $\theta_L$ of

\[ W_H^D(n, I) = W_H^D(n, I, g_H^D(n, I), g_L^D(n, I)) \]

and

\[ W_L^D(n, I) = W_L^D(n, I, g_H^D(n, I), g_L^D(n, I)). \]

The last step is to calculate the ex-ante welfare under the decentralized regime. But this is simply the welfare of countries when $n$ high types are realized out of $I$ total draws, added over the probability of each level of $n$, namely

\begin{equation}
V_D(I) = \sum_{n=0}^{I} \binom{I}{n} q^n(1 - q)^{I-n} \left[ nW_H^D(n, I) + (I - n)W_L^D(n, I) \right]
\end{equation}

Figure 2 shows that the key implications for the case of perfectly correlated signals carry over to the case of imperfectly correlated signals. That is, as Figure 2A shows, as the information becomes less precise, in that $\phi$ decreases, the cutoff level $I^*$ for when the fiscal union is preferred increases. Moreover, as Figure 2B shows, as the externality $\gamma$ decreases, the cutoff level $I^*$ below which the decentralized regime is preferred also increases.

2. A Monetary Union

We turn now to embedding this fiscal union in a monetary union with a single currency which manifests it as a single price level in all countries. We also abstract from the direct externalities considered above. As we will show, if the monetary authority has commitment then there will be no indirect fiscal externalities from the monetary authority and, hence, the Oates logic on the benefits of decentralization will immediately apply. In sharp contrast, if the monetary authority lacks commitment, then when any fiscal authority increases it spending and nominal government debt the monetary authority will respond by increasing inflation. This force will introduce an indirect fiscal externality from the monetary authority and the analysis will have many parallels to that of the real model considered above with a direct externality.
An important difference between the monetary economy without commitment and the real economy with direct externalities is that in the monetary case the externalities are induced solely by the equilibrium behavior of the monetary authority, hence, they depend on some general equilibrium tradeoffs, such as the cost of distortionary taxes on labor supply and the detrimental effect of ex-post inflation on productivity. As such they are not simply controlled by a single parameter, such as the spillover effect $\gamma$ in the simple real model with externalities that we have just analyzed.

A. The Monetary Economy

Consider a two period monetary model with $I$ countries united in a monetary union. Each country $i = 1, \ldots, I$ has a representative consumer, firms, and government. Consumers in different countries differ from each other only in their preferences for government spending. In particular, the utility of a representative consumer in country $i$ is

\[ u(c_{i0}) + \theta_i h(g_i) + \beta u[c_{i1} - v(\ell_i)] \]

where we note that utility in the second period has the GHH form which will imply that there are no income effects on labor supply. Here $c_{i0}$ and $c_{i1}$ are consumption in periods 0 and 1, $g_i$ is government spending in period 0, and $\ell_i$ is labor in period 1. We assume that $u$ and $h$ are increasing and concave and $v(\ell)$ is increasing and convex. In each country the taste $\theta_i$ for government consumption is random. We consider the same two information structures as in the real model, namely the perfectly correlated case and the independent case with the same notation.

We model monetary policy as the choice of the price level in period 1, $p_1$, that produces the period 1 gross inflation rate $\pi = p_1/p_0$ where $p_0$ is the given price level in period 0 normalized to 1. Output in period 0 is a constant $y_0$. Output in period 1 is produced using labor $\ell_i$ using a production function $A(x)\ell_i$ where $x$ is an input $x$ bought from abroad by firms that enhances the productivity of labor, in that $A'(x) > 0$. In order to capture the costs of inflation, we assume that each firm has an initial amount of money $M$ it can use it to buy this input $x$ at a nominal cost in period 1 of $\pi x$ units of money.
Firms. Competitive producers in country $i$ solve

\[(30) \quad \max_{x_i, \ell_i} A(x_i) \ell_i - w_i \ell_i \]

subject to $x_i \leq \frac{M}{\pi}$. So in a competitive equilibrium

\[(31) \quad w_i = A(x_i) \quad \text{and} \quad x_i = \frac{M}{\pi} \]

These producers pay labor a wage of $w = A$ where $A = A(x)$.

Governments. As before the government of each country $i$ observes its citizens taste $\theta_i \in \{\theta_H, \theta_L\}$ perfectly whereas for each country $i$ the central authority only sees a noisy signal $s_i \in \{s_H, s_L\}$ of the taste of consumers there, where the informativeness $\phi$ is as in (4), Bayes’ rule (5) holds, and the probability of a high signal is $p_H$ and given by (6). For simplicity, we assume that the government consumption $g_i$ is financed entirely by nominal debt issued to country $i$ consumers in period 0 and paid for by distortionary taxation of those consumers in period 1. In particular, in period 0 the government of each country $i$ issues to consumers claims to $B_i$ units of currency payable in period 1. Let $1 + R$ denote the nominal interest rate on this debt. Then the budget constraint of government $i$ in period 0 is

\[(32) \quad g_i = \frac{B_i}{1 + R} \]

In period 1, each government $i$ collects (real) tax revenues $T_i$ defined as $\tau_i A \ell_i$ to pay off the real value of its debt obligations $B_i/\pi$ subject to the period 1 budget constraint

\[(33) \quad T_i = \frac{B_i}{\pi} \]

Consumers. The consumer in country $i$ can save either by saving $d_i$ in its government’s nominal debt or by saving $k_i$ units in a real storage technology that has a real rate of return $1 + r$. The consumer problem is then to maximize (29) subject to the budget
constraints

\[ c_{0i} = y_0 - d_i - k_i \]

where \( y_0 \) is the endowment in period 0, and

\[ c_{1i} = (1 - \tau_i)w\ell_i + (1 + r)k_i + (1 + R)\frac{d_i}{\pi}. \]

Substituting out the budget constraints, the consumer in country \( i \)'s problem is

\[ \max_{k_i, d_i, \ell_i} u(y_0 - d_i - k_i) + \theta_i h(g_i) + \beta u \left( (1 - \tau_i)A\ell_i + (1 + r)k_i + (1 + R)\frac{d_i}{\pi} - v(\ell_i) \right) \]

with corresponding first order conditions for real storage, nominal government bonds, and labor given by

\[ k_i : u'(c_{0i}) = \beta(1 + r)u'(c_{1i} - v(\ell_i)) \]

\[ d_i : u'(c_{0i}) = \beta \frac{1 + R}{\pi} u'(c_{1i} - v(\ell_i)) \]

\[ \ell_i : v'(\ell_i) = A(1 - \tau_i) \]

Throughout we assume that \( 1 + r \) and \( y_0 \) are sufficiently large such that the representative consumer in each country saves a strictly positive amount.

