Workshop -3d

Physical Chemistry II

Exercises Set 2

1. Concepts in Complex Dynamics

Briefly answer the following questions:

a) What properties of Edward Lorenz' mathematical weather model make it non-linear, and in what components (variables, parameters,..)

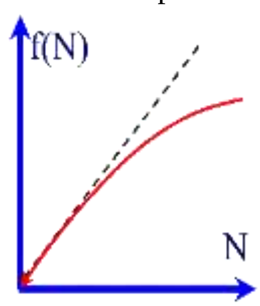
b) How can one discover from a variation of starting values in positions and momenta of a particle system whether or not it would obey deterministic dynamics?

c) What is a positive or negative feed-back effect? Are there some feed-back effects that could influence Earth' climate?

d) What does the phase curve for a given 1D dynamical system illustrate? Does the phase curve of a gravitational pendulum depend on initial conditions, if so, how?

2. Biological Population Growth

Consider a species such as mayflies, whose life cycle is one year, in an environment of

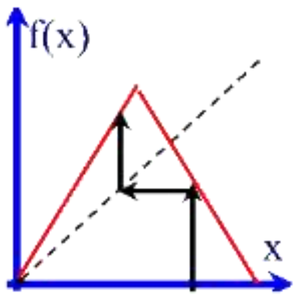


constant food supply. Assume a starting generation of N_0 individual flies. It is reasonable to assume that the number (N_1) of descendants is proportional to the number of parents. However, such linear growth law may apply to relatively small populations, but it cannot go on growing like that forever. If the number of individuals becomes larger, the limited food supply and an increased danger by predators, who acquire a taste for these flies, will reduce the size of the population. Therefore, the population will depend in a non-linear fashion on the size of the previous population. A simple correction term to the linear growth law will be of the form $\Delta N_n \propto N_{n-1}^2$.

a) Argue that the actual growth law corresponds to a logistic map **f(N)**. Draw a connection between the above proportionality constants and the map parameters.

b) Give a condition for a steady-state fly population.

3. <u>Triangular Map</u>



A system described by the "Triangular Map"

$$x_{n+1} = f(x_n) = r \cdot \left(1 - 2\left|\frac{1}{2} - x_n\right|\right)$$

defined over the interval $0 \le x \le 1$ is capable of orderly and chaotic motion, depending on the magnitude of the amplification parameter r.

a) Discuss and ***illustrate graphically***, for which values of r all trajectories are attracted to $x = 0$, regardless of initial conditions.

b) For chaotic motion, two trajectories that start at very similar initial points, e.g., at x_0 and $x_0 + \varepsilon$, become exponentially separated from each other, with increasing number N of iterations. Under what conditions on the Liapunov coefficient $\lambda = \int_0^1 dx \ln\left|f'(x)\right|$ does the motion become chaotic, and what range in r values does the chaotic regime correspond to?

4. <u>Numerical Evaluation</u> (AC)

Modify the provided MS Excel spreadsheet to model the Logistic Map behavior for the range of input variable $x \in [0,1]$ with several gain parameters μ. For the following tasks, compute at least 100 iterations.

a) For μ = 2.5 check for initial values $x_0 \approx 0.3$ and $x_0 \approx 0.7$, if and how iterations depend on initial conditions. Plot (in MS Excel) the iterates x_n *vs.* iteration number n.

b) For μ = 0.5, check for initial values around $x_0 \approx 0.6$, if and how iterations depend on initial conditions. Plot (in MS Excel) the iterates x_n *vs.* iteration number n.

c) For μ = 3.80 check on sensitivity on initial conditions. Use values around $x_0 \approx 0.8$. Evaluate and discuss the quality of the dynamics, i.e., does the map for this amplification parameter have intermittent or chaotic character?

d) For the tasks in *c*) above, plot the frequency of values of the iterates x_n visited by the map.