Due: Workshop

Physical Chemistry II

Exercises Set 5

1. Conceptual

a) What are the distinctive characteristics of the different forms of energy, potential energy, kinetic energy, and thermal energy?

b) How is heat energy generated and transmitted by conduction and convection, as compared to radiative transfer?

c) What are criteria for assessing combinations of probabilities for separate event types.

d) What types of data are needed in application of Bayes' Theorem? What are its advantages?

2. Cellular Automaton Propagation Rules

For the one-dimensional (*d=1*) cellular automata considered in class which had k=2 possible states (0 and 1), the laws of propagation in time depended on the status of the cells in the immediate neighborhood domain (radius R=1) and the previous status of the cell itself. The entirety of possible rules turned out to be N_R=256.

a) Write down an equation for N_R in terms of domain size, dimension of the CA, and the multiplicity k of states for each cell.

b) How many propagation rules are possible for a CA in 3D, where each cell in 3-dimensional space can attain any of the 7 colors of the rainbow?

3. Heat Loss by Radiation

A solid red-hot iron ball of radius $\boldsymbol{R}$ =10 cm is placed in a room of dimensions 5m $\boldsymbol{x}$ 4m $\boldsymbol{x}$ 3.5m. It is exposed to the ambient air (18^0C), cooling down from its original temperature of $\boldsymbol{T}$ = **1273K** by radiating heat energy $\boldsymbol{Q}$ according to the Stefan-Boltzmann Law (assume emissivity $\boldsymbol{\varepsilon}$ =1, neglect the effect of the ambient temperature).

a) Write down the differential equation for the cooling rate, d$\boldsymbol{Q}$ /dt.

b) What is the temperature of the iron ball after $\boldsymbol{t}$ =1hr?

c) How long does it take for the ball to cool down to 25^0C?

d) What is the final room temperature?

Data: The density of iron is ρ = 2.7g/cm^3, the specific heat of iron is C_V=**0.451 J/g °C**.
The density of air is ρ =1.2g/L, its specific heat at constant volume is C_V = **0.718 kJ/kg°C**.

4. Probabilities for Combined Events

Consider a game using the outcome of rolling two perfect dice *A* and *B*. Calculate the probabilities for the following events:

a) $P(A \wedge B)$ *for* $[A] = 6$ *and* $[B] = 6$

b) $P(A \vee B)$ *for* $[A] = 6$ *or* $[B] = 6$

c) $P(A \oplus B)$ *for* either $[A] = 6$ *or* $[B] = 6$, *but excluding* $[A] = [B] = 6$

d) $P(\neg A \wedge \neg B)$ *for* neither $[A] = 6$ *nor* $[B] = 6$

5. Permutations

Take the sequence of different integer numbers S_4 ={1,2,3,4}.

a) Write down all cyclic permutations $P_c(S_4)$ of this set. Is there any difference between cw and ccw permutations?

b) From the set of cyclic permutations constructed in a), generate all non-cyclic permutations $P_{nc}(S_4)$.

c) What is the number of all different permutations of the set S_4?

d) By complete induction, prove that the number of all permutations of a set of N objects equals $P_c(S_N) = N!$

e) Assume that, in a set S_N of N integer numbers, m numbers are equal. What is the number of different permutations $P'(S_N)$?

6. <u>Conditional Probabilities</u> (AC)

A drug test is mandatory for airline pilots. A general estimate is that perhaps a small 5% of them may actually be users. The available drug test has been shown to be highly effective: It will indicate with 95% probability a true positive result for actual drug users, and will produce a true negative result for non-drug users in 97% of the cases. For the following assume a sample of N=1000 pilots are tested for the specific drug use. Start with your definition of the two events E1 and E2.

a) How many of these persons are expected to be non-users?

b) How many false positive test results can be expected from this group?

c) How many true positive tests can be expected from this group?

d) One person in the group tests positive. What is the likelihood of that person being an actual user of drugs? Apply Bayes' Theorem.