From now on we are going to let the government choose tax revenues \( T_i = \tau_iA_i\ell_i \) instead of tax rates \( \tau_i \) and substitute out the static first order condition for labor to define a labor function as a function of these revenues. To do so, multiply the consumer’s first order condition for labor (39) by \( \ell_i \) and rewrite the resulting condition as

\[ T_i = (A_i - v'(\ell_i))\ell_i. \]

Under suitable conditions on \( v \) namely that \( v \), is increasing and convex in \( \ell \) and that revenues \( T_i \) are low enough that we are on the left side of the Laffer curve, in which case the right side of (40) is monotone decreasing in \( \ell \) so that higher tax revenues are associated with lower
levels of labor. Hence we can invert this condition to obtain optimal labor supplied by a consumer as a function of the tax revenues the government will raise and the productivity $A_i$ denoted by

\[(41) \quad \ell_i = \ell(T_i, A_i).\]

From now on, we take it as understood that this function is constructed from the left-side of the Laffer curve.

Notice that as long as the savings of each consumer if positive, these constraints yield the same outcome as if the market clearing constraint holds only across all countries so that are instead written

$$\sum_i (c_{0i} + k_{0i} + g_i) = \sum_i y_0 \quad \text{and} \quad \sum_i c_{i1} = \sum_i (A \ell_i + (1 + r) k_{0i}).$$

The reason is simply that since throughout we have made assumptions that all consumers are saving, it does not matter if they save with each other or use the storage technology.

**B. With Commitment**

Here the monetary authority moves before any information or signals occur and simply chooses a number $\pi$ for inflation. Consumers, firms, and the fiscal authorities, move later and take $\pi$ as given. We set up and solve the equilibrium backwards from the end of period 1. To do so we set up the problems and equilibrium notion for both the decentralized fiscal regime and the fiscal union and for each solve for the continuation equilibrium from period 0 given $\pi$ and the some arbitrary fiscal policies chosen by either the governments or the fiscal union. Then we solve for the optimal fiscal policies in the two regimes and after substituting them into the problems of the consumers and firms problems have the continuation equilibrium in period 0 given $\pi$. Finally, we consider the problem of the monetary authority at the beginning of period 0, who understands how the actions of fiscal authorities, private agents, and prices in the continuation equilibrium from period 0 depend on $\pi$. It will soon become clear that the optimal choice of monetary authority with commitment in either fiscal regime is zero inflation in that $\pi^* = 1$. 

29
Continuation competitive equilibrium in period 0

Facing a history in which the monetary authority has chosen some arbitrary policy \( \pi \), the realizations of types and signals for each country and authority have occurred, and government policies for each country \( i \), \( \{ \tau_i, g_i, B_i \} \) have been set, a continuation competitive equilibrium in period 0, is a nominal interest rate \( R \) and allocations \( \{ c_{0i}, c_{1i}, k_i, d_i, \ell_i, y_{1i} \} \), for all countries such that i. the consumer allocations in country \( i \) solve the consumer problem (36), ii. the firm policies solve the firm problem (30) and zero profits holds

\[(42) \quad w_i = A(x_i),\]

where \( x_i = M/\pi \), iii. the government budget constraint holds in periods 0 and 1

\[(43) \quad g_i = \frac{B_i}{1 + R} \quad \text{and} \quad T_i = \frac{B_i}{\pi},\]

iv. the arbitrage condition across storage and bonds holds

\[(44) \quad 1 + r = \frac{1 + R}{\pi},\]

v. the resource constraints in periods 0 and 1 hold

\[(45) \quad c_{0i} + k_i + g_i = y_0 \quad \text{and} \quad c_{1i} = A(x_i)\ell_i,\]

where \( x_i = M/\pi \), and vi. the bond market clears

\[(46) \quad d_i = \frac{B_i}{1 + R}.\]

In both the decentralized regime and the fiscal union we write the relevant fiscal authority’s problem as a Ramsey problem in which the fiscal authority understands that future will evolve according the continuation equilibrium. Specifically, we use the equilibrium conditions to set up this Ramsey problem of maximizing consumer utility (36) as follows. First, use conditions (44), (46), and the period 0 budget constraints of the government and
the consumer to write consumption in period 0 as
\[ c_{0i} = y_0 - \frac{T_i}{1 + r} - k_i. \]

Then use these same two conditions together with the zero profit condition (42), that \( T_i = \tau_i w \ell_i \), and that \( \ell_i = \ell(T_i, A) \) from (41) to write the consumer’s period 1 budget constraint as
\[ c_{1i} = A\ell(T_i, A) - T_i + (1 + r)k_i + \frac{B_i}{\pi}, \]

which using the government’s period 1 budget constraint \( T_i = B_i/\pi \) can be written
\[ c_{1i} = A\ell(T_i, A) + (1 + r)k_i. \]

where \( A = A(M/\pi) \) follows from firm optimality (31). Then use the government budget constraint in periods 0 and 1, the arbitrage condition, and bond market clearing to write \( g = T_i/(1 + r)\pi \).

Hence, the utility in period 1 in a continuation equilibrium is
\[ u(y_0 - \frac{T_i}{1 + r} - k_i) + \theta_i h\left( \frac{T_i}{1 + r} \right) + \beta u[A\ell(T_i, A) + (1 + r)k_i - v(\ell(T_i, A))] \]

where \( T_i = B_i/\pi \) and \( A = A(M/\pi) \). Notice that we have used up all the equilibrium conditions except for the first order condition for savings (37) which is given by
\[ u'(c_{0i}) = \beta(1 + r)u'(c_{1i} - v(\ell_i)). \]

**Decentralized regime**

Consider first the case in which fiscal policy is chosen in a decentralized way in each country. Here the history facing the fiscal authority includes the inflation rate \( \pi \) chosen by the monetary authority as well as the realization of types \( \theta_i \) observed perfectly by each country’s consumers and fiscal authority. Importantly, given that the monetary authority has already set inflation \( \pi \), the problem of each country \( i \) can be solved in isolation. The reason for this is twofold. First, we have purposely chosen to abstract from any direct externalities of the
type considered earlier. Second, under commitment there are no indirect externalities arising from monetary policy.

We start by positing a given $\theta_i$ as well as types $\bar{\theta}_{-i}$, and then consider decentralized authority’s problem given inflation $\pi$ and taking as given the policies of all other fiscal authorities. We then calculate ex-ante utility by taking the expectation over $\theta_i$. After we do this we have the objective function of the monetary authority and we then calculate the optimal inflation rate.

We write the fiscal authority’s problem as a (type of) Ramsey problem in which the government of country $i$ understands that future will evolve according the continuation equilibrium and takes as given other government policies $\{\tau_j, g_j, B_j\}_{j \neq i}$. Using (49) we have that the fiscal authority’s original problem in the decentralized regime is

\begin{equation}
W_D(\theta_i, \pi) = \max_{k_i, B_i} \left\{ u \left( y_0 - \frac{B_i}{(1 + r)\pi} - k_i \right) + \theta_i h \left( \frac{B_i}{(1 + r)\pi} \right) \right.

\left. + \beta u \left[ A\ell \left( \frac{B_i}{\pi}, A \right) + (1 + r)k_i - v \left( \ell \left( \frac{B_i}{\pi}, A \right) \right) \right] \right\}
\end{equation}

where $A = A(M/\pi)$ subject to the first order condition for savings

\begin{equation}
u'(c_{0i}) = \beta(1 + r)u'(c_{1i} - v(\ell_i)).
\end{equation}

Consider a relaxed version of this problem in which we drop the first order condition for saving and simply maximize (51) without the constraint (52).

**Lemma 1.** The fiscal authority’s original problem is equivalent to the relaxed problem.

**Proof.** The first order condition to the relaxed problem with respect to storage $k_i$ coincides with the first order condition for savings in the constraint (52). Hence, a solution to the relaxed problem is feasible for the original problem. So it must yield at least as high utility and hence solve the original problem. \textit{Q.E.D.}

In the real model considered above, we considered two different information structures: a perfectly correlated case in which all countries draw the same type $\theta$, and an independent case in each country independently draws its own type. We do the same here. To understand intuitively what is the level of inflation that maximizes ex-ante welfare under either
information structure substitute out for $A$ and replace $B_i/\pi$ as a control with real revenues $T_i = B_i/\pi$ raised in period 1 to get that ex-post value for a given country given a type $\theta_i$ is

$$W^D(\theta_i, \pi) = \max_{k_i, T_i} \left\{ u \left( y_0 - \frac{T_i}{1+r} - k_i \right) + \theta_i h \left( \frac{T_i}{1+r} \right) ight. \\
+ \beta u \left[ A \left( \frac{M}{\pi} \right) \ell(T_i, A \left( \frac{M}{\pi} \right)) + (1+r)k_i - v \left( \ell(T_i, A \left( \frac{M}{\pi} \right)) \right) \right] \right\}$$

A moment’s inspection makes clear that if productivity did not depend on $\pi$ then inflation would have no effect on the value $W^D$. The reason is simply that all inflation does is change the nominal interest rate $1+R = (1+r)\pi$ for a given real rate $1+r$ where $r$ is a technologically given parameter from the storage technology. As (53) makes clear then as $\pi$ is raised the nominal interest rate increases, but the same real revenues $T_i$ are needed to finance whatever government spending $g_i = T_i/(1+r)$ is chosen in period 0. From an examination of this problem the following result is obvious.

**Lemma 2.** Under commitment for a given $\pi$ the model is equivalent to a real model of selling real bonds at a fixed real interest rate $1+r$ and a fixed productivity level $A(M/\pi)$.

Now once we consider the case of interest in which $A(x)$ decreases in the input $x = M/\pi$ all that raising the (gross) inflation rate $\pi$ above 1 does is to decrease productivity, which lowers the value of $W^D$. Since we have built into the environment the assumption that it is only feasible for the monetary authority to engineer non-negative net inflation, in that the gross inflation rate $\pi \in [1, \infty)$, then regardless of the realized types, the ex-post welfare is maximized at $\pi^* = 1$. Since the holds for every realized type, it is clear that ex-ante welfare is also maximized at $\pi^* = 1$. We summarize this discussion as follows.

**Proposition 3.** Under commitment regardless of the information structure, the optimal inflation in the decentralized regime is zero.

**Fiscal Union**

Clearly, the identical argument that we have just developed applies in the fiscal union: under commitment the optimal $\pi^* = 1$ for every realization of ex-post states and therefore, ex-ante welfare is maximized at zero inflation. Hence, an analogous proposition to that in Proposition 3 is immediate.
Proposition 4. Under commitment regardless of the information structure, the optimal inflation in fiscal union is zero.

Exactly as we discussed in the previous section we can then reduce the monetary economy to an equivalent real economy with productivity $A = A(M)$ which is invariant to any fiscal choices by the union. Hence, the monetary authority with noisy signals is reduces to real economy with noisy signals and no fiscal externalities. Since we have argued that under commitment by the monetary authority, the monetary economies under a decentralized fiscal regime and a fiscal union both reduced to their equivalent real counterparts with no fiscal externality, a version of the earlier decentralization immediately applies to the monetary economy.

Proposition 5. (A Generalized Decentralization Theorem under monetary commitment) As long as signals are not perfectly informative, in that $\phi < 1$, the decentralized regime yields higher ex-ante welfare than the fiscal union. The difference in welfare between these regimes increases as the informativeness of the signal falls.

C. Without Commitment

We now turn to the more subtle case in which the monetary authority does not have commitment. Here the equilibrium will not reduce to a real economy and, importantly, the interaction between fiscal policy and monetary authority will be critical in determining welfare.

The key difference with the commitment case is the timing of when the monetary authority makes its decision. Now the monetary authority moves at the beginning of period 1 and chooses inflation after consumers in each country have chosen their real saving $\bar{k} = (k_1, \ldots, k_I)$ and the fiscal authorities have chosen their spending and nominal debts $\bar{B} = (B_1, \ldots, B_I)$. Faced with the state variables $(\bar{B}, \bar{k})$ the monetary authority understands that for any history of such state variables and its choice of inflation, the choices in period 1 by consumer, firms and governments will constitute a continuation equilibrium in period 1.

Here again we solve for the equilibrium by working our way backwards from the end of period 1 to the beginning of period 1 and then solve for the optimal policy for the monetary authority for any vector of state variables, say $\hat{\pi}(\bar{B}, \bar{k})$, and the work back to the beginning.
of period 1 and solve for optimal choice of fiscal policies in the two fiscal regimes.

**Continuation equilibrium in period 1**

Consider now the beginning of period 1. In period 0 the vectors of types $\tilde{\theta}$, signals $\tilde{s}$, spending levels $\tilde{g}$, nominal debts $\tilde{B}$, and real savings $\tilde{k}$ have been realized for all countries. Now, if we had considered a general time and period non-separable utility function of the form $U(c_0, g, \theta, c_1, \ell_1)$ the union-wide state would consist of all of them. However, since our utility function is separable between periods 0 and 1, in that it is a special case of the form $u(c_0, g, \theta) + \beta u(c_1, \ell_1)$ at the beginning of period 1, the vectors $(\tilde{\theta}, \tilde{s}, \tilde{g})$ have no effect on period 1 utility and, from the point of view of the monetary authority, are irrelevant constants.

**Firm problem.** Taking as given the inflation rate $\pi$, the firm problem is the same as before, namely to solve (30) with optimality condition (31).

**Governments.** Taking as given the inflation rate and the nominal debt it has issued in period 0 the budget constraint of the government in period 1 is that real tax revenues $T_i = \tau_i A \ell_i$ must equal the real value of this debt, so that

$$T_i = \frac{B_i}{\pi}.$$  \hspace{1cm} (54)

**Consumer problem in period 1.** Given the real savings $k_i$ and nominal bond holdings $d_i$, and facing a current tax rate $\tau_i$, we can substitute out the period 1 budget constraint into consumer utility, drop the period 0 utility, which at this point is just a constant, to write the problem as simply as choosing labor to solve

$$\max_{\ell_i} \beta u \left( (1 - \tau_i) A \ell_i + (1 + r) k_i + (1 + R) \frac{d_i}{\pi} - v(\ell_i) \right)$$
with a first order condition

$$\ell_i : v'(\ell_i) = A(1 - \tau_i)$$

$$\max_{\ell_i} \beta u \left( (1 - \tau_i) A \ell_i + (1 + r) k_i + (1 + R) \frac{d_i}{\pi} - v(\ell_i) \right)$$

with a first order condition

$$\ell_i : v'(\ell_i) = A(1 - \tau_i)$$
where $A = A(M/\pi)$. Now, facing a history in which the monetary authority has chosen some arbitrary policy $\pi$, the realizations of types and signals for each country and authority have occurred, and government policies for each country $i$, $\{\tau_i, g_i, B_i\}$ have been set, a *continuation competitive equilibrium in period 0*, is an continuation allocation $\{c_{1i}, \ell_i, y_{1i}\}$, for all countries such that i. the consumer allocations in country $i$ solve the consumer problem in period 1 (55), iii. the government budget constraint holds in period 1 (54) and iv. the resource constraint in 1 holds $c_{1i} = A(x)\ell_i$ where $x = M/\pi$.

The continuation equilibrium in period 1 is the same in both fiscal regimes. The way we solve back for the equilibrium of the decentralized regime and the fiscal union differ and we consider each separately.

**Fiscal choices in period 1**

Consider choices of period 1 policy in both regimes given the vectors of types $\tilde{\vartheta}$, spending levels $\tilde{g}$, nominal debts $\tilde{B}$, and real savings $\tilde{k}$. In the decentralized regime the problem of the government of country $i$ is to choose tax rates to maximize the continuation consumer utility (55) subject to its period 1 budget constraint. In the fiscal union the problem is to choose tax rates to maximize an equally weighted average of the continuation utility of each country subject to its budget constraint. Since utility is additively separable between periods and the types only enter the period 0 utility, the information and types realized in period 0 are irrelevant for period 1. Moreover since there are no fiscal externalities in period 1, the countries are not connected. Hence, the problem of maximizing the sum of utilities can be broken into the problem of maximizing each country’s utility one at a time.

To set up the Ramsey type problem of maximizing the period 1 utility of country $i$ we can follow the same steps we did to arrive at (49), except now, since period 0 values are just constants that can be dropped, the utility in period 1 in a continuation equilibrium can be written

\[
(57) \quad \beta u [A\ell(T_i, A) + (1 + r)k_i] - v(\ell(T_i, A))]
\]
and dropping the constant $\beta$, the problem of government $i$ is to choose $T_i$ so solve

\begin{equation}
V_1(B_i, k_i, \pi) = \max_{\ell_i} u \left[ A \ell_i + (1 + r) k_i - v(\ell(T_i, A)) \right]
\end{equation}

subject to $T_i = B_i/\pi$ where $A = A(M/\pi)$. Clearly, the solution is to choose $T_i$ on the left-side of the Laffer curve in revenues to satisfy the period 1 budget constraint.

**The Monetary Authority’s Problem**

Here also since the utility function is additively separable across periods and the types enter only into period 0 utility, given the relevant vector of states $(\bar{B}, \bar{k})$, the choices of the monetary authority do not depend on either the types or the information structure in period 1. The monetary authority’s problem is to choose $\pi$ to maximize an equally weighted sum of $V_1(B_i, k_i, \pi)$ over $i$. Substituting in the optimal fiscal choice this problem can be written as

\[
W_{MA}(\bar{B}, \bar{k}) = \max_{\pi \geq 1} \frac{1}{I} \sum_{i=1}^{I} u [c_{1i} - v(\ell_i)]
\]

where $c_{1i} = A\ell_i + (1 + r) k_i$, $\ell_i = \ell(\frac{B_i}{\pi}, A)$, and $A = A(M/\pi)$. This problem defines the optimal monetary policy function $\hat{\pi}(\bar{B}, \bar{k})$.

**Perfectly Correlated Preferences**

As earlier we begin with the simpler case in which signals are perfectly correlated. Here we start with the fiscal union and then turn to the decentralized regime.

**Fiscal Union.** Consider a fiscal union in which the common signal for all countries, $s \in \{s_H, s_L\}$, has been realized at the beginning of period 0. We can write this problem as a type of Ramsey problem following similar steps as before. The problem of this fiscal union in period 0 is to choose

\begin{equation}
W(s, \hat{\pi}(\cdot)) = \max_{\{B_i, k_i\}} \frac{1}{I} E \left[ \sum_{i=1}^{I} u(c_{0i}) + \theta_i h \left( \frac{B_i}{(1 + r) \pi} \right) + \beta u [c_{1i} - v(\ell(T_i, A))] \bigg| s \right]
\end{equation}
subject to the first order condition for real saving in each country

\[ u'(c_{0i}) = \beta u'(c_{1i} - \nu'(\ell_i)) \]

where \( c_{0i} = y_0 - B_i/[1 + r] - k_i \), \( c_{1i} = A\ell(B_i/\pi, A) + (1+r)k_i \), \( \ell = \ell(B_i/\pi, A) \), \( A = A(M/\pi) \), and \( \pi = \hat{\pi}(\bar{B}, \bar{k}) \).

Given that all countries are symmetric we focus on a class of symmetric allocations in which \( B_i = B \) and \( k_i = k \) for all \( i \). Hence, the fiscal union need only evaluate the monetary authority’s policy at the symmetric allocation which can be written \( \hat{\pi}(\bar{B}(B), \bar{k}(k)) \) where \( \bar{B}(B) = (B, .., B) \) and \( \bar{k}(k) = (k, .., k) \). We claim that the following result holds for such histories.

**Lemma 3.** When faced with a symmetric history \((\bar{B}(B), \bar{k}(k))\) the optimal policy of the monetary authority does not depend on \( k \).

**Proof.** Let us define

\[
F(B, \pi) = \bar{A}(\pi)\ell(B_i/\pi, \bar{A}(\pi)) - \nu(\ell(B_i/\pi, \bar{A}(\pi))
\]

where for notational simplicity we define \( \bar{A}(\pi) = A(M/\pi) \). Hence, \( c_1 - \nu(\ell) = F(B, \pi) + (1 + r)k \). For an arbitrary history \((\bar{B}, \bar{k})\) the problem of the monetary authority is

\[
W_{MA}(\bar{B}, \bar{k}) = \max_{\pi \geq 1} \frac{1}{I} \sum_{i=1}^{I} u(F(B_i, \pi) + (1 + r)k_i)
\]

which has a first order condition of

\[
\frac{1}{I} \sum_{i=1}^{I} u'[c_{1i} - \nu(\ell_i)] F_{i\pi} = 0.
\]

Now for a symmetric history \( c_{1i} - \nu(\ell_i) \) is independent of \( i \), and imposing symmetry this first order condition reduces to \( F_\pi = 0 \). Since neither \( F \) nor \( \ell \) depend on \( k \), neither does the optimal policy \( \hat{\pi} \). Q.E.D.

From this lemma we know that the optimal policy of the monetary authority is \( \pi = \hat{\pi}(\bar{B}(B)) \) and, in particular, it does not depend on \( k \). Now given this result, and imposing
symmetry in the choices of the fiscal union, we can reduce the problem of the fiscal union (59) to

\[(62) \ W^C(s, \hat{\pi}(\cdot)) = \max_{B, k} u \left( y_0 - \frac{B_i}{(1 + r) \pi} - k_i \right) + \hat{\vartheta}_s h \left( \frac{B}{(1 + r) \pi} \right) + \beta u \left( F(B, \pi) + (1 + r)k \right)\]

subject to the first order condition for real saving in each country

\[u'(c_0) = \beta u'(c_1 - v'(\ell))\]

where \(\hat{\vartheta}_s \equiv E(\theta|s)\) and \(\pi = \hat{\pi}(\bar{B}(B))\). Then the logic of our earlier lemma applies, and this problem reduces to a relaxed one in which we drop the savings constraint, that is

\[(63) \ W^C(s, \hat{\pi}(\cdot)) = \max_{B, k} u \left( y_0 - \frac{B_i}{(1 + r) \pi} - k_i \right) + \hat{\vartheta}_s h \left( \frac{B}{(1 + r) \pi} \right) + \beta u \left( F(B, \pi) + (1 + r)k \right)\]

Thus the ex-post value in the fiscal union given the signal \(s\) is (63), and the ex-ante value is

\[(64) \ V^C = p_H W^C(s_H, \hat{\pi}(\cdot)) + (1 - p_H) W^C(s_{-H}, \hat{\pi}(\cdot))\]

Notice that this value does not depend on the number of countries, but obviously depends on the informativeness of the signal \(\phi\).

**Decentralized Regime.** Here we also focus on symmetric allocations. To define the problem of any given government \(i\), however, we need to consider situations in which an individual country \(i\) considers what choices to make given that all other countries besides it has made the same choices. That is we need to consider \textit{almost symmetric} of the form \((B_i, k_i; \bar{B}_{-i}, \bar{k}_{-i})\) where \(\bar{B}_{-i} = \bar{B}(B_{-i})\) and \(\bar{k}_{-i} = \bar{k}(k_{-i})\) denote histories in which all other countries besides \(i\) have chosen \((B_{-i}, k_{-i})\) respectively.

Faced with such histories, it is not longer true that the policy of the monetary authority does not depend on \(\bar{k}\). The reason is that the first order condition (61) becomes

\[(65) \ \frac{1}{I} u' [c_{1i} - v(\ell_i)] F_{i\pi} + \frac{I - 1}{I} u' [c_{1, -i} - v(\ell_{-i})] F_{-i\pi} = 0.\]
where \( c_{1,-i}, \ell_{-i}, F_{-i,\pi} \) and \( \ell_{-i,\pi} \) denote the symmetric choices of all countries besides \( i \). Here the monetary authority puts a higher marginal utility weight \( u' \) on country \( i \)'s distortions relative to the other countries, the lower is the value of \( c_{1i} - v(\ell_{i}) \) for that country. This will tend to occur when the debt issued by country \( i \) is higher and the real saving done by this country is lower than all other countries. This force leads to some complicated interactions across fiscal authorities.

Under this regime we consider the best response of the fiscal authority of country \( i \), given that the \( I - 1 \) other countries have chosen \( B_{-i}, k_{-i} \). We can write the state of other countries as \((B_{-i}, k_{-i}, I)\) where the \( I \) reminds us that there are \( I \) countries in total so that the problems is

\[
W^D(\theta, B_{-i}, k_{-i}, I) = \max_{B,k} u\left(y_0 - \frac{B}{(1+r)\pi} - k\right) + \theta h\left(\frac{B}{(1+r)\pi}\right) + \beta u\left(F(B, \pi) + (1+r)k\right)
\]

subject to the first order condition for savings, where we have suppressed the dependence on the policy function of the monetary authority. From this problem we define the best response of the fiscal authority

\[
\tilde{B}(\theta, B_{-i}, k_{-i}, I) \text{ and } \tilde{k}(\theta, B_{-i}, k_{-i}, I),
\]

and the equilibrium allocations for the given state \( \theta \) are defined as the fixed point \((\tilde{B}(\theta, I), \tilde{k}(\theta, I))\) of the best responses, namely,

\[
(B, k) = (\tilde{B}(\theta, B, k, I), \tilde{k}(\theta, B, k, I)).
\]

The value in the decentralized regime given \( \theta \) is

\[
W^D(\theta, I) = W^D(\theta, \tilde{B}(\theta, I), \tilde{k}(\theta, I), I)
\]
and the ex-ante value is

\[ V^D(I) = qW^D(\theta_H, I) + (1-q)W^D(\theta_L, I). \]

For the rest of the current draft we will abstract from the complicated forces discussed above under (65) by assuming that the \( u(\cdot) \) function is linear and that \( \beta(1+r) > 1 \). Under these conditions, we can drop the first order condition for savings as a constraint and replace it by the condition that all countries save all of their first period endowment. In particular, they set \( c_{0i} = 0 \) and \( k_i = y_0 - B_i/[(1+r)\pi] \). Note that we retain nonlinearity by having the disutility of labor being nonlinear in \( \ell \) and the utility of government spending being nonlinear.

**Results.** The general forces in such an economy are as follows. Consider first how the attractiveness of a decentralized regime relative to a fiscal union varies with the number of countries in the union. Because of the indirect fiscal externality discussed earlier, the (ex-ante) value in the decentralized regime falls with the number of countries. Of course, there are no such externalities in the fiscal union and, hence, its (ex-ante) value is independent of the number of countries.

For this economy there are three general cases. At one extreme, the indirect fiscal externality in the decentralized regime is sufficiently strong relative to the information disadvantage in the fiscal union, that the fiscal union is preferred regardless of the size of the monetary union. At the other extreme, the fiscal externality is small relative to the information disadvantage and the decentralized regime is preferred regardless of the size of the monetary union. We find most interesting the last case, in which it is optimal to pair a sufficiently small monetary union with a decentralized fiscal regime but to pair a sufficiently large monetary union with a fiscal union. Figure 3 shows these three possibilities when \( u(c) = c \), \( h(g) = \log g \), and \( v(\ell) = \ell^{1+1/\eta}/(1+1/\eta) \), and

\[ A(\pi) = a + d \left( \frac{M}{\pi} - 1 \right) - \frac{e}{2} \left( \frac{M}{\pi} - 1 \right)^2. \]

We set \( \eta = 1/2, a = 3, d = .05, e = .5, M = 1 \). In this figure we vary the dispersion of \( \theta \), holding fixed its mean at \( \mu_\theta = 1 \). In particular, in Figure 3A, \( \theta_H = 1.04 \) and \( \theta_L = .96 \), in
Figure 3B, $\theta_H = 1.02$ and $\theta_L = .98$, and in Figure 3C, $\theta_H = 1.07$ and $\theta_L = .93$.

Now consider this last case in which a cutoff rule exists. As the information content of signals falls, in that $\phi$ falls, the cutoff level for which a fiscal union is preferred falls. Clearly, as this information gets worse the value of the fiscal union falls, but since countries in the decentralized regime see their type, the value in the decentralized equilibrium is unchanged. Figure 4 shows that with a moderately informative signal of $\phi = 3/4$, a fiscal union is optimal in any monetary union with at least 4 countries but with an uninformative signal of $\phi = 1/2$, it takes 5 countries before that is true.

In Figures 5A and 5B we show how the policies in a fiscal union change as the informativeness of signals improves. As $\phi$ increases towards 1, Figure 5A shows that government spending better mimics the state and becomes less uniform across countries and, as Figure 5B shows the resulting inflation rates also become more volatile. Notice that with $\phi = 1/2$, the model implies the Oates’ assumption that centralized policy is uniform across member states.

In Figures 5C and 5D we show how policies in the decentralized fiscal regime change with the size of the monetary union. Figure 5C shows that government spending increases under both states and Figure 5D shows how this increase in spending ends up generating more inflation as the size of the monetary union increases. Of course, this force is the main reason why welfare falls in the decentralized regime as the size of the monetary union grows: the free-riding problem fiscal policy becomes worse. In particular, as it grows the induced inflation costs from each government’s own fiscal policy fall, so governments increase their spending and hence induce higher inflation, which hurts all other countries.

**Analytics.** Here we focus on an even simpler case in which the functional forms have been specifically chosen, actually reverse-engineered, so that the elasticity of induced inflation with respect to borrowing is a constant in both regimes. Before we captured a negative effect of inflation on output by assuming that productivity falls with inflation. Here instead, we assume that $A$ is constant and follow Aguiar et al. (2015) by assuming that there is a direct negative cost of inflation on utility of $\psi \pi$. Specifically, we assume that utility over
consumption and government spending is linear, and that disutility over labor is given by

$$v(\ell) = A\ell - \kappa_0 + \left(\frac{\alpha - 1}{\alpha}\right) \frac{1}{\ell} \left[ \log \frac{\ell}{\bar{\ell}} \right]^{\frac{\alpha}{\alpha - 1}}$$

where $\alpha > 1$ and $\bar{\ell}$ is the most labor a consumer can supply. As in (40) we can solve for the level of labor associated with raising $T$ in revenues with a linear income tax to get

$$(67) \quad \ell(T) = \bar{\ell} \exp(-\bar{\ell}T^{\alpha-1}).$$

Here the analog of (60) is

$$(68) \quad F\left(\frac{B}{\pi}, \pi\right) \equiv A\ell - \psi\pi - v(\ell),$$

where $\ell = \ell(B/\pi)$, so utility in period 1 is $u(c_{i1} - v(\ell_i)) = F\left(\frac{B}{\pi}, \pi\right) + (1 + r)k_i$. Substituting our functional form for $v$ and the equilibrium labor allocation (67) we can write (68)

$$F\left(\frac{B}{\pi}, \pi\right) = \kappa_0 - \frac{\kappa_1}{\alpha} \left(\frac{B}{\pi}\right)^\alpha - \psi\pi,$$

where $\kappa_1 = (\alpha - 1)\bar{\ell}$. As we shall soon see, this form for $F$ implies that it is optimal for the monetary authority to increase inflation by a given percent as nominal debt increases a given percent.

**Perfectly correlated preferences.** Consider first the problem of the monetary authority in period 1 in an economy where preferences of countries are perfectly correlated. In the centralized case, the only relevant history for the monetary authority is the symmetric one in which countries have the same borrowing $B$ and savings $k$, which with equal weights across countries implies that the monetary authority solves

$$W_{MA}(B, k) = \max_{\pi \geq 1} \left[ F(B, \pi) + (1 + r)k \right]$$
which assuming interiority yields the first order condition

\[(69) \quad F_\pi(B, \pi) = \kappa_1 \pi^{-\alpha - 1} - \psi = 0.\]

This implies that optimal inflation rule is

\[\pi^c(B) = \left( \frac{\kappa_1}{\psi} \right)^{\frac{1}{1+\alpha}} B^{\frac{\alpha}{1+\alpha}},\]

which has constant elasticity \(\eta^C\) given by

\[(70) \quad \eta^C \equiv \frac{B}{\pi} \frac{\partial \pi^c(B)}{\partial B} = \frac{\alpha}{\alpha + 1}.\]

**Fiscal Union.** Given this policy from the monetary authority, the fiscal union solves

\[
W^C(s_i, I) = \frac{1}{I} \max_B \sum \left\{ \frac{\tilde{\theta}_i B}{(1 + r) \pi} + \beta \left[ F(B, \pi) + (1 + r)y_0 - \frac{B}{(1 + r)\pi} \right] \right\},
\]

where \(\pi = \pi^c(B)\) with first order condition

\[-\frac{\tilde{\gamma}_i B_i \partial \pi^c}{\pi^2} \frac{\partial \pi^c}{\partial B_i} + \frac{\tilde{\gamma}_i}{\pi} + \beta F_B + \beta F_\pi \frac{\partial \pi^c}{\partial B_i} = 0,
\]

where we refer to \(\tilde{\gamma}_i = \tilde{\theta}_i/(1 + r) - \beta\) as the expected desire to borrow, given the posterior taste \(\tilde{\theta}_i\). Using our constant elasticity formula (70), substituting for \(F_B\), and using that under optimal monetary policy (69) implies that \(F_\pi = 0\), the optimal debt in the fiscal union given the signal \(s_i\) is

\[(71) \quad B^C(s_i) = \left( \frac{\kappa_1}{\psi} \right) \left[ \frac{\tilde{\gamma}_i (1 - \eta^C)}{\beta \kappa_1} \right]^{\frac{1+\alpha}{\alpha-1}}.
\]

Hence, borrowing in a fiscal union is increasing in the \(\tilde{\theta}_i\) and, as before, not dependent on the size of the union. This policy implies that

\[(72) \quad W^C(s_i) = \tilde{\gamma}_i^{\frac{\alpha}{1-\alpha}} \left( \frac{1}{\beta \kappa_1} \right)^{\frac{1}{\alpha-1}} \left[ (1 - \eta^C)^{\frac{1}{\alpha-1}} - \left( \frac{\alpha + 1}{\alpha} \right) (1 - \eta^C)^{\frac{\alpha}{\alpha-1}} \right] + \chi\]
where \( \chi \equiv \beta(\kappa_0 + (1 + r)y_0) \). Hence, ex-ante welfare is just \( p_H W^C(s_H) + (1 - p_H) W^C(s_L) \).

**Decentralized Fiscal Policy.** Face with an almost symmetric history in which country \( i \) has borrowing \( B_i \), and the remaining \( I - 1 \) countries have borrowing \( B_{-i} \), then the first order condition for the monetary authority

\[
F_\pi(B_i, \pi) + (I - 1) F_\pi(B_{-i}, \pi) = 0
\]

implies that

\[
\pi(B, B_{-i}, I) = \frac{\kappa_1}{\psi} \left[ \frac{B^\alpha + (I - 1)B^\alpha_{-i}}{I} \right]^{\frac{1}{1+\pi}}
\]

and, hence, that the response of inflation to debt has a constant elasticity,

\[
\frac{\partial \hat{\pi}(B_i, B_{-i}, I)}{\partial B_i} = -\frac{F_{\pi B}(B_i, \pi)}{F_{\pi \pi}(B_i, \pi) + (I - 1) [F_{\pi \pi}(B_{-i}, \pi)]} = \frac{1}{I} \frac{\alpha}{\alpha + 1} \frac{\pi}{B}.
\]

When the decentralized fiscal authority chooses its level of borrowing \( B_i \), it takes into account that by increasing borrowing in period 0, inflation in period 1 will increase following the monetary authority rule. In the decentralized regime, the fiscal authority only considers the effect on \( \pi \) of its increment in \( B_i \), whereas in the centralized regime, the fiscal union takes into account the total effect on \( \pi \) of increasing \( B \) in all countries, so the percentage change in inflation when one country increases its nominal debt by 1% is

\[
\eta^D(I) \equiv \frac{B \partial \hat{\pi}(B_i, B_{-i}, I)}{\pi} \bigg|_{B_i = B_{-i} = B} = \frac{\alpha}{I(\alpha + 1)}.
\]

Since utility over consumption is linear and \( \beta(1 + r) > 1 \), consumers have incentives to save everything they can. Then, \( k_{i0} = y_0 - B_i / [(1 + r)\pi] \), and the best response of the decentralized fiscal authority when all other countries choose \( B_{-i} \) is

\[
W^D(\theta_i, B_{-i}, I) = \max_{B_i} \left\{ \frac{\theta_i B_i}{(1 + r)\pi} + \beta \left[ F(B_i, \pi) + (1 + r)y_0 - \frac{B_i}{(1 + r)\pi} \right] \right\}
\]
subject to the monetary policy rule

\[ \pi = \hat{\pi}(B_i, B_{-i}, I). \]

The first order condition is

\[
- \frac{\gamma_i B_i}{\pi^2} \frac{\partial \hat{\pi}}{\partial B_i} + \frac{\gamma_i}{\pi} + \beta F_B + \beta F_\pi \frac{\partial \hat{\pi}}{\partial B_i} = 0
\]

where \( \gamma_i = \theta_i/(1 + r) - \beta \). Using our constant elasticity formula (73), substituting for \( F_B \), and using that, after imposing symmetry \( F_\pi = 0 \), the solution of the decentralized economy is

\[
B^D(\theta_i, I) = \left( \frac{\kappa_1}{\psi} \right) \left[ \frac{\gamma_i (1 - \eta^D(I))}{\beta \kappa_1} \right]^{\frac{1+\alpha}{\alpha-1}}
\]

and,

\[
\pi^D(I) = \left( \frac{\kappa_1}{\psi} \right) \left[ \frac{\gamma_i (1 - \eta^C/I)}{\beta \kappa_1} \right]^{\frac{\alpha}{\alpha-1}}.
\]

Unlike in the centralized regime, the borrowing under a decentralized regime is increasing in the number of countries in the union. The reason is that as \( I \) increases, the externality becomes worse in the sense that the decentralized fiscal authority only takes into account a fraction \( 1/I \) of the total effect of increasing \( B \) on inflation. Then, evaluating ex-post welfare in the decentralized regime at its optimal allocations gives

\[
W^D(\theta_i, I) = \gamma_i \left( \frac{1}{\beta \kappa_1} \right)^{\frac{1}{\alpha-1}} \left[ (1 - \eta^C/I)^{\frac{1}{\alpha-1}} - \left( \frac{\alpha + 1}{\alpha} \right) (1 - \eta^C/I)^{\frac{\alpha}{\alpha-1}} \right] + \chi,
\]

where we used that \( \eta^D(I) = \eta^C/I \). Ex-ante welfare in the decentralized regime is then defined as

\[
V^D(I) = qW^D(\theta_H, I) + (1-q)W^D(\theta_L, I).
\]

Notice that as the number of countries increases to infinity ex-post welfare in the decentralized equilibrium tends to a constant given by
CHECK AND FILL IN DETAILS

\[(77) \quad \lim_{I \to \infty} W^D(\theta_i, I) = -\gamma_i^{\frac{\alpha}{\alpha - 1}} \left( \frac{1}{\beta K_1} \right)^{\frac{1}{\alpha - 1}} \frac{1}{\alpha} + \chi \]

Comparing the value \(W^D(\theta_i, I)\) as \(I\) tends to infinity with that of the centralized regime \(W^C(s_i)\) given by (72), which is constant in \(I\), we can see that as the number of countries increase the centralized ex-post welfare will tend to be higher than the decentralized. It can be shown that this is the case by comparing (77) and (72) if the following is satisfied

\[
\tilde{\gamma}_i^{\frac{\alpha}{\alpha - 1}} \left( \frac{1}{\beta K_1} \right)^{\frac{1}{\alpha - 1}} \left[ (1 - \eta^C)^{\frac{1}{\alpha - 1}} - \left( \frac{\alpha + 1}{\alpha} \right) (1 - \eta^C)^{\frac{\alpha}{\alpha - 1}} \right] > -\gamma_i^{\frac{\alpha}{\alpha - 1}} \left( \frac{1}{\beta K_1} \right)^{\frac{1}{\alpha - 1}} \frac{1}{\alpha}.
\]

Then if \(\gamma_i\) and \(\tilde{\gamma}_i\) are positive this condition is satisfied if

\[(1 - \eta^C)^{\frac{1}{\alpha - 1}} - \left( \frac{\alpha + 1}{\alpha} \right) (1 - \eta^C)^{\frac{\alpha}{\alpha - 1}} > 0 \text{ iff } \eta^C > \frac{1}{\alpha + 1},\]

which is true because \(\alpha > 1\) and \(\eta^C = \frac{\alpha}{\alpha + 1}\). Therefore, we can conclude that at \(I \to \infty\), the centralized regime is preferred to the decentralized because \(W\)

To see how welfare decreases with the number of countries, we take the derivative of (76) with respect to \(I\), that is

\[
\frac{\partial W^D}{\partial I} = \gamma^{\frac{\alpha}{\alpha - 1}} \left( \frac{1}{\beta K_1} \right)^{\frac{1}{\alpha - 1}} \eta^C I^2 \left[ \frac{1}{\alpha - 1} (1 - \eta^C / I)^{\frac{1}{\alpha - 1} - 1} - \left( \frac{\alpha + 1}{\alpha} \right) \frac{\alpha}{\alpha - 1} (1 - \eta^C / I)^{\frac{\alpha}{\alpha - 1} - 1} \right],
\]

which is negative if

\[
\frac{1}{\alpha - 1} (1 - \eta^C / I)^{\frac{1}{\alpha - 1} - 1} - \left( \frac{\alpha + 1}{\alpha} \right) \frac{\alpha}{\alpha - 1} (1 - \eta^C / I)^{\frac{\alpha}{\alpha - 1} - 1} < 0
\]

that is if,

\[\eta^C < I \frac{\alpha}{\alpha + 1}\]

which is true for \(I > 1\) because from (70) we have that \(\eta^C = \frac{\alpha}{\alpha + 1}\). Then, given that ex-post welfare in the centralized regime is higher than ex-post welfare in the decentralized regime
for any signal and types when $I$ grows to infinity, we can then state the following proposition.

**Independent Preferences Across Countries**

3. Conclusion

4. References


Figure 1: Real Economy, Perfectly Correlated

A. Ex-ante welfare comparison varying $\phi$

B. Ex-ante welfare comparison varying $\gamma$
Figure 2: Real Economy, Independent Signals

A. Ex-ante welfare comparison varying $\phi$

B. Ex-ante welfare comparison varying $\gamma$
Figure 3: Monetary Without Commitment, Perfectly correlated

A. Case 1: Cutoff Rule

B. Case 2: Centralized Is Always Better

C. Case 3: Decentralized Is Always Better

Notes: in this plot we use $\theta_L=0.96$ and $\theta_H=1.04$. The probability of $\theta_H$, $q$, is set to 0.5.
Figure 4: Monetary Without Commitment, Perfectly correlated

Ex-ante welfare comparison

(Perfectly correlated θ)

Notes: in this plot we use θ_L=0.96 and θ_H=1.04, and q is set to 0.5
Figure 5: Monetary Without Commitment, Perfectly correlated

A. Government Spending Centralized

B. Inflation under Centralized

C. Government Spending Decentralized

B. Inflation under Decentralized

Information, $\phi$

Low signal | High signal

Number of countries, I

$\theta_L$ | $\theta_U$
Figure 6: Monetary Without Commitment, Heterogeneous $\theta$

Ex-ante welfare comparison
(Heterogeneous $\theta_i$ across countries)

Notes: in this plot we use $\theta_L = 0.96$ and $\theta_H = 1.04$, and $q$ is set to 0.5